AMO-Advanced Modelling and Optimization, Volume 16, Number 1, 2014

A Hybrid Optimization Technique for Solving Non-Linear Multi-Objective Optimization Problem .

Rashmi Ranjan Ota† , Jayanta Kumar Dash* and A.K. Ojha**
† ITER,SOA University, Bhubaneswar,India
Email: otapink@gmail.com
* ITER,SOA University, Bhubaneswar, India
Email: jkdash@gmail.com
** School Basic Sciences,IIT, Bhubaneswar, India
Email: akojha57@yahoo.com

Abstract

A multi-objective optimization problem contains more than one objective function that needs to be considered simultaneously to find the compromise solution. Such situation arises in many optimization problems where two, sometimes more than two conflicting objective functions have to be minimized at once. Our strategy in this paper to combine different optimization techniques to find the pareto optimal solutions of the multi-objective optimization problems known as hybrid method. The discussed hybrid method not only provides a set of pareto optimal solution but also enhance the optimizers over all performance. In this paper we have combined ϵ-constraint and weighted mean method . The solution procedure of the proposed hybrid method is illustrated by the numerical examples.

Keywords : Geometric Programming, ϵ-constraint method, Weighted mean method, Hybrid method, Optimization.

AMO - Advanced Modeling and Optimization. ISSN: 1841-4311

1 Introduction

In engineering and sometimes in economics, the design problems are generally characterised by the presence of many conflicting objectives. So it is a challenging question, how to combine different objectives to yield a pareto optimal solution for design problems. In this paper, we have tried to answer the above question by hybriding different optimization techniques. There are certain areas, where mathematical modelling of physical problems are nonlinear having several conflicting objectives. So that Instead of a single objective optimization, we have to construct a multi-objective optimization problem(MOOP). A MOOP consists of several objective functions whose component cost under certain constraints are in the form of posynomials or polynomials and it's solution consisting of those vectors whose components can not be all improved simultaneously known as pareto optimal solution. So far, several methods have been proposed by the authors[1, 2, 3, 4, 5] to solve various multi-objective non-linear programming problems subject to linear and nonlinear constraints. Now-a-days, geometric programming technique have been used extensively to solve various engineering design problems which are in the form of multi-objective functions. Since the objective functions of a MOOP with given constraints are conflicting with each other, therefore a MOOP does not have a single solution that could optimize all objective functions simultaneously. But the decision makers are always in search of a most compromise solution which could optimize all objective functions simultaneously. In recent past, Ojha et al.[16, 17, 18] have shown in their research paper, how to solve multi-objective geometric programming optimization problems related to different real world situations using various optimization techniques and also study the convergence of optimal solutions. The weighted mean method, is one of the widely used classical method for solving a multi-objective optimization problem. This method scalarizes a set of objectives into a single objective by pre-multiplying each objective with user supplied weights. Another most used classical method is ϵ-constrained method. It remove the difficulties solving a problem having non-convex objective space with respect to weighted mean method. In this paper we have discussed a kind of hybrid method by combining weighted mean with ϵ-constraint method for solving multi-objective geometric programming problems. The two vital reasons for which hybrid method is more useful for solving multi-objective optimization problem than any other method. First, it not only ensure better convergence to pareto optimal front but also demands smaller computational effort than each individual method applied alone. The other focusing part of this paper is whether objective should be aggregated first before getting optimal solution or we search for a optimal solution before articulating our preference. Deb and Goel[7], in their recent paper have shown how posteriori approach of hybridization is better than online approach as former can obtain better convergence as well as better diversity. It is also clear from their observation that optimum balance between the local search and the evolutionary search is essential to achieve best results-good diversity and convergence to pareto optimal front. In a research paper, Kaveh et al. [19] developed a new hybrid method for optimal design of truss structures. This method is based on a modified multi-objective particle swarm optimization, tournament decision making process, and a local search algorithm. Lin et al.[8] in their paper used mixed coding to represent continuous and discrete variables. They demonstrate mixed integer hybrid differential equation is superior to other method in terms of solution quality and robustness property. Muye et al.[9] developed aerodynamic shape optimization tools for complex industrial flows based on hybrid method which couples a stochastic genetic algorithm and deterministic BFGS Hill climbing method. A hybrid method for optimal scheduling of short term electric power generation of cascaded hydro electric plant based on particle swarm optimization and chance constrained programming developed by Jiekang et al.[10]. A hybrid method for solving global optimization problems have been presented by Gil et al.[11] in their recent paper. Dai et al. [12] in their paper have shown how non-linear hybrid conjugate gradient method produces a discrete search directives at every iteration and converge globally. Similarly Youn et al.[13] have shown the involvement of hybrid analysis method in evaluation of probabilistic constraints which can be done in two different ways. One is reliability index approach and other is performance measure approach. In a similar development Ghiasi et al.[14] presented non-dominated sorting hybrid algorithm for multi-objective optimization problem. P.Xu[15] in his paper also try to solve global optimization problem using hybridization concept. Cook et al.[20], in their paper developed a new hybrid technique combining a neural network with a genetic algorithm for process of parameter optimization.

The organization of the paper is as follows: Following the introduction, concept of multi-objective optimization problem is given in sec-2 and in sec-3, weighted mean method has been discussed. Subsequently, ϵ-constraint and hybrid method have been discussed in sec-4 and sec-5 respectively. Where as a concept regarding the convergence of optimal solution has been discussed in sec-6. An illustrative example has been incorporated in Sec-7 and Finally some conclusions drawn from the result have been presented in Section-8.

2 Multi-Objective Optimization Problem(MOOP):

The method of optimizing systematically and simultaneously a collection of objective function is called multi-objective optimization or vector optimization. A multi-objective optimization problem can be stated as:
Find $x = (x_1, x_2, ...x_n)^T$, so as to

$$\min : f_k(x), \quad k = 1, 2, ...p \tag{2.1}$$

Subject to

$$g_i(x) \leq 0, \quad i = 1, 2, ...m \tag{2.2}$$

$$x = (x_1, x_2, ...x_n) \geq 0 \tag{2.3}$$

In this multi-objective optimization problem, there are p number of minimization type objective functions, m number of inequality type constraints and n number of strictly positive decision variables.
The multi-objective optimization problem defined in (2.1)-(2.3) is considered as a vector-minimization problem. It is assumed that the problem has an optimal compromise solution.

3 Weighted Mean Method:

Weighted mean method is the widely used simplest method which convert a set of objectives into a single objective by multiplying each objective with user defined weights to find the non-inferior optimal solution of a multi-objective optimization problem within the convex objective space.

If $f_1(x), f_2(x), ...f_p(x)$ are 'p' objective functions for any vector $x = (x_1, x_2, ...x_n)^T$,then we can define weighting mean method is as follows.

$$Let \;\; W = \{w : w \in R^n, w_k > 0, \sum_{k=1}^{n} w_k = 1\} \tag{3.1}$$

be the set of non-negative weights. Using weighted method the multi-objective optimization problem given in sec-2 can be defined as:

$$Q(w) = \min_{x \in X} \sum_{k=1}^{p} w_k f_k(x) \tag{3.2}$$

subject to

$$g_i(x) \leq 0, i = 1, 2, ...m \tag{3.3}$$

$$x_j > 0, j = 1, 2, ...n \tag{3.4}$$

4 ϵ-Constraint Method:

A method which overcomes some of the convexity problems of the weighted sum technique is known as ϵ-constraint method. The ϵ-constraint method was proposed by Haimes et al.[6] for generating Pareto optimal solutions for the multi-objective optimization problem. The ϵ-constraint method is defined as:

$$\min: \; f_0^k(x), \quad where \; k \in \{1, 2, ...p\} \tag{4.1}$$

subject to

$$f_0^j(x) \leq \epsilon_j, \;\; j = 1, 2, ...p, \;\; j \neq k \tag{4.2}$$

$$g_i(x) \leq 0, \quad i = 1, 2, ...m \tag{4.3}$$

we define

$$L_j \leq \epsilon_j \leq U_j, j = 1, 2, ...p, j \neq k$$

where

$$L_j = \min_{\forall x \in X} \;\; f_0^j(x), j = 1, 2...., p$$

and

$$U_j = \max_{\forall x \in X} \;\; f_0^j(x), j = 1, 2, ..., p$$

$$x \in X, \quad X \; being \; the \; feasible \; region$$

5 Hybrid Method:

In the proposed hybrid method, we have combined two important optimization techniques for solving multi-objective optimization problem. One of them is weighted mean and other is ϵ-constraint method. The mathematical formulation of hybrid method is as follows:
If $f_1(x), f_2(x), ...f_p(x)$ are 'p' objective functions for any vector $x = (x_1, x_2, ...x_n)^T$,then we can define hybrid method is as follows.

$$Let \;\; W = \{w : w \in R^n, w_k > 0, \sum_{k=1}^{n} w_k = 1\} \tag{5.1}$$

be the set of non-negative weights. Using hybrid method, the multi-objective optimization problem given in sec-2 can be defined as:

$$Q(w) = \min_{x \in X} \sum_{k=1}^{p} w_k f_k(x) \tag{5.2}$$

subject to

$$f_j(x) \leq \epsilon_j, \quad j = 1, 2, ...p, \quad j \neq k \tag{5.3}$$

$$g_i(x) \leq 0, \qquad i = 1, 2, ...m \tag{5.4}$$

$$x \in X, \quad X \; being \; the \; feasible \; region$$

6 Convergence of ϵ-constraint Method:

Till now, no specific method has been derived for the convergence of the pareto optimal solutions of the Multi-objective programming problem or that is available in any form. Since many years, the decision makers are trying their best in order to find the most compromise solutions using some of the existing methods like fuzzy programming, goal programming and weighting methods. In the present paper, we have used ϵ-constraint method as defined in the section-4 to find the pareto optimal solution. The following adopted procedures can be considered to show, how the set of pareto optimal solutions are converging to certain point.
Step 1: Find the bounds of the objective functions ($f_0^{(k)}(x), k = 1, 2, ...p$) with the help

of obtained ideal solutions $X^{(1)}, X^{(2)}, ...X^{(p)}$ by using Multi-objetive programming algorithms as discussed in the section-2, such that L_k and U_k are the best and worst values of $f_0^{(k)} i.e L_k \leq f_0^{(k)}(x) \leq U_k, k = 1, 2...p$

Step 2: Let ϵ_k, be a point in the interval such that $L_k \leq \epsilon_k \leq U_k, k = 1, 2...p$

Step 3: Change the value of ϵ_k in the interval $[L_k, U_k]$ to generate a set of pareto optimal solution.

Step 4: Compare the pareto optimal solution with the solution obtained by Hybrid programming method.

Step 5: If the pareto optimal solution obtained in step 3 is equal to the optimal compromise solution obtained in step 4, then stop and accept the pareto optimal solution of the problem. This indicate that the set of solution generated by ϵ-constraint method converges to this particular solution. Finally, the decision maker has to choose his/her solution from the pareto set according to their satisfaction.

The following illustrative examples explain, How to find the pareto optimal solutions and convergence of the proposed method.

7 Numerical example:

We consider the following example to illustrate the proposed method for solving MOOP.

Example

Find x_1, x_2 so as to

$$\min f_1(x) = 2x_1^2 + x_1x_2 \tag{7.1}$$

$$\min f_2(x) = 2x_1 - x_2^2 \tag{7.2}$$

subject to

$$x_1 + 3x_2 \leq 10 \tag{7.3}$$

$$x_1 - x_2 \leq 4 \tag{7.4}$$

$$where \;\; x_1, x_2 \geq 0 \tag{7.5}$$

Primal Solution of $f_1(x)$:

Find x_1, x_2 so as to

$$\min f_1(x) = 2x_1^2 + x_1x_2 \tag{7.6}$$

subject to

$$x_1 + 3x_2 \leq 10 \tag{7.7}$$

$$x_1 - x_2 \leq 4 \tag{7.8}$$

$$where \;\; x_1, x_2 \geq 0 \tag{7.9}$$

Solution of primal $f_1(x) = 68.75$ for $x_1 = 5.5$ and $x_2 = 1.5$.

Primal Solution of $f_2(x)$:

Find x_1, x_2 so as to

$$\min f_2(x) = 2x_1 - x_2^2 \tag{7.10}$$

subject to

$$x_1 + 3x_2 \leq 10 \tag{7.11}$$

$$x_1 - x_2 \leq 4 \tag{7.12}$$

$$where \;\; x_1, x_2 \geq 0 \tag{7.13}$$

Solution of primal $f_2(x) = 9.000$ for $x_1 = 5.000$ and $x_2 = 0.99999$.
Using the value of f_1 in f_2 and f_2 in f_1, we can find the region of convergence for both the objective function as: $54.99999 \leq f_1 \leq 68.75$ and $8.75 \leq f_2 \leq 9$.

Solution by weighted mean method:

Using weighted mean method we can write the given multi-objective optimization problem as follows:

$$\min Z = w_1(2x_1^2 + x_1x_2) + w_2(2x_1 - x_2^2) \tag{7.14}$$

subject to

$$x_1 + 3x_2 \leq 10 \tag{7.15}$$

$$x_1 - x_2 \leq 4 \tag{7.16}$$

$$w_1 + w_2 = 1 \tag{7.17}$$

$$w_1, w_2, x_1, x_2 \geq 0 \tag{7.18}$$

Solution of the problem is given in following table-1.

Table-1

(Primal solution)

w_1	w_2	x_1	x_2	Z
1	0	5.5	1.5	68.75
0.8	0.2	5.5	1.5	56.75
0.6	0.4	5.5	1.5	44.75
0.2	0.8	5.5	1.5	20.75
0	1	5.0	0.99999	9.0

From the above table-1, we can observe that the optimal solution of Z varies form optimal solution of f_2 to f_1 on changing the value of weights between 0 and 1. For simplification we have taken only five cases. However, the result can be verified for any point between 0 and 1.

Solution of the problem by Hybrid method:

Using the hybrid method we can write the problem as follows:

$$\min Z = w_1(2x_1^2 + x_1x_2) + w_2(2x_1 - x_2^2) \tag{7.19}$$

subject to

$$2x_1^2 + x_1x_2 \le \epsilon_1, \quad (54.999999 \le \epsilon_1 \le 68.75) \tag{7.20}$$

$$2x_1 - x_2^2 \le \epsilon_2, \quad (8.75 \le \epsilon_2 \le 9.0) \tag{7.21}$$

$$x_1 + 3x_2 \le 10 \tag{7.22}$$

$$x_1 - x_2 \le 4 \tag{7.23}$$

$$w_1 + w_2 = 1 \tag{7.24}$$

$$w_1, w_2, x_1, x_2 \ge 0 \tag{7.25}$$

Table-2

(Solution by Hybrid method)

w_1	w_2	ϵ_1	ϵ_2	x_1	x_2	Z
1	0	55	8.8	4.8309	1.7230	55.00
–	–	68	9.0	5.4652	1.5115	68.00
–	–	68.75	9.0	5.5	1.5	68.75
0.8	0.2	55	8.8	4.9813	1.078	45.76
–	–	68	9.0	5.474	1.474	56.155
–	–	68.75	9.0	5.5	1.5	56.75
0	1	55	8.8	4.552	0.552	8.8
–	–	68	9.0	4.776	0.776	9.0
–	–	68.75	9.0	5.0	1.0	9.0

From the above table-2, it is observed that changing the values of w_1 and w_2 between 0 and 1, as well as the values of ϵ_1 and ϵ_2 within their required range, the optimal solution of f_1 and f_2 remain same as we got by weighted mean method. But the hybrid method ensure a better optimal solution which converges fast as compared to obtained by other methods like weighted mean and ϵ-constraint method.

8 Conclusion:

In this paper, the proposed hybrid method analysed and examined for different test problems in different space region for solving MOOP. Our initial motivation for hybrid presentation was the observation of the role of different optimization techniques as par with hybrid technique. We found hybrid representation ensure a stable and superior performance as compared to other optimization methods. In this paper we have made an implementation of both weighted mean and ϵ-constraint method and compare their performance using hybrid representation. It is evident that hybrid representation being good for solving real world multi-objective optimization problem.

Acknowledgement:
The authors are highly grateful to the anonymous referees and editor for their valuable suggestions, comments and necessary advice for corrections to improve the quality of presentation and solution procedure of the paper. The author(A.K.Ojha) is thankful to CSIR for the financial support of this work through grant No:25(0201)/12/EMR-II.

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