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Capacitated Transportation Problem under 2-Vehicle

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Abstract

In this paper we represent a two-vehicle cost varying transportation model to solve capacitated transportation problem. In this model the transportation cost varies due to capacity of vehicles as well as amount of transport quantity. At first we propose an algorithm to determine unit transportation cost with initial allocation to the basic cells by North-West corner rule. Then solve it. The unit transportation cost vary during optimality test when allocations are changed. Numerical example is presented to illustrate the two-vehicle cost varying transportation problem(TVCVTP). Finally, comparison is given to show better effective of this model.

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1 Introduction

Transportation problem of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destination of demand in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values but in real life applications unit transportation cost may vary.
A capacitated transportation problem is such a transportation problem in which the supply and demand constraints are equality type and capacity restriction on each route are specified.
In transportation problem unit transportation cost is constant from each source to each destination. But in reality, it is not constant; it depends on amount of transport quantity and capacity of vehicles. If amount of quantity is small then small(capacity) vehicle is sufficient for deliver. Where as if amount of quantity is large then big(capacity) vehicle is needed. So, depend on amount of transport quantity and the capacity of vehicles, the unit transportation cost is not constant. The cost varying transportation problem is such a transportation problem where unit transportation cost is varied depending on the selection of vehicles and number of vehicles.
The basic transportation problem was originally developed by Hitchock [14] and letter by Dantzig [6]. Many researchers [13,15,17] did work on fixed charge transportation problem. Gupta and Arora [8] presented a capacitated fixed charge bi-criterion indefinite quadratic transportation problem, giving the same priority to cost as well as time is studied. They developed an algorithm which is based on the concept of solving the indefinite quadratic fixed charge transportation problem. Gupta and Arora [11] discussed on a paradox in a

capacitated transportation problem where the objective function is a ratio of two linear functions consisting of variable costs and profits respectively. In another paper, Gupta and Arora [9,10] discussed on restricted flow in a fixed charge capacitated transportation problem with bounds on total source availabilities and total destination requirements. Dahiya and Verma [5] considered a class of the capacitated transportation problems with bounds on total availabilities at sources and total destination requirements. In this paper, unbalanced capacitated transportation problems have been discussed in the present paper as a particular case of original problem. In addition, they have discussed paradoxical situation in a balanced capacitated transportation problem and have obtained the paradoxical solution by solving one of the unbalanced problems. Arora and Ahuja [1] discussed a paradox in fixed charge transportation problem. Then Arora and Khurana [2] introduced three-dimensional fixed charge transportation problem is an extension of the classical three-dimensional transportation problem in which a fixed cost is incurred for every origin. Basu et. al. [3] represented an algorithm for finding the optimum solution of solid fixed charge transportation problem. Then Bit, et. al. developed fuzzy programming technique for multi objective capacitated transportation problem. Singh and Saxena [16] introduced the multiobjective time transportation problem with additional restrictions. Recently, Dutta and Murthy [7] developed fuzzy transportation problem with additional restrictions.

Here we present some capacitated transportation problem. To solve this problem we consider two vehicles whose capacities are less then the capacity restrictions i.e., capacity of each vehicle is less than or equal to the all route restrictions. Since vehicles are fixed (Contract with fixed Price) so unit transportation cost is varied. This type of transportation problem is named as cost varying transportation problem. In our model we consider this type of capacitated transportation problems and solved these under two vehicles cost varying transportation problem.

2 Mathematical Formulation

2.1 Preliminaries

A transportation problem can be stated in **Model 1** as follows:
Model 1

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}, \\ \text{subject to} \quad \sum_{i=1}^{m} x_{ij} &= a_i, \quad i = 1,\ldots,m \\ \sum_{j=1}^{n} x_{ij} &= b_j, \quad j = 1,\ldots,n \\ \sum_{i=1}^{m} a_i &= \sum_{j=1}^{n} b_j \\ x_{ij} &\geq 0 \;\; \forall i, \;\; \forall j \end{aligned}$$

A transportation problem can be represent in the following tabulated form.

	D_1	D_2	..	D_n	*stock*
O_1	c_{11}	$c_{12},$	....	$c_{1n},$	a_1
O_2	c_{21}	$c_{22},$	....	$c_{2n},$	a_2
....	....	....	....	....	....
O_m	c_{n1}	$c_{n2},$	....	$c_{nn},$	a_m
Demand	b_1	b_2	....	b_n	

Table: Tabular representation of a transportation problem.

where a_i is the quantity of material available at source $O_i, i = 1,\ldots,m$
b_j is the quantity of material required at destination $D_j, j = 1,\ldots,n$
c_{ij} is the unit cost of transportation from st source O_i to destination D_j.
The following terms are to be defined with reference to the transportation problems.

Definition 1: Feasible Solution (F.S.):

A set of non-negative allocations $x_{ij} \geq 0$ *which satisfies* $(1), (2)$ *is known as feasible solution.*

Definition 2: Basic Feasible Solution (B.F.S.):

A feasible solution to a m*-origin and* n*-destination problem is said to be basic feasible solution if number of positive allocations are* $(m + n - 1)$.

If the number of allocations in a basic feasible solutions are less than (m+n-1), it is called degenerate basic feasible solution (DBFS) otherwise non-degenerate basic feasible solution (NDBFS).

Definition 3: Optimal Solution:

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

Theorem 2.1: *The number of basic variables in a Transportation Problem(T.P.) is at most* $(m + n - 1)$

Theorem 2.2: *There exits a F.S. in each Transportation Problem(T.P.)*

Theorem 2.3: *In each T.P. there exits at least one B.F.S. which makes the objective function a minimum*

Theorem 2.4: *The solution of a T.P. is never unbounded*

Definition 4: Loop:

In the Transportation table, a sequence of cells is said to form a loop, if

(i) each adjacent pair of cells either lies in the same column or in the same row;

(ii) not more than two consecutive cells in the sequence lie in the same row or in the same column;

(iii) the first and the last cells in the sequence lie either in the same row or in the same column;

(iv) the sequence must involve at least two rows or two columns of the table.

Theorem 2.5: *A sub-set of the columns of the coefficient matrix of a T.P. are linearly dependent, iff, the corresponding cells or a sub-set of them can be sequenced to form a loop.*

2.2 North-West corner rule

Step 1. Compute min (a_1, b_1). If $a_1 < b_1$, min $(a_1, b_1) = a_1$ and if $a_1 > b_1$, min $(a_1, b_1) = b_1$. Select $x_{11} =$ min (a_1, b_1) allocate the value of x_{11} in the cell $(1, 1)$.
Step 2. If $a_1 < b_1$, compute min $(a_2, b_1 - a_1)$. Select $x_{21} =$ min $(a_2, b_1 - a_1)$ and allocate the value of x_{21} in the cell $(2, 1)$.
If $a_1 > b_1$, compute min $(a_1 - b_1, b_2)$. Select $x_{12} =$ min $(a_1 - b_1, b_2)$ and allocate the value of x_{12} in the cell $(1, 2)$.
Let us now make an assumption that $a_1 - b_1 < b_2$. With this assumption the next cell for which some allocation is to made, is the cell $(2, 2)$.
If $a_1 = b_1$ then allocate 0 only in one of two cells $(2, 1)$ or $(1, 2)$. The next allocation is to be made cell $(2, 2)$.
In general, if an allocation is made in the cell $(i + 1, j)$ in the current step, the next allocation will be made either in cell (i, j) or $(i, j + 1)$.
The feasible solution obtained by this away is always a B.F.S.

2.3 optimality test:

In order to test for optimality we should follow the procedure as given bellow:
Step 1. Start with B.F.S. consisting of $m + n - 1$ allocation in independent positions.
Step 2. Determine a set of $m+n$ numbers $u_i, i = 1, \ldots, m$ and $v_j, j = 1, \ldots, n$ such that in each cell (i, j) $c_{ij} = u_i + v_j$
Step 3. Calculate cell evaluations (unit cost difference) d_{ij} for each empty cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
Step 4. Examine the matrix of cell evaluation d_{ij} for negative entries and conclude that
(i) If all $d_{ij} > 0$, then Solution is optimal and unique.
(ii) If all $d_{ij} \geq 0$ and at least one $d_{ij} = 0$, then solution is optimal and alternative solution also exists.
(iii) If at least one $d_{ij} < 0$, then solution is not optimal. If it is so, further improvement is required by repeating the above process after Step 5 and onwards.

Step 5. (i) See the most negative cell in the matrix $[d_{ij}]$.
(ii) Allocate θ to this empty cell in the final allocation table. Subtract and add the amount of this allocation to other corners of the loop in order to restore feasibility.
(iii) This value of θ, in general is obtained by equating to zero the minimum of the allocations containing $-\theta$ (not $+\theta$) only at the corners of the closed loop.
(iv) Substitute the value of θ and find a fresh allocation table.
Step 6. Again, apply the above test for optimality till we find $d_{ij} \geq 0$.

2.4 capacitated transportation problem:

Consider m origins and n destinations. At each origin O_i, let a_i be the amount of a homogeneous product which we want to transport to n destinations D_j to satisfy the demand for b_j units of the product there. The unit transportation cost is c_{ij} from source i to destination j. A variable x_{ij} represents the unknown quantity to transported from origin O_i to destination D_j. Let r_{ij} be the capacitated restrictions on route i, j for capacitated transportation problem.
A capacitated transportation problem can be stated in **Model 2** as follows:
Model 2

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}, & \\ \text{subject to} \quad \sum_{i=1}^{m} x_{ij} &= a_i, \quad i = 1,\ldots,m & \\ \sum_{j=1}^{n} x_{ij} &= b_j, \quad j = 1,\ldots,n & \\ \sum_{i=1}^{m} a_i &= \sum_{j=1}^{n} b_j & (1) \\ 0 \leq x_{ij} &\leq r_{ij} \quad \forall i, \quad \forall j & \end{aligned}$$

The condition (1) is necessary condition for the **Model 2** to have a feasible solution, however, this is not sufficient because of the capacity restriction on each route.

2.5 2-vehicle cost varying transportation problem

Suppose there are two types off vehicles V_1, V_2 from each source to each destination. Let C_1 and $C_2(> C_1)$ are the capacities(in unit) of the vehicles V_1 and V_2 respectively,$C_1 \leq r_{ij}$ and $C_2 \leq r_{ij}$ $\forall i, j$. $R_{ij} = (R^1_{ij}, R^2_{ij})$ represent transportation cost for each cell (i, j); where R^1_{ij} is the transportation cost from source $O_i, i = 1, \ldots, m$ to the destination $D_j, j = 1, \ldots, n$ by the vehicle V_1. And R^2_{ij} is the transportation cost from source $O_i, i = 1, \ldots, m$ to the destination $D_j, j = 1, \ldots, n$ by the vehicle V_2. So, cost varying transportation problem can be represent in the following tabulated form.

	D_1	D_2	..	D_n	*stock*
O_1	R^1_{11}, R^2_{11}	R^1_{12}, R^2_{12}	....	R^1_{1n}, R^2_{1n}	a_1
O_2	R^1_{21}, R^2_{21}	R^1_{22}, R^2_{22}	....	R^1_{2n}, R^2_{2n}	a_2
....	....	....	....	....	....
O_m	R^1_{m1}, R^2_{m1}	R^1_{m2}, R^2_{m2}	....	R^1_{mn}, R^2_{mn}	a_m
Demand	b_1	b_2	....	b_n	

Table: Tabular representation of cost varying transportation problem.

2.6 Solution procedure capacitated transportation problem

To solve **model 2** we consider such type of vehicles V_1, V_2 such that the capacities Let C_1 and $C_2(> C_1)$ of the vehicles V_1 and V_2 respectively, satisfies following restrictions.

$$C_1 \leq r_{ij} \quad \forall i, j \tag{2}$$

$$\text{and} \quad C_2 \leq r_{ij} \quad \forall i, j \tag{3}$$

so that **Model 2** converted to a two vehicle cost varying transportation problem. Then apply following rule.

2.7 Solution procedure 2-vehicle cost varying transportation problem

2.7.1 Determination of c_{ij}

To solve this problem, apply our proposed **Algorithms** stated as follows:

2.7.2 Algorithm

Step 1. Since unit cost is not determined (because it depends on quantity of transport), so North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S.

Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{p1_{ij}R1_{ij}+p2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ \\ 0 & \text{if } x_{ij} = 0 \end{cases} \tag{4}$$

$$\begin{aligned} \text{where} \quad & p1_{ij},\ p2_{ij}, i = 1, \ldots, m; j = 1, \ldots, n \text{ are integer solution of} \\ \min \quad & p1_{ij}R1_{ij} + p2_{ij}R2_{ij} \\ \text{s.t. } x_{ij} \leq \quad & p1_{ij}C_1 + p2_{ij}C_2 \end{aligned}$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij}, then for non-basic cell c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{p1_{ij}R1_{ij}+p2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ \\ 0 & \text{if } x_{ij} = 0 \end{cases} \tag{5}$$

$$\begin{aligned} \text{where} \quad & p1_{ij},\ p2_{ij}, i = 1, \ldots, m; j = 1, \ldots, n \text{ are integer solution of} \\ \min \quad & p1_{ij}R1_{ij} + p2_{ij}R2_{ij} \\ \text{s.t. } x_{ij} \leq \quad & p1_{ij}C_1 + p2_{ij}C_2 \end{aligned}$$

In this manner we convert cost varying transportation problem to a usual transportation problem but c_{ij} is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix c_{ij} by **Step 2** and for non-basic we fix c_{ij} by **Step 3**.

Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

2.7.3 Bi-level Mathematical Programming for 2-vehicle cost varying transportation problem

The Bi-level mathematical programming for 2-vehicle cost varying transportation problem is formulated in **Model 3** as follows:

Model 3

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}, \qquad (6)\\
\text{where,} \quad & c_{ij} \text{ is determined by following mathematical programming}\\
c_{ij} \quad =& \begin{cases} \frac{p1_{ij}R1_{ij}+p2_{ij}R2_{ij}}{x_{ij}}, & \text{if } x_{ij} \neq 0 \\ \\ 0 & \text{if } x_{ij} = 0 \end{cases}\\
\min \quad & p1_{ij}R1_{ij} + p2_{ij}R2_{ij} \qquad (7)\\
\text{s. t. } x_{ij} \quad \leq& \quad p1_{ij}C_1 + p2_{ij}C_2\\
\sum_{i=1}^{m} x_{ij} \quad =& \quad a_i, \quad i = 1,\ldots,m\\
\sum_{j=1}^{n} x_{ij} \quad =& \quad b_j, \quad j = 1,\ldots,n\\
\sum_{i=1}^{m} a_i \quad =& \quad \sum_{j=1}^{n} b_j\\
0 \leq x_{ij} \quad \leq& \quad r_{ij} \quad \forall i, \quad \forall j\\
\text{where} \quad & p1_{ij},\ p2_{ij}, i = 1,\ldots,m; j = 1,\ldots,n \text{ are integer solution of}
\end{aligned}$$

3 Numerical Example

Consider a capacitated transportation problem as

	D_1	D_2	D_3	*stock*
O_1	.95	1.5	1.3	48
O_2	1.1	.92	2.1	52
O_3	1.2	1.3	1.3	25
Demand	75	30	20	

with capacity of route restrictions are as follows:

$$r_{11} = 20, r_{12} = 23, r_{13} = 25;$$
$$r_{21} = 35, r_{22} = 24, r_{23} = 27;$$
$$r_{31} = 28, r_{32} = 30, r_{33} = 26;$$

To solve this capacitated transportation problem, first select the two vehicles where, the capacities of vehicles of V_1 and V_2 are respectively, $C_1 = 6$ and $C_2 = 18$. So that the route restriction is violated. Also, the fixed cost for each vehicles are shown in each cell (i, j) in following tabulated form of 2-vehicle cost varying transportation problem.

	D_1	D_2	D_3	*stock*
O_1	$4, 8$	$5, 10$	$10, 20$	48
O_2	$2, 3$	$8, 16$	$6, 12$	52
O_3	$7, 14$	$3, 6$	$9, 18$	25
Demand	75	30	20	

Determination of c_{ij} for both basic and non-basic cell

Step 1. By North-west corner rule initial B.F.S. is

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 48$ $4, 8$	$5, 10$	$10, 20$	48
O_2	$x_{21} = 27$ $2, 3$	$x_{22} = 25$ $8, 16$	$6, 12$	52
O_3	$7, 14$	$x_{32} = 5$ $3, 6$	$x_{33} = 20$ $9, 18$	25
Demand	75	30	20	

Step 2. Using (5), we determine $c_{11} = \frac{24}{48}$, $C_{21} = \frac{6}{27}$, $c_{22} = \frac{32}{25}$,$C_{32} = \frac{3}{5}$ $c_{33} = \frac{27}{20}$,
Step 3. Using (6), we determine$c_{12} = \frac{20}{25}$, $c_{23} = \frac{18}{20}$, $C_{13} = \frac{30}{20}$, $C_{31} = \frac{7}{5}$
With these c_{ij} the transportation problem converted to

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 48$ $c_{11} = \frac{24}{48}$ $4, 8$	$c_{12} = \frac{20}{25}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 27$ $c_{21} = \frac{6}{27}$ $2, 3$	$x_{22} = 25$ $c_{22} = \frac{32}{25}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{7}{5}$ $7, 14$	$x_{32} = 5$ $c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20$ $c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

optimality test

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 48$ $c_{11} = \frac{24}{48}$ $4, 8$	$c_{12} = \frac{20}{25}$ $5, 10$ $d_{12} < 0$	$c_{13} = \frac{30}{20}$ $10, 20$ $d_{13} > 0$	0
O_2	$x_{21} = 27$ $c_{21} = \frac{6}{27}$ $2, 3$	$x_{22} = 25$ $c_{22} = \frac{32}{25}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$ $d_{23} > 0$	$-\frac{5}{18}$
O_3	$c_{31} = \frac{7}{5}$ $7, 14$ $d_{31} < 0$	$x_{32} = 5$ $c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20$ $c_{33} = \frac{27}{20}$ $9, 18$	$\frac{2077}{900}$
v_j	$\frac{1}{2}$	$\frac{701}{450}$	$-\frac{431}{450}$	

Since $d_{12} < 0$ so, solution is not optimal. So a loop occurred in cells$(1, 1), (1, 2), (2, 2), (2, 1), (1, 1)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 23$ $c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 25$ $c_{12} = \frac{20}{25}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 52$ $c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{32}{25}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{7}{5}$ $7, 14$	$x_{32} = 5$ $c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20$ $c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11}=23$ $c_{11}=\frac{12}{23}$ $4,8$	$x_{12}=25$ $c_{12}=\frac{20}{25}$ $5,10$	$c_{13}=\frac{30}{20}$ $10,20$ $d_{13}<0$	 0
O_2	$x_{21}=52$ $c_{21}=\frac{9}{52}$ $2,3$	$c_{22}=\frac{32}{25}$ $8,16$ $d_{22}>0$	$c_{23}=\frac{18}{20}$ $6,12$ $d_{23}<0$	 $-\frac{417}{1196}$
O_3	$c_{31}=\frac{7}{5}$ $7,14$ $d_{31}>0$	$x_{32}=5$ $c_{32}=\frac{3}{5}$ $3,6$	$x_{33}=20$ $c_{33}=\frac{27}{20}$ $9,18$	 $-\frac{1}{5}$
v_j	$\frac{12}{23}$	$\frac{20}{25}$	$\frac{31}{20}$	

Since $d_{13} < 0$ (i.e, most negative) so, solution is not optimal. So a loop occurred in cells$(1,2),(3,2),(3,3),(1,3),(1,2)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11}=23$ $c_{11}=\frac{12}{23}$ $4,8$	$x_{12}=5$ $c_{12}=\frac{5}{5}$ $5,10$	$x_{13}=20$ $c_{13}=\frac{30}{20}$ $10,20$	 48
O_2	$x_{21}=52$ $c_{21}=\frac{9}{52}$ $2,3$	$c_{22}=\frac{8}{5}$ $8,16$	$c_{23}=\frac{18}{20}$ $6,12$	 52
O_3	$c_{31}=\frac{7}{5}$ $7,14$	$x_{32}=25$ $c_{32}=\frac{6}{25}$ $3,6$	$x_{33}=20$ $c_{33}=\frac{27}{20}$ $9,18$	 25
Demand	75	30	20	

Determine a set of 6 numbers $u_i, i=1,2,3$ and $v_j, j=1,2,3$ such that in each cell basic (i,j) $c_{ij}=u_i+v_j$,
each non-basic cell (i,j) by using formula $d_{ij}=c_{ij}-(u_i+v_j)$
So the tabular representation of $u_i, i=1,2,3,\;\; v_j, j=1,2,3$ and d_{ij} non-basic cell (i,j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 23$ $c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 5$ $c_{12} = \frac{5}{5}$ $5, 10$	$x_{13} = 20$ $c_{13} = \frac{30}{20}$ $10, 20$	0
O_2	$x_{21} = 52$ $c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16$ $d_{22} > 0$	$c_{23} = \frac{18}{20}$ $6, 12$ $d_{23} < 0$	$-\frac{417}{1196}$
O_3	$c_{31} = \frac{7}{5}$ $7, 14$ $d_{31} > 0$	$x_{32} = 25$ $c_{32} = \frac{6}{25}$ $3, 6$	$c_{33} = \frac{27}{20}$ $9, 18$ $d_{33} > 0$	$-\frac{19}{25}$
v_j	$\frac{12}{23}$	$\frac{25}{25}$	$\frac{3}{2}$	

Since $d_{23} < 0$ (i.e, most negative) so, solution is not optimal. So a loop occurred in cells$(1, 1), (1, 3), (2, 3), (2, 1), (1, 1)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	$stock$
O_1	$x_{11} = 43$ $c_{11} = \frac{28}{43}$ $4, 8$	$x_{12} = 5$ $c_{12} = \frac{5}{5}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 32$ $c_{21} = \frac{6}{32}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16$	$x_{23} = 20$ $c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{28}{25}$ $7, 14$	$x_{32} = 25$ $c_{32} = \frac{6}{25}$ $3, 6$	$c_{33} = \frac{27}{20}$ $9, 18$	25
$Demand$	75	30	20	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 43$ $c_{11} = \frac{28}{43}$ $4, 8$	$x_{12} = 5$ $c_{12} = \frac{5}{5}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$ $d_{13} > 0$	0
O_2	$x_{21} = 32$ $c_{21} = \frac{6}{32}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16$ $d_{22} > 0$	$x_{23} = 20$ $c_{23} = \frac{18}{20}$ $6, 12$	$-\frac{319}{688}$
O_3	$c_{31} = \frac{28}{25}$ $7, 14$ $d_{31} > 0$	$x_{32} = 25$ $c_{32} = \frac{6}{25}$ $3, 6$	$c_{33} = \frac{27}{20}$ $9, 18$ $d_{33} > 0$	$-\frac{19}{25}$
v_j	$\frac{28}{43}$	$\frac{25}{25}$	1.364	

Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution. $x_{11} = 43, x_{12} = 5, x_{21} = 32, x_{23} = 20, x_{32} = 25$. Minimum cost $Z^* = 28 + 5 + 6 + 18 + 6 = 61$ unit(Rs.)

3.1 Comparison:

3.1.1 Comparison with capacitated transportation problem

To solve the above example(capacitated transportation) we have from **Model 2** as following **Model 4**

Model 4

$$\begin{aligned} \min z = &.95x_{11} + 1.5x_{12} + 1.3x_{13} + 1.1x_{21} + .92x_{22} \\ &+2.1x_{23} + 1.2x_{31} + 1.3x_{32} + 1.3x_{33} \\ \text{subject to} \quad & x_{11} + x_{12} + x_{13} = 48; x_{21} + x_{22} + x_{23} = 52; x_{31} + x_{32} + x_{33} = 25; \\ & x_{11} + x_{21} + x_{31} = 75; x_{12} + x_{22} + x_{23} = 30; x_{13} + x_{23} + x_{33} = 20; \\ & 0 \leq x_{11} \leq 20; 0 \leq x_{12} \leq 23; 0 \leq x_{13} \leq 25; \\ & 0 \leq x_{21} \leq 35; 0 \leq x_{22} \leq 24; 0 \leq x_{23} \leq 27; \\ & 0 \leq x_{31} \leq 28; 0 \leq x_{32} \leq 30; 0 \leq x_{33} \leq 26; \end{aligned}$$

Solve **Model 4** by Lingo Package we get the following result: $x_{11} = 20, x_{12} = 8, x_{13} = 20, x_{21} = 30, x_{22} = 22, x_{31} = 25$. Minimum cost $Z^* = 140.24$ unit(Rs.)

Clearly, 2-vehicles cost varying transportation model gives better result than usual capacitated transportation model.

3.1.2 Comparison with single vehicle cost varying transportation problem

If the **Example** is solved by considering only single vehicle either V_1 or V_2 then minimum transportation cost is increased. The results of **Example** for V_1, for V_2 and for both V_1, V_2 are given in the following **Table-T1**.

Problems	*Single Vehicle CVTP*		*Two-vehicle CVTP*
	V_1	V_2	V_1, V_2
Example	$Z^* = 25$ $x_{11} = 43, x_{12} = 5$ $x_{21} = 32, x_{23} = 20$ $x_{32} = 25$	$Z^* = 54$ $x_{11} = 23, x_{12} = 25$ $x_{22} = 52$ $x_{32} = 5, x_{33} = 20$	$Z^* = 25$ $x_{11} = 43, x_{12} = 5$ $x_{21} = 32, x_{23} = 20$ $x_{32} = 25$

Table - $T1$: The computational results of Example

It is seen from **table** $T1$ that two-vehicle cost varying transportation model give more efficient result than a single vehicle cost varying transportation model.

4 Conclusion

In this paper we have developed two-vehicle cost varying transportation problem. Using vehicles whose capacities are not exceed to the route capacities of the capacitated transportation problem we convert capacitated transportation problem to the cost varying transportation problem. North-west corner rule plays a role to allocate initial basic cell. Then by our proposed algorithm of determination of c_{ij} we transfer this cost varying transportation problem to usual transportation problem. Then apply optimality test where unit transportation cost vary from one table to another table. Finally, achieve optimal solution. Comparing numerically, it is seen that two-vehicle cost varying transportation model gives more efficient result than single objective capacitated transportation problem. This problem is more real life problem than usual transportation problem.

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