Capacitated Transportation Problem under 2-Vehicle

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Abstract
In this paper we represent a two-vehicle cost varying transportation model to solve capacitated transportation problem. In this model the transportation cost varies due to capacity of vehicles as well as amount of transport quantity. At first we propose an algorithm to determine unit transportation cost with initial allocation to the basic cells by North-West corner rule. Then solve it. The unit transportation cost vary during optimality test when allocations are changed. Numerical example is presented to illustrate the two-vehicle cost varying transportation problem (TVCVTP). Finally, comparison is given to show better effective of this model.
Keywords: Capacitated Transportation Problem, Cost Varying Transportation Problem, Basic Cell, Non-basic Cell, North West Corner Rule.

1 Introduction

Transportation problem of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destination of demand in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values but in real life applications unit transportation cost may vary.

A capacitated transportation problem is such a transportation problem in which the supply and demand constraints are equality type and capacity restriction on each route are specified.

In transportation problem unit transportation cost is constant from each source to each destination. But in reality, it is not constant; it depends on amount of transport quantity and capacity of vehicles. If amount of quantity is small then small(capacity) vehicle is sufficient for deliver. Where as if amount of quantity is large then big(capacity) vehicle is needed. So, depend on amount of transport quantity and the capacity of vehicles, the unit transportation cost is not constant. The cost varying transportation problem is such a transportation problem where unit transportation cost is varied depending on the selection of vehicles and number of vehicles.

The basic transportation problem was originally developed by Hitchcock [14] and letter by Dantzig [6]. Many researchers [13,15,17] did work on fixed charge transportation problem. Gupta and Arora [8] presented a capacitated fixed charge bi-criterion indefinite quadratic transportation problem, giving the same priority to cost as well as time is studied. They developed an algorithm which is based on the concept of solving the indefinite quadratic fixed charge transportation problem. Gupta and Arora [11] discussed on a paradox in a
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capacitated transportation problem where the objective function is a ratio of two linear functions consisting of variable costs and profits respectively. In another paper, Gupta and Arora [9,10] discussed on restricted flow in a fixed charge capacitated transportation problem with bounds on total source availabilities and total destination requirements. Dahiya and Verma [5] considered a class of the capacitated transportation problems with bounds on total availabilities at sources and total destination requirements. In this paper, unbalanced capacitated transportation problems have been discussed in the present paper as a particular case of original problem. In addition, they have discussed paradoxical situation in a balanced capacitated transportation problem and have obtained the paradoxical solution by solving one of the unbalanced problems. Arora and Ahuja [1] discussed a paradox in fixed charge transportation problem. Then Arora and Khurana [2] introduced three-dimensional fixed charge transportation problem is an extension of the classical three-dimensional transportation problem in which a fixed cost is incurred for every origin. Basu et. al. [3] represented an algorithm for finding the optimum solution of solid fixed charge transportation problem. Then Bit, et. al. developed fuzzy programming technique for multi objective capacitated transportation problem. Singh and Saxena [16] introduced the multiobjective time transportation problem with additional restrictions. Recently, Dutta and Murthy [7] developed fuzzy transportation problem with additional restrictions.

Here we present some capacitated transportation problem. To solve this problem we consider two vehicles whose capacities are less then the capacity restrictions i.e., capacity of each vehicle is less than or equal to the all route restrictions. Since vehicles are fixed (Contract with fixed Price) so unit transportation cost is varied. This type of transportation problem is named as cost varying transportation problem. In our model we consider this type of capacitated transportation problems and solved these under two vehicles cost varying transportation problem.
2 Mathematical Formulation

2.1 Preliminaries

A transportation problem can be stated in Model 1 as follows:

Model 1

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij},$$

subject to

$$\sum_{i=1}^{m} x_{ij} = a_i, \quad i = 1, \ldots, m$$

$$\sum_{j=1}^{n} x_{ij} = b_j, \quad j = 1, \ldots, n$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$x_{ij} \geq 0 \quad \forall i, \forall j$$

A transportation problem can be represented in the following tabulated form.

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>..</th>
<th>$D_n$</th>
<th>stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1$</td>
<td>$c_{11}$</td>
<td>$c_{12}$</td>
<td>....</td>
<td>$c_{1n}$</td>
<td>$a_1$</td>
</tr>
<tr>
<td>$O_2$</td>
<td>$c_{21}$</td>
<td>$c_{22}$</td>
<td>....</td>
<td>$c_{2n}$</td>
<td>$a_2$</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>$O_m$</td>
<td>$c_{m1}$</td>
<td>$c_{m2}$</td>
<td>....</td>
<td>$c_{mn}$</td>
<td>$a_m$</td>
</tr>
<tr>
<td>Demand</td>
<td>$b_1$</td>
<td>$b_2$</td>
<td>....</td>
<td>$b_n$</td>
<td></td>
</tr>
</tbody>
</table>

Table: Tabular representation of a transportation problem.

where $a_i$ is the quantity of material available at source $O_i, i = 1, \ldots, m$

$b_j$ is the quantity of material required at destination $D_j, j = 1, \ldots, n$

$c_{ij}$ is the unit cost of transportation from source $O_i$ to destination $D_j$.

The following terms are to be defined with reference to the transportation problems.
Definition 1: Feasible Solution (F.S.):
A set of non-negative allocations \( x_{ij} \geq 0 \) which satisfies (1), (2) is known as feasible solution.

Definition 2: Basic Feasible Solution (B.F.S.):
A feasible solution to a \( m \)-origin and \( n \)-destination problem is said to be basic feasible solution if number of positive allocations are \( (m + n - 1) \).
If the number of allocations in a basic feasible solutions are less than \( (m+n-1) \), it is called degenerate basic feasible solution (DBFS) otherwise non-degenerate basic feasible solution (NDBFS).

Definition 3: Optimal Solution:
A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

Theorem 2.1: The number of basic variables in a Transportation Problem (T.P.) is at most \( (m + n - 1) \)

Theorem 2.2: There exits a F.S. in each Transportation Problem (T.P.)

Theorem 2.3: In each T.P. there exits at least one B.F.S. which makes the objective function a minimum

Theorem 2.4: The solution of a T.P. is never unbounded

Definition 4: Loop:
In the Transportation table, a sequence of cells is said to form a loop, if
(i) each adjacent pair of cells either lies in the same column or in the same row;
(ii) not more than two consecutive cells in the sequence lie in the same row or in the same column;
(iii) the first and the last cells in the sequence lie either in the same row or in the same column;
(iv) the sequence must involve at least two rows or two columns of the table.

Theorem 2.5: A sub-set of the columns of the coefficient matrix of a T.P. are linearly dependent, iff, the corresponding cells or a sub-set of them can be sequenced to form a loop.
2.2 North-West corner rule

**Step 1.** Compute $\min (a_1, b_1)$. If $a_1 < b_1$, $\min (a_1, b_1) = a_1$ and if $a_1 > b_1$, $\min (a_1, b_1) = b_1$. Select $x_{11} = \min (a_1, b_1)$ allocate the value of $x_{11}$ in the cell $(1, 1)$.

**Step 2.** If $a_1 < b_1$, compute $\min (a_2, b_1 - a_1)$. Select $x_{21} = \min (a_2, b_1 - a_1)$ and allocate the value of $x_{21}$ in the cell $(2, 1)$.

If $a_1 > b_1$, compute $\min (a_1 - b_1, b_2)$. Select $x_{12} = \min (a_1 - b_1, b_2)$ and allocate the value of $x_{12}$ in the cell $(1, 2)$.

Let us now make an assumption that $a_1 - b_1 < b_2$. With this assumption the next cell for which some allocation is to made, is the cell $(2, 2)$.

If $a_1 = b_1$ then allocate 0 only in one of two cells $(2, 1)$ or $(1, 2)$. The next allocation is to be made cell $(2, 2)$.

In general, if an allocation is made in the cell $(i + 1, j)$ in the current step, the next allocation will be made either in cell $(i, j)$ or $(i, j + 1)$.

The feasible solution obtained by this away is always a B.F.S.

2.3 optimality test:

In order to test for optimality we should follow the procedure as given bellow:

**Step 1.** Start with B.F.S. consisting of $m + n - 1$ allocation in independent positions.

**Step 2.** Determine a set of $m + n$ numbers $u_i, i = 1, \ldots, m$ and $v_j, j = 1, \ldots, n$ such that in each cell $(i, j)$ $c_{ij} = u_i + v_j$.

**Step 3.** Calculate cell evaluations (unit cost difference) $d_{ij}$ for each empty cell $(i, j)$ by using formula $d_{ij} = c_{ij} - (u_i + v_j)$.

**Step 4.** Examine the matrix of cell evaluation $d_{ij}$ for negative entries and conclude that

(i) If all $d_{ij} > 0$, then Solution is optimal and unique.

(ii) If all $d_{ij} \geq 0$ and at least one $d_{ij} = 0$, then solution is optimal and alternative solution also exists.

(iii) If at least one $d_{ij} < 0$, then solution is not optimal. If it is so, further improvement is required by repeating the above process after Step 5 and onwards.
Step 5. (i) See the most negative cell in the matrix \([d_{ij}]\).

(ii) Allocate \(\theta\) to this empty cell in the final allocation table. Subtract and add the amount of this allocation to other corners of the loop in order to restore feasibility.

(iii) This value of \(\theta\), in general is obtained by equating to zero the minimum of the allocations containing \(-\theta\) (not \(+\theta\)) only at the corners of the closed loop.

(iv) Substitute the value of \(\theta\) and find a fresh allocation table.

Step 6. Again, apply the above test for optimality till we find \(d_{ij} \geq 0\).

2.4 capacitated transportation problem:

Consider \(m\) origins and \(n\) destinations. At each origin \(O_i\), let \(a_i\) be the amount of a homogeneous product which we want to transport to \(n\) destinations \(D_j\) to satisfy the demand for \(b_j\) units of the product there. The unit transportation cost is \(c_{ij}\) from source \(i\) to destination \(j\). A variable \(x_{ij}\) represents the unknown quantity to transported from origin \(O_i\) to destination \(D_j\). Let \(r_{ij}\) be the capacitated restrictions on route \(i, j\) for capacitated transportation problem.

A capacitated transportation problem can be stated in Model 2 as follows:

Model 2

\[
\begin{align*}
\text{min} & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}, \\
\text{subject to} & \quad \sum_{i=1}^{m} x_{ij} = a_i, \quad i = 1, \ldots, m \\
& \quad \sum_{j=1}^{n} x_{ij} = b_j, \quad j = 1, \ldots, n \\
& \quad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \\
& \quad 0 \leq x_{ij} \leq r_{ij} \quad \forall i, \forall j
\end{align*}
\]

The condition (1) is necessary condition for the Model 2 to have a feasible solution, however, this is not sufficient because of the capacity restriction on each route.
2.5 2-vehicle cost varying transportation problem

Suppose there are two types of vehicles $V_1, V_2$ from each source to each destination. Let $C_1$ and $C_2 (> C_1)$ are the capacities (in unit) of the vehicles $V_1$ and $V_2$ respectively, $C_1 \leq r_{ij}$ and $C_2 \leq r_{ij}$ $\forall i, j$. $R_{ij} = (R_{1ij}, R_{2ij})$ represent transportation cost for each cell $(i, j)$; where $R_{1ij}$ is the transportation cost from source $O_i, i = 1, \ldots, m$ to the destination $D_j, j = 1, \ldots, n$ by the vehicle $V_1$. And $R_{2ij}$ is the transportation cost from source $O_i, i = 1, \ldots, m$ to the destination $D_j, j = 1, \ldots, n$ by the vehicle $V_2$. So, cost varying transportation problem can be represented in the following tabulated form.

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_n$</th>
<th>Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1$</td>
<td>$R_{111}$, $R_{112}$</td>
<td>$R_{121}$, $R_{122}$</td>
<td>$\ldots$</td>
<td>$R_{1n1}$, $R_{1n2}$</td>
</tr>
<tr>
<td>$O_2$</td>
<td>$R_{121}$, $R_{122}$</td>
<td>$R_{221}$, $R_{222}$</td>
<td>$\ldots$</td>
<td>$R_{2n1}$, $R_{2n2}$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>$O_n$</td>
<td>$R_{m11}$, $R_{m12}$</td>
<td>$R_{m21}$, $R_{m22}$</td>
<td>$\ldots$</td>
<td>$R_{mn1}$, $R_{mn2}$</td>
</tr>
<tr>
<td>Demand</td>
<td>$b_1$</td>
<td>$b_2$</td>
<td>$\ldots$</td>
<td>$b_n$</td>
</tr>
</tbody>
</table>

Table: Tabular representation of cost varying transportation problem.

2.6 Solution procedure capacitated transportation problem

To solve model 2 we consider such type of vehicles $V_1, V_2$ such that the capacities $C_1$ and $C_2 (> C_1)$ of the vehicles $V_1$ and $V_2$ respectively, satisfies following restrictions.

$$C_1 \leq r_{ij} \hspace{1em} \forall i, j$$  \hspace{1em} (2)  

and  

$$C_2 \leq r_{ij} \hspace{1em} \forall i, j$$  \hspace{1em} (3)  

so that Model 2 converted to a two vehicle cost varying transportation problem. Then apply following rule.
2.7 Solution procedure 2-vehicle cost varying transportation problem

2.7.1 Determination of $c_{ij}$

To solve this problem, apply our proposed Algorithms stated as follows:

2.7.2 Algorithm

Step 1. Since unit cost is not determined (because it depends on quantity of transport), so North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S.

Step 2. After the allocate $x_{ij}$ by North-west corner rule, for basic cell we determine $c_{ij}$ (unit transportation cost from source $O_i$ to destination $D_j$) as

$$ c_{ij} = \begin{cases} \frac{p_{1ij}R_{1ij} + p_{2ij}R_{2ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases} \tag{4} $$

where $p_{1ij}, p_{2ij}, i = 1, \ldots, m; j = 1, \ldots, n$ are integer solution of

$$ \min p_{1ij}R_{1ij} + p_{2ij}R_{2ij} \quad \text{s.t. } x_{ij} \leq p_{1ij}C_1 + p_{2ij}C_2 $$

Step 3. For non-basic cell $(i, j)$ possible allocation is the minimum of allocations in $i^{th}$ row and $j^{th}$ column (for possible loop). If possible allocation be $x_{ij}$, then for non-basic cell $c_{ij}$ (unit transportation cost from source $O_i$ to destination $D_j$) as

$$ c_{ij} = \begin{cases} \frac{p_{1ij}R_{1ij} + p_{2ij}R_{2ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases} \tag{5} $$

where $p_{1ij}, p_{2ij}, i = 1, \ldots, m; j = 1, \ldots, n$ are integer solution of

$$ \min p_{1ij}R_{1ij} + p_{2ij}R_{2ij} \quad \text{s.t. } x_{ij} \leq p_{1ij}C_1 + p_{2ij}C_2 $$
In this manner we convert cost varying transportation problem to a usual transportation problem but \( c_{ij} \) is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

**Step 4.** During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix \( c_{ij} \) by **Step 2** and for non-basic we fix \( c_{ij} \) by **Step 3**.

**Step 5.** Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

### 2.7.3 Bi-level Mathematical Programming for 2-vehicle cost varying transportation problem

The Bi-level mathematical programming for 2-vehicle cost varying transportation problem is formulated in **Model 3** as follows:

**Model 3**

\[
\begin{align*}
\text{min} & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, \\
\text{where,} & \quad c_{ij} \text{ is determined by following mathematical programming} \\
& \quad c_{ij} = \begin{cases} 
\frac{p_{1ij} R_{1ij} + p_{2ij} R_{2ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\
0, & \text{if } x_{ij} = 0
\end{cases} \\
\text{min} & \quad p_{1ij} R_{1ij} + p_{2ij} R_{2ij} \\
\text{s. t.} & \quad x_{ij} \leq p_{1ij} C_{1} + p_{2ij} C_{2} \\
& \quad \sum_{i=1}^{m} x_{ij} = a_{i}, \quad i = 1, \ldots, m \\
& \quad \sum_{j=1}^{n} x_{ij} = b_{j}, \quad j = 1, \ldots, n \\
& \quad \sum_{i=1}^{m} a_{i} = \sum_{j=1}^{n} b_{j} \\
& \quad 0 \leq x_{ij} \leq r_{ij} \quad \forall i, \quad \forall j
\end{align*}
\]

where \( p_{1ij}, p_{2ij}, i = 1, \ldots, m; j = 1, \ldots, n \) are integer solution of
3 Numerical Example

Consider a capacitated transportation problem as

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1$</td>
<td>.95</td>
<td>1.5</td>
<td>1.3</td>
<td>48</td>
</tr>
<tr>
<td>$O_2$</td>
<td>1.1</td>
<td>.92</td>
<td>2.1</td>
<td>52</td>
</tr>
<tr>
<td>$O_3$</td>
<td>1.2</td>
<td>1.3</td>
<td>1.3</td>
<td>25</td>
</tr>
<tr>
<td>Demand</td>
<td>75</td>
<td>30</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

with capacity of route restrictions are as follows:

- $r_{11} = 20, r_{12} = 23, r_{13} = 25$;
- $r_{21} = 35, r_{22} = 24, r_{23} = 27$;
- $r_{31} = 28, r_{32} = 30, r_{33} = 26$;

To solve this capacitated transportation problem, first select the two vehicles where, the capacities of vehicles of $V_1$ and $V_2$ are respectively, $C_1 = 6$ and $C_2 = 18$. So that the route restriction is violated. Also, the fixed cost for each vehicles are shown in each cell $(i,j)$ in following tabulated form of 2-vehicle cost varying transportation problem.

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1$</td>
<td>4, 8</td>
<td>5, 10</td>
<td>10, 20</td>
<td>48</td>
</tr>
<tr>
<td>$O_2$</td>
<td>2, 3</td>
<td>8, 16</td>
<td>6, 12</td>
<td>52</td>
</tr>
<tr>
<td>$O_3$</td>
<td>7, 14</td>
<td>3, 6</td>
<td>9, 18</td>
<td>25</td>
</tr>
<tr>
<td>Demand</td>
<td>75</td>
<td>30</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

**Determination of $c_{ij}$ for both basic and non-basic cell**

**Step 1.** By North-west corner rule initial B.F.S. is
Step 2. Using (5), we determine $c_{11} = \frac{24}{48}$, $C_{21} = \frac{6}{27}$, $c_{22} = \frac{32}{25}, C_{32} = \frac{3}{5}, c_{33} = \frac{27}{20}$.

Step 3. Using (6), we determine $c_{12} = \frac{20}{25}, c_{23} = \frac{18}{20}, C_{13} = \frac{20}{20}, C_{31} = \frac{7}{5}$

With these $c_{ij}$ the transportation problem converted to

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1$</td>
<td>$x_{11} = 48$</td>
<td>$c_{11} = \frac{24}{48}$</td>
<td>$c_{12} = \frac{20}{25}$</td>
<td>$c_{13} = \frac{30}{20}$</td>
</tr>
<tr>
<td></td>
<td>4, 8</td>
<td>5, 10</td>
<td>10, 20</td>
<td></td>
</tr>
<tr>
<td>$O_2$</td>
<td>$x_{21} = 27$</td>
<td>$c_{21} = \frac{6}{27}$</td>
<td>$x_{22} = 25$</td>
<td>$c_{22} = \frac{32}{25}$</td>
</tr>
<tr>
<td></td>
<td>2, 3</td>
<td>8, 16</td>
<td>6, 12</td>
<td></td>
</tr>
<tr>
<td>$O_3$</td>
<td>$c_{31} = \frac{7}{5}$</td>
<td>$x_{32} = 5$</td>
<td>$c_{32} = \frac{3}{5}$</td>
<td>$x_{33} = 20$, $c_{33} = \frac{27}{20}$</td>
</tr>
<tr>
<td></td>
<td>7, 14</td>
<td>3, 6</td>
<td>9, 18</td>
<td></td>
</tr>
</tbody>
</table>

**Demand** 75 30 20

**Optimality test**

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic $(i, j)$ $c_{ij} = u_i + v_j$,

each non-basic cell $(i, j)$ by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

So the tabular representation of $u_i, i = 1, 2, 3$, $v_j, j = 1, 2, 3$ and $d_{ij}$ non-basic cell $(i, j)$

is given in the following table
Capacitated Transportation Problem under 2-Vehicle

Since $d_{12} < 0$ so, solution is not optimal. So a loop occurred in cells $(1, 1), (1, 2), (2, 2), (2, 1), (1, 1)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_1$</td>
<td>$x_{11} = 48$</td>
<td>$c_{11} = \frac{24}{48}$</td>
<td>$c_{12} = \frac{20}{25}$</td>
<td>$c_{13} = \frac{30}{20}$</td>
</tr>
<tr>
<td></td>
<td>4, 8</td>
<td>5, 10</td>
<td>$d_{12} &lt; 0$</td>
<td>10, 20</td>
</tr>
<tr>
<td>$O_2$</td>
<td>$x_{21} = 27$</td>
<td>$c_{21} = \frac{6}{27}$</td>
<td>$x_{22} = 25$</td>
<td>$c_{22} = \frac{32}{25}$</td>
</tr>
<tr>
<td></td>
<td>2, 3</td>
<td>8, 16</td>
<td>6, 12</td>
<td>$d_{23} &gt; 0$</td>
</tr>
<tr>
<td>$O_3$</td>
<td>$c_{31} = \frac{7}{5}$</td>
<td>$x_{32} = 5$</td>
<td>$c_{32} = \frac{3}{5}$</td>
<td>$x_{33} = 20$</td>
</tr>
<tr>
<td></td>
<td>7, 14</td>
<td>3, 6</td>
<td>9, 18</td>
<td>2077</td>
</tr>
</tbody>
</table>

$\text{Demand}$ | 75  | 30  | 20  | 25  |

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic $(i, j)$ $c_{ij} = u_i + v_j,$ each non-basic cell $(i, j)$ by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

So the tabular representation of $u_i, i = 1, 2, 3, \ v_j, j = 1, 2, 3$ and $d_{ij}$ non-basic cell $(i, j)$ is given in the following table
Since \( d_{13} < 0 \) (i.e., most negative) so, solution is not optimal. So a loop occurred in cells \((1, 2), (3, 2), (3, 3), (1, 3), (1, 2)\) and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

<table>
<thead>
<tr>
<th></th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
<th>( u_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_1 )</td>
<td>( x_{11} = 23 ) ( c_{11} = \frac{12}{23} ) ( 4,8 )</td>
<td>( x_{12} = 25 ) ( c_{12} = \frac{20}{25} )</td>
<td>( c_{13} = \frac{30}{20} ) ( 10,20 ) ( d_{13} &lt; 0 )</td>
<td>0</td>
</tr>
<tr>
<td>( O_2 )</td>
<td>( x_{21} = 52 ) ( c_{21} = \frac{9}{52} ) ( 2,3 )</td>
<td>( c_{22} = \frac{32}{25} ) ( 8,16 ) ( d_{22} &gt; 0 )</td>
<td>( c_{23} = \frac{18}{20} ) ( 6,12 ) ( d_{23} &lt; 0 )</td>
<td>(-\frac{417}{1196})</td>
</tr>
<tr>
<td>( O_3 )</td>
<td>( c_{31} = \frac{7}{5} ) ( 7,14 ) ( d_{31} &gt; 0 )</td>
<td>( x_{32} = 5 ) ( c_{32} = \frac{3}{5} )</td>
<td>( x_{33} = 20 ) ( c_{33} = \frac{27}{20} )</td>
<td>(-\frac{1}{5})</td>
</tr>
<tr>
<td>( v_j )</td>
<td>( \frac{12}{23} )</td>
<td>( \frac{20}{25} )</td>
<td>( \frac{31}{20} )</td>
<td></td>
</tr>
</tbody>
</table>

Determine a set of 6 numbers \( u_i, i = 1, 2, 3 \) and \( v_j, j = 1, 2, 3 \) such that in each cell basic \((i, j)\) \( c_{ij} = u_i + v_j \), each non-basic cell \((i, j)\) by using formula \( d_{ij} = c_{ij} - (u_i + v_j) \).

So the tabular representation of \( u_i, i = 1, 2, 3, \) \( v_j, j = 1, 2, 3 \) and \( d_{ij} \) non-basic cell \((i, j)\) is given in the following table.
Capacitated Transportation Problem under 2-Vehicle

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic $(i, j) c_{ij} = u_i + v_j$,
each non-basic cell $(i, j)$ by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3, v_j, j = 1, 2, 3$ and $d_{ij}$ non-basic cell $(i, j)$
is given in the following table

Since $d_{23} < 0$ (i.e, most negative) so, solution is not optimal. So a loop occurred in

\[
\begin{array}{|c|c|c|c|}
\hline
O_1 & D_1 & D_2 & D_3 \\
\hline
x_{11} = 23 & c_{11} = \frac{12}{23} & x_{12} = 5 & c_{12} = \frac{5}{5} \\\n4, 8 & 5, 10 & & 10, 20 \\\n\hline
O_2 & x_{21} = 52 & c_{21} = \frac{9}{32} & \begin{cases} c_{22} = \frac{8}{5} \\
2, 3 & \text{if } d_{22} > 0 \\
8, 16 & \text{if } d_{23} < 0 \\
\end{cases} \\\n\hline
O_3 & c_{31} = \frac{7}{5} & x_{32} = 25 & c_{32} = \frac{6}{25} \\
7, 14 & d_{31} > 0 & 3, 6 & \text{if } d_{33} > 0 \\
\hline
v_j & \frac{12}{23} & \frac{25}{25} & \frac{3}{2} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Demand} & D_1 & D_2 & \text{stock} \\
\hline
\text{Demand} & 75 & 30 & 20 \\
\hline
\end{array}
\]
Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution. $x_{11} = 43, x_{12} = 5, x_{21} = 32, x_{23} = 20, x_{32} = 25$. Minimum cost $Z^* = 28 + 5 + 6 + 18 + 6 = 61$ unit(Rs.)

### 3.1 Comparison:

#### 3.1.1 Comparison with capacitated transportation problem

To solve the above example(capacitated transportation) we have from Model 2 as following Model 4

**Model 4**

\[
\begin{align*}
\text{min } z &= .95x_{11} + 1.5x_{12} + 1.3x_{13} + 1.1x_{21} + .92x_{22} \\
&+ 2.1x_{23} + 1.2x_{31} + 1.3x_{32} + 1.3x_{33} \\
\text{subject to } x_{11} + x_{12} + x_{13} &= 48; x_{21} + x_{22} + x_{23} = 52; x_{31} + x_{32} + x_{33} = 25; \\
x_{11} + x_{21} + x_{31} &= 75; x_{12} + x_{22} + x_{23} = 30; x_{13} + x_{23} + x_{33} = 20; \\
0 &\leq x_{11} \leq 20; 0 \leq x_{12} \leq 23; 0 \leq x_{13} \leq 25; \\
0 &\leq x_{21} \leq 35; 0 \leq x_{22} \leq 24; 0 \leq x_{23} \leq 27; \\
0 &\leq x_{31} \leq 28; 0 \leq x_{32} \leq 30; 0 \leq x_{33} \leq 26;
\end{align*}
\]

Solve Model 4 by Lingo Package we get the following result: $x_{11} = 20, x_{12} = 8, x_{13} = 20, x_{21} = 30, x_{22} = 22, x_{31} = 25$. Minimum cost $Z^* = 140.24$ unit(Rs.)

Clearly, 2-vehicles cost varying transportation model gives better result than usual capacitated transportation model.
3.1.2 Comparison with single vehicle cost varying transportation problem

If the Example is solved by considering only single vehicle either $V_1$ or $V_2$ then minimum transportation cost is increased. The results of Example for $V_1$, for $V_2$ and for both $V_1$, $V_2$ are given in the following Table-T1.

<table>
<thead>
<tr>
<th>Problems</th>
<th>Single Vehicle CVTP</th>
<th>Two-vehicle CVTP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_1$</td>
<td>$V_2$</td>
</tr>
<tr>
<td>Example</td>
<td>$Z^* = 25$</td>
<td>$Z^* = 54$</td>
</tr>
<tr>
<td></td>
<td>$x_{11} = 43, x_{12} = 5$</td>
<td>$x_{11} = 23, x_{12} = 25$</td>
</tr>
<tr>
<td></td>
<td>$x_{21} = 32, x_{23} = 20$</td>
<td>$x_{22} = 52$</td>
</tr>
<tr>
<td></td>
<td>$x_{32} = 25$</td>
<td>$x_{32} = 5, x_{33} = 20$</td>
</tr>
</tbody>
</table>

Table - T1 : The computational results of Example

It is seen from table T1 that two-vehicle cost varying transportation model give more efficient result than a single vehicle cost varying transportation model.

4 Conclusion

In this paper we have developed two-vehicle cost varying transportation problem. Using vehicles whose capacities are not exceed to the route capacities of the capacitated transportation problem we convert capacitated transportation problem to the cost varying transportation problem. North-west corner rule plays a role to allocate initial basic cell. Then by our proposed algorithm of determination of $c_{ij}$ we transfer this cost varying transportation problem to usual transportation problem. Then apply optimality test where unit transportation cost vary from one table to another table. Finally, achieve optimal solution. Comparing numerically, it is seen that two-vehicle cost varying transportation model gives more efficient result than single objective capacitated transportation problem. This problem is more real life problem than usual transportation problem.
References


Capacitated Transportation Problem under 2-Vehicle


