Multi-Criteria Model for Optimal Number and Placement of Sensors for Structural Health Monitoring: Lexicographic Method Implementation

Ivan Mustakerov and Daniela Borissova

Institute of Information and Communications Technologies at Bulgarian Academy of Sciences,
Department of Information Processes and Decision Support Systems
Bulgaria, Sofia 1113, Acad. G. Bonchev, St. bl. 2.

E-mail: mustakerov@iit.bas.bg, dborissova@iit.bas.bg

Abstract: The predictive maintenance based on dynamic response information obtained by sensors is important to identify the possibility of equipment damage. Optimal number and placement of sensors is an essential problem in structural health monitoring. The paper refers to this problem by formulation of multi-criteria model. The formulated model takes into account all sensors locations and corresponding to a time-varying distributed load mode shapes. Based on this model, a multi-criteria optimization task is formulated where one of the criteria maximizes information supplied from sensors and the other minimizes sensors number. The lexicographic method is used to solve multi-criteria optimization task. The sensors' number and locations are defined taking into account some given acceptable tolerance about information loss compared with the ideal case when all sensors are present. The results of numerical validation show the possibility for practical application of this approach in predictive maintenance structural health monitoring.

Keywords: *combinatorial optimization model, Lexicographic method, multi-criteria problem, optimal sensors' number and locations, structural health monitoring.*

1. INTRODUCTION

Predictive maintenance helps to determine the condition of in-service equipment in order to predict when proper maintenance should be performed. Implementation of condition monitoring and fault detection system entail initial investment but these costs are being offset by the benefits of continuous production, minimum downtimes and early planning to replace the defected parts [Hameed, et al., 2009)]. Sensors placement for structural health monitoring plays a key role in structural control and damage detection. It has increased interest in development of methods for determination of number and locations of sensors for characterizing the dynamic behaviour of a given structure. Optimization of sensors placement has an essential effect on the structural health monitoring system due to the need to collect sufficient data and associated costs. The ultimate goal is to use a minimum number of sensors, placed at the right locations, so that both data loss can be minimal and presence of

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damage of the structure system can be identified effectively. The system of sensors is a fundamental for mechanical analyses of the structure and condition evaluation. Structural health monitoring relies on the data acquired from the sensors. The number of sensors installed in a structure could be intensely increased but it is constrained by the cost associated with data acquisition systems as well as the initial installation of the sensors. This means that how to deploy an optimal number of sensors is of crucial importance in the design and construction of an effective structural health monitoring [Li et al., 2011].

The problem of determining the optimum number of sensors for a particular application, together with their best possible locations, received considerable attention recently. For the optimal sensor placement problem it is important not only to find the best positions of sensors for a specific task but also to estimate the required number of sensors for the best sensor performance. An efficient method for optimal sensor placements can dramatically reduce the computational efforts for optimization [Li et al., 2004]. A number of different optimization techniques are developed over the last decades including heuristic approaches, classical and combinatorial optimization. Many early optimization methods are based on rough and ready ideas without much use of theoretical background. The state of the technology for structure fault detection is reviewed in [Doebling et all., 1996; Farrar & Worden, 2007]. Integrating the advances in various disciplines for optimum sensor layout design under uncertainty can be defined [Guratzsch & Mahadevan, 2006]. By improved genetic algorithm it is possible to find the optimal placement of sensors [Liu et al, 2008]. Various methods share a common basis on sensor placement in the field of structural dynamics and especially concerned with fault detection [Kaveh, et al., 2014; Wang et al., 2012; Yi et al., 2011; Worden & Burrows, 2001]. Stepwise techniques can add or remove one or more sensor at the time in order to find the best combination [Staszewski & Worden, 2001]. Optimal positions deciding and optimal number of sensors defining are two separate problems. The knowledge and experience of engineers are combined with signal processing for the proper solving of optimal sensors locations problem. The problem of optimal number of sensors relies very much on advanced signal processing techniques [Staszewski & Worden, 2001].

In the paper a multi-criteria combinatorial approach that optimizes sensors number and locations is proposed. The sensors are defined in such way that loss of the required information is minimal in some specified limits. For the goal, multi-criteria optimization problem is formulated to define minimal sensors number and maximal data acquisition toward numerous given dynamic response curves. The proposed optimal sensor placement approach is based on dropping out the sensors with smallest information loss to provide the real functions of structural dynamic response to be the closest ones to the functions of structural dynamic response curve with all sensors present.

2. OPTIMAL SENSORS LOCATION PROBLEM

The predictive maintenance monitoring is essential for any successful system application. Maintenance monitoring is based on dynamic response information (mode shape curves) to identify the possibility of system faults. In structural vibration measurements, the locations of sensors are essential for determination of structure response to dynamic loads. The structure could be simplified as a system with more degrees-of-freedom or as a lumped mass system. The goal is to have more nodal points for detailed data of structural responses and a part of these nodes could be used as sensors locations. In general the more sensors are used; the more detailed information of the structure can be obtained. However, the more number of sensors are used, the more instruments and workloads are required and in practice a fixed number of sensors should be located on the structure positions that best characterize the structure response to dynamic loading.

As a result of dynamic analysis the natural frequencies and mode shapes of the structure with dumping neglected can be defined. The natural frequencies of the structure are the frequencies at which structure vibrates if it is subjected to a disturbance. The deformed shape of structure at a specific frequency of vibration is known as mode shape.

Let us consider a vibrating cantilever beam that is subject to a time-varying distributed load. By the method of dynamic equilibrium are determined the corresponding mode shapes shown on Fig. 1 [Tejada, 2009].

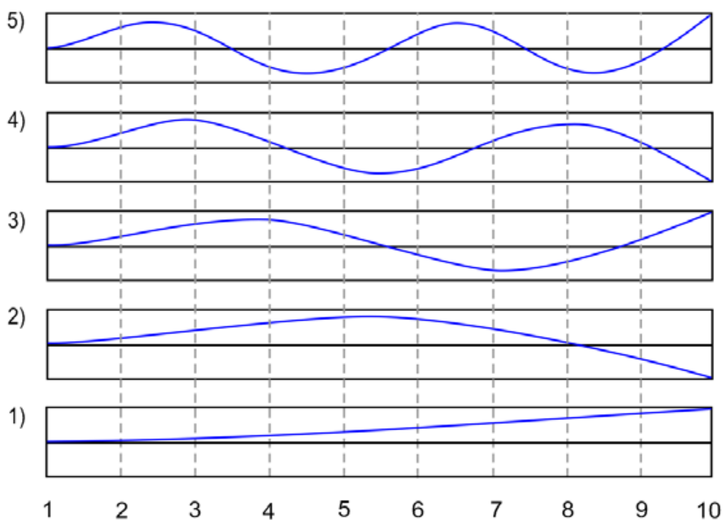

Fig. 1. Cantilever beam vibration mode shape functions

If more modal points for data structural responses are used as sensors locations, the more detailed information of the structure status can be obtained. In practice, some fixed number of sensors is to be located on the optimal positions. The optimal location of sensors is determined by past experience of knowledge of structure or by finite elements analysis. In the paper, for the sake of clarity we assume 10 sensors evenly distributed on the cantilever beam (Fig. 1).

The problem of sensors optimization can be described as: which sensors to be dropped out without essential loss of information accuracy taking into account all vibration mode shape functions simultaneously. For example, if sensor #7 is dropped out its data will be replaced by linear interpolation of the two neighbouring point's data (Fig. 2).

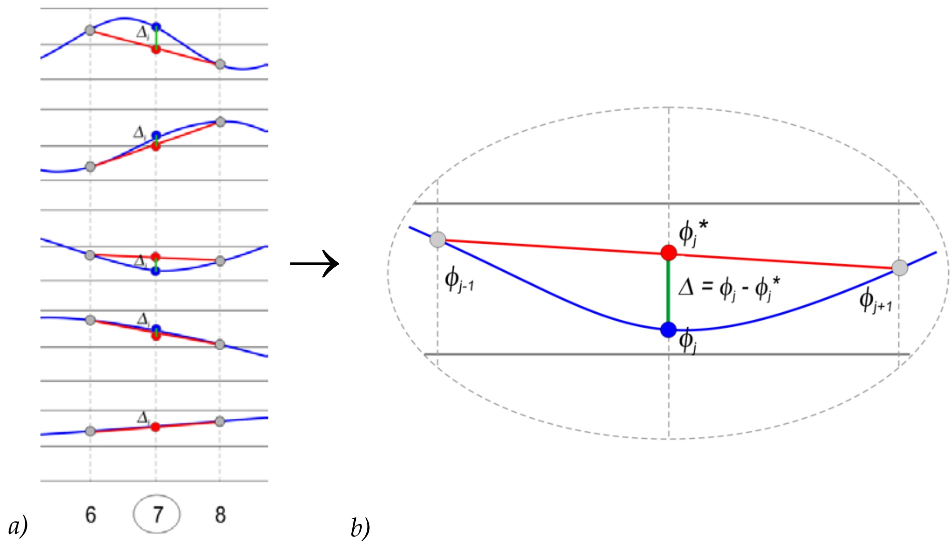

Fig. 2. Linear interpolation when sensor #7 is dropped out (a) and corresponding data deviation for 3-th mode shape (b)

As it can be seen from Fig. 2, the data deviation for omitted sensor #7 is negligibly small for mode shapes 1, 2, and 4, but is considerable for mode shapes 3 and 5. Taking into account all mode shapes simultaneously and all sensors as candidates to be dropped out while keeping deviation as $\Delta_j = \phi_j - \phi_j^*$ in all modes and for all 10 locations as minimum as possible, the sensors optimization problem turn out to be a complex combinatorial optimization problem. Having in mind the requirement of sensors' number minimization and sensors' data maximization, this problem can be considered as multi-criteria problem.

3. MULTI-CRITERIA MODEL FOR OPTIMAL NUMBER AND PLACEMENT OF SENSORS

The mode shape matrix of a structure for *p* modes can be represented as [Li et al., 2004]:

$$\Phi = [\phi_1, \phi_2, \phi_3, ..., \phi_p] \tag{1}$$

Let assume that the structure has *n* degrees of freedom or nodes where the sensors can be located. The goal is to define optimal number of sensors and their locations considering all p mode shapes of a structure simultaneously. To define optimal number of sensors some of them are to be dropped without considerable loss of information. If *j-th* sensor for *i-th* mode shape is missing, its data can be calculated as linear interpolation of data of its neighbours as $\phi_j^{i*} = \frac{(\phi_{j-1}^i + \phi_{j+1}^i)}{2}$. The deviation of data if *j-th* sensor for *i-th* mode shape is missing Δ_j^i is calculated as $\Delta_j^i = \phi_j^i - \phi_j^{i*}$. The choice of sensors to be present or to be dropped out is done by assigning of binary integer variables x_j to each of sensors.

Using all of these assumptions a multi-criteria problem can be formulated as follows:

$$\max \sum_{i=1}^{p} \sum_{j=1}^{n} \left| x_j \phi_{i,j} + y_j \phi_{i,j}^* \right| \tag{2}$$

$$\min \sum_{j=1}^{n} x_j \tag{3}$$

subject to

$$\phi_{i,j}^* = \left| \frac{(\phi_{i,j-1} + \phi_{i,j+1})}{2} \right| \tag{4}$$

$$x_j + y_j = 1 \tag{5}$$

$$x_j \in \{0,1\} \tag{6}$$

$$y_j \in \{0,1\} \tag{7}$$

where x_j are binary integer variables assigned to sensors; $\phi_{i,j}^*$ is linear interpolation value if sensor j is missing; y_j are binary integer variables assigned to $\phi_{i,j}^*$; p is the number of mode shapes; n is the number of sensors. The relation (5) represents the fact that if sensor j is present there is no interpolation of data and vice versa.

There exist different approaches for solving of multi-criteria problems. Multi-criteria decision making approach is characterized by the use of mathematical programming techniques and some decision making method. In most multi-criteria decision making methods, the decision maker plays a major role in providing information for his preferences that influence on the final solution [Miettinen, 2008]. In general, the multi-criteria problems can be handled in different ways depending on when the decision-maker expresses his preference on the different objectives: never, before, during or after the actual optimization procedure. Depending on this, different multi-criteria solution methods can be used. The most widely used methods are based on a priori expressing of the decision makers preferences. The specificity of the investigated problem requires minimize the number of sensors while keeping the structure health monitoring information as close to the maximal (when all sensors are available) as it is possible. For the goal, the lexicographic method is chosen as most appropriate. This method is a priory aggregation of preference information method that requires arranging of objective functions in order of importance. Then multi-criteria problem is solved as a sequence of single objective optimization problems solved one at a time.

4. DETERMINATION OF OPTIMAL NUMBER OF SENSORS AND THEIR PLACEMENT BY LEXICOGRAPHIC METHOD

In order to verify the developments in the paper, the example of vibrating cantilever beam structure [Tejada, 2009] shown in Fig. 1 is used. The adapted for the example data for first 5 mode shapes of structure are shown in Table 1. Every mode shape is normalized to have maximum value of each mode shape equal to 1.

Table 1. Data for first 5 mode shapes of structure

Node, j	Mode 1, ($\phi_{1,j}$)	Mode 2, ($\phi_{2,j}$)	Mode 3, ($\phi_{3,j}$)	Mode 4, ($\phi_{4,j}$)	Mode 5 ($\phi_{5,j}$)
1	0,102	0,294	0,370	0,576	0,610
2	0,202	0,581	0,766	0,820	0,722
3	0,335	0,770	0,872	0,370	-0,090
4	0,467	0,870	0,604	-0,384	-0,724
5	0,590	0,807	-0,020	-0,810	-0,020
6	0,720	0,579	-0,670	-0,430	0,726
7	0,810	0,269	-0,854	0,410	0,050
8	0,890	-0,187	-0,465	0,820	-0,735
9	0,950	-0,603	0,251	0,230	-0,037
10	1,000	-1,000	1,000	-1,000	1,000

4.1. Lexicographic Method Implementation

Implementation of lexicographic method based on the data from Table 1 for the formulated model (2) – (7), requires solving in the first place single criterion optimization task toward the most preferable criterion for maximizing of structure health monitoring information:

$$\max \sum_{i=1}^{5} \sum_{j=1}^{10} \left| x_j \phi_{i,j} + y_j \phi^*_{i,j} \right| \tag{8}$$

subject to

$$\phi^*_{i,j} = \left| \frac{(\phi_{i,j-1} + \phi_{i,j+1})}{2} \right| \tag{9}$$

$$x_j + y_j = 1,\ j = 1,2,\ldots,10 \tag{10}$$

$$x_j \in \{0,1\} \tag{11}$$

$$y_j \in \{0,1\} \tag{12}$$

$$x_1 = 1,\ x_{10} = 1 \tag{13}$$

In current example, it is assumed that the first and last sensors have to be present always (13). The solution of this problem represents the ideal case where all 10 sensors are present and will provide the maximal value of objective function (8) that can be used on the next step of the method.

On the second step of the lexicographic method implementation, a single criterion task for second by importance criterion of multi-criteria problem is solved. The objective function of the task solved in the first step is set as restriction in second single criterion task with some acceptable tolerance toward its optimal value. The corresponding optimization problem solved on the second step is:

$$\min \sum_{j=1}^{10} x_j,\ \forall\ x_j \in \{0,1\} \tag{14}$$

subject to

$$\sum_{i=1}^{5}\sum_{j=1}^{10}\left|x_j\phi_{i,j} + y_j\phi^*_{i,j}\right| \geq \alpha\left(\sum_{i=1}^{5}\sum_{j=1}^{10}\left|x_j\phi_{i,j} + y_j\phi^*_{i,j}\right|\right)^{optimum} \tag{15}$$

$$\phi^*_{i,j} = \left|\frac{(\phi_{i,j-1} + \phi_{i,j+1})}{2}\right| \tag{16}$$

$$x_j + y_j = 1 \tag{17}$$

$$x_j \in \{0,1\}\ \ y_i \in \{0,1\} \tag{18}$$

$$x_1 = 1, x_{10} = 1 \tag{19}$$

where $\left(\sum_{i=1}^{5}\sum_{j=1}^{10}\left|x_j\phi_{i,j} + y_j\phi^*_{i,j}\right|\right)^{optimum}$ is optimal value of (8) defined on the first step and α is a coefficient that defines the required data accuracy when some sensors are to be dropped out.

4.2. Lexicographic Method Results

The results of lexicographic method implementation for definition of optimal number and placement of sensors are illustrated in Fig. 3.

Fig. 3a represents the case when all 10 sensors are present. In this case all of the information for health structure monitoring is available. Fig. 3b, 3c and 3d represent the solutions for α set to be equal to 98%, 97% and 95%, respectively. Setting α = 98% results in dropping out sensor #9. In Fig. 3b the corresponding mode shapes curves are plotted with dotted line for the missing sensor. Analogically, for α = 97%

sensors #3 and #9 are dropped out (Fig 3c) and for $\alpha = 95\%$ sensors #3, #5 and #9 are dropped out (Fig 3d).

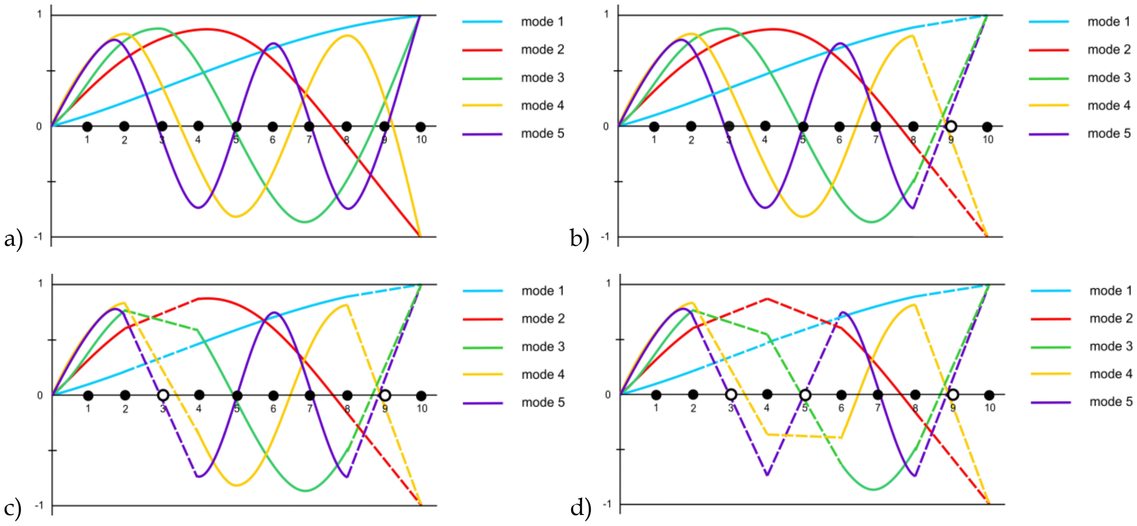

Fig. 3. Results of lexicographic method implementation: a) ideal case for 10 sensors; b) 9 sensors and $\alpha = 0.98\%$; c) 8 sensors and $\alpha = 0.97\%$; d) 7 sensors and $\alpha = 0.95\%$.

The formulated optimization tasks are solved by LINGO[2] ver. 12 on a desktop PC with Intel® Celeron® 2.93 GHz CPU and 2 GB of RAM under MS© Windows XP operating system. The solution times for the described numerical examples are of order of seconds but obviously depend on the size of the problems.

4.3. Discussion of Results

The main problem of sensors' number and locations optimization for structure health monitoring is to provide as minimum loss of information as possible when some of sensors are dropped out. This problem is mathematical formalized by the proposed combinatorial optimization model (2) – (5). The formulated mixed-integer nonlinear model takes into account both different sensors locations and mode shapes corresponding to a time-varying distributed load. Based on this model, a multi-criteria optimization task is formulated where one of the criteria maximizes information supplied from sensors and the other minimizes sensors number. The lexicographic method is used to solve multi-criteria optimization task. An essential advantage of using this method is its easiness and intuitive understanding from engineers that are not familiar with multi-criteria optimization. Along with this, the implementation of lexicographic method for described problem allows setting of acceptable accuracy for sensors' information as a required deviation from the ideal

[2] http://www.lindo.com

case when all sensors are present. The multi-criteria task solution based on real data shows that sensors corresponding to minimal data loss are dropped out. The described approach defines sensors on particular locations that can be dropped out in order to provide best fit to the given acceptable accuracy. In this way, both of sensors' number and their locations are defined considering all mode shapes simultaneously. Regardless of the various mode shape curves, the solutions results provide an accuracy of 98, 97 and 95 percents when dropping out one, two or three sensors. The results of the proposed approach approbation prove its practical applicability.

5. CONCLUSION

The current paper describes an approach to determination of optimal number and locations of sensors for structure health monitoring application. This approach is based on combinatorial optimization modelling and multi-criteria optimization task formulation. The lexicographic method is used for solution of multi-criteria task. The solution of formulated multi-objective nonlinear discrete mixed-integer optimization task provides Pareto-optimal configuration of sensors locations. The sensors' number and locations are defined taking into account given acceptable tolerance about information loss toward the ideal case when all sensors are present. Pareto-optimal solution of the formulated multi-criteria task considers all of the mode shapes and sensors locations simultaneously. The proposed approach is illustrated on the example of vibrating cantilever beam structure. The results of numerical experiments show the possibility for practical application of this approach for structural health monitoring sensors optimization. From a practical prospective, an interesting further research is to consider bigger sets of mode shapes and sensors locations and to test the corresponding large scale optimization problems.

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