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Intuitionistic Fuzzy Optimization Technique in EOQ Model with Two Types of Imperfect Quality Items

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Abstract

This paper discusses an Economic Order Quantity (EOQ) model with two types of imperfect quality items : scrap and reworkable, where the set-up cost, the holding cost and the demand are considered as fuzzy numbers. The fuzzy parameters are then transformed into corresponding interval numbers. Minimization of the interval objective function (obtained by using interval parameters) has been transformed into a classical multi-objective EOQ problem. The order relation that represents the decision maker's preference among the interval objective functions has been defined by the right limit, left limit, center and half-width of an interval. This concept is used to minimize the interval objective function. The problem has been solved by intuitionistic fuzzy programming technique. Finally, the proposed method is illustrated with a practical numerical example. The Pareto Optimality test is used for verification of optimality of the solution.

Keywords : Inventory, Interval Number, Fuzzy Sets, Pareto Optimal Solution, Intuitionistic Fuzzy Optimization Technique, Multi-objective Programming.

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1 Introduction

In traditional mathematical problems, the parameters are always treated as deterministic in nature. However, in practical engineering problem, uncertainty always exists. In order to deal with such uncertain situations stochastic approach is most commonly used, where it is assumed that the cost component possesses a known probability distribution function [1], [2], [3]. However, many parameters of the problem with uncertainties do not have any past information. To overcome this difficulty fuzzy model is used [4], [5], [6], [7], [8]. In such cases, fuzzy set theory, introduced by Zadeh [9] is acceptable. Also, in various real world engineering problems such as, portfolio investment, project planning management, transportation etc. involves multi-dimension and multi-objective optimization. Zimmermann [10] proposed a fuzzy linear programming for multi objective optimization. Recently, Jiang et. al. [11] has described a non-linear interval number programming method for two objectives optimization. Multi-objective optimization with fuzzy objective is mostly preferred by several researchers [12], [13], because, this approach is more realistic than that of deterministic or probabilistic one.

There are several studies on fuzzy EOQ model. Lin et al. [14] have developed a fuzzy model for production inventory problem. Katagiri and Ishii [15] have proposed an inventory problem with shortage cost as fuzzy quantity. Li et al. [16] have formulated fuzzy models for single period inventory problem. Vujosevic [17] has developed an EOQ model inventory costs are fuzzy. Also, a class of research work has been done using multi-objective optimization. Ishibuchi and Tanaka [18] utilizes multi-objective optimization in an interval objective function. Nayak and Pal [12] introduced intuitionistic fuzzy optimization technique for multi-objective bi-matrix game. Jana and Roy [20] formulated a transportation model and solved it by multi-objective intuitionistic fuzzy optimization. Some more works are also available in [28, 29, 31, 32].

In the above discussion, it is assumed that the ordered quantity or the manufactured items are perfect. However, it is quite natural that lot may contain some defective items. Some of the defective items are re-workable and after making over these can be sold at the same market price but some items are scrap and it must be sold in a secondary market at a discount price. Various research has also done in this field. Lee, Rosenblatt [21] first introduced imperfect quality items in inspection and ordering policies for products. Salameh and Jaber [22] have proposed an Economic Production Quantity model for imperfect quality items. A parallel approach is done by Chen et al. [23] with fuzzy sense. Al-Salameh [24] has proposed an EOQ model with previous mentioned two types of imperfect quality items, where the parameters are deterministic. However, in practical business problem parameters are not fixed rather they are different due to various circumstances. For example, demand of a particular commodity vary due to various reason. Inventory costs (ordering cost, holding cost, carrying cost etc.) fluctuates due to changes in season, transportation cost, mailing or telephonic charges etc.

This paper discusses a fuzzy EOQ model with two types of imperfect quality items : scrap and re-workable. After placing the order the total lot goes through 100 % inspection. Two types of defective items are found out. Customer's demand is meet up by good and re-worked items whereas the scrap items are sold at discount price in a secondary market. No shortages are allowed here i. e. the good and re-worked items are sufficient to satisfy customer's demand. Demand, holding cost and ordering cost are taken as intuitionistic fuzzy numbers, and expression for fuzzy cost is established. For minimizing the cost function we transformed the fuzzy objective function into interval objective function. Now, this single objective function is then converted to multi-objective problem by defining left limit, right limit and center of the objective function. This multi-objective function is then solved by intuitionistic fuzzy optimization technique. Exponential membership and quadratic non-membership function is considered here. This model is illustrated by a practical numerical example and lastly Pareto-Optimality test is performed.

The article is organized as follows : In Section 1 preliminary definitions of intuitionistic fuzzy set, interval number, basic interval arithmatic optimization in interval situation and nearest interval approximation is briefly described. Section 2 contains model formulation. The fuzzy optimization technique is described in Section 3. In Section 4 the process is illustrated by a numerical example and in the last section the entire work is concluded.

2 Preliminaries

Definition 1 Atanassov [34], [36] Let $X = \Big\{x_1, x_2, \dots, x_n\Big\}$ be a finite universal set. An Atanassov's intuitionistic fuzzy set (IFS) in a given universal set X is an expression $\widehat{A}$ is given by

$$\widehat{A} = \Big\{\langle x_i, \mu_{\widehat{A}}(x_i), \nu_{\widehat{A}}(x_i)\rangle : x_i \in X\Big\}, \tag{1}$$

where the functions $\mu_{\widehat{A}} : X \to [0,1]$ *i.e.* $x_i \in X \to \mu_{\widehat{A}}(x_i) \in [0,1]$ and $\nu_{\widehat{A}} : X \to [0,1]$ *i.e.* $x_i \in X \to \nu_{\widehat{A}}(x_i) \in [0,1]$ define the degree of membership and the degree of non-membership respectively of an element $x_i \in X$ satisfy the condition: for every $x_i \in X$, $0 \leq \mu_{\widehat{A}}(x) + \nu_{\widehat{A}}(x) \leq 1$.

Definition 2 Let $\widehat{A}$ and $\widehat{B}$ be two Atanassov's IFSs in the set X. The intersection of $\widehat{A}$ and $\widehat{B}$ is defined as follows :

$$\widehat{A} \cap \widehat{B} = \Big\{\langle x_i, \min(\mu_{\widehat{A}}(x_i), \mu_{\widehat{B}}(x_i)), \max(\nu_{\widehat{A}}(x_i), \nu_{\widehat{B}}(x_i))\rangle | x_i \in X\Big\}.$$

Definition 3 Let $\Re$ be the set of all real numbers. An interval, Moore [25], may be expressed as

$$\overline{a} = [a_L, a_R] = \{x : a_L \leq x \leq a_R, a_L \in \Re, a_R \in \Re\}, \tag{2}$$

where a_L and a_R are called the lower and upper limits of the interval $\overline{a}$, respectively.

If $a_L = a_R$ then $\overline{a} = [a_L, a_R]$ is reduced to a real number a, where $a = a_L = a_R$. Alternatively an interval $\overline{a}$ can be expressed in mean-width or center-radius form as $\overline{a} = \langle m(\overline{a}), w(\overline{a})\rangle$, where $m(\overline{a}) = \frac{1}{2}(a_L + a_R)$ and $w(\overline{a}) = \frac{1}{2}(a_R - a_L)$ are respectively the mid-point and half-width of the interval $\overline{a}$. The set of all interval numbers in $\Re$ is denoted by $I(\Re)$.

Basic interval arithmetic

Let $\overline{a} = [a_L, a_R] = \langle m(\overline{a}), w(\overline{a})\rangle$ and $\overline{b} = [b_L, b_R] = \langle m(\overline{b}), w(\overline{b})\rangle \in I(\Re)$, then

$$\overline{a} + \overline{b} = [a_L + b_L, a_R + b_R]; \quad \overline{a} + \overline{b} = \langle m(\overline{a}) + m(\overline{b}), w(\overline{a}) + w(\overline{b})\rangle. \tag{3}$$

The multiplication of an interval by a real number $c \neq 0$ is defined as

$$\begin{aligned} c\overline{a} &= [ca_L, ca_R]; \quad \text{if } c > 0 \\ &= [ca_R, ca_L]; \quad \text{if } c < 0 \end{aligned} \tag{4}$$

$$c\overline{a} = c\langle m(\overline{a}), w(\overline{a})\rangle = \langle cm(\overline{a}), |c|w(\overline{a})\rangle \tag{5}$$

The difference of these two interval numbers is

$$\overline{a} - \overline{b} = [a_L - b_R, a_R - b_L]. \tag{6}$$

The product of these two distinct interval numbers is given by

$$\overline{a}.\overline{b} = \Big[\min\Big\{a_L.b_L, a_L.b_R, a_R.b_L, a_R.b_R\Big\}, \max\Big\{a_L.b_L, a_L.b_R, a_R.b_L, a_R.b_R\Big\}\Big]. \tag{7}$$

The division of these two interval numbers with $0 \notin \overline{b}$ is given by

$$\overline{a}/\overline{b} = \left[\min\left\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\right\}, \max\left\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\right\}\right]. \tag{8}$$

Optimization in interval environment

Now we define a general nonlinear objective function with coefficients of the decision variables as interval numbers as

$$\text{Minimize } \overline{Z}(x) = \frac{\sum_{i=1}^{n}[a_{L_i}, a_{R_i}]\prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l}[b_{L_i}, b_{R_i}]\prod_{j=1}^{n} x_j^{q_j}} \tag{9}$$

subject to $x_j > 0$, $j = 1, 2, \cdots, n$ and $x \in S \subset \Re$ where S is a feasible region of x, $0 < a_{L_i} < a_{R_i}$, $0 < b_{L_i} < b_{R_i}$ and r_i, q_j are positive numbers. Now we exhibit the formulation of the original problem (9) as a multi-objective non-linear problem.

Now $\overline{Z}(x)$ can be written in the form $\overline{Z}(x) = \Big[Z_L(x), Z_R(x)\Big]$ where

$$Z_L(x) = \frac{\sum_{i=1}^{n} a_{L_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{R_i} \prod_{j=1}^{n} x_j^{q_j}}, \tag{10}$$

$$Z_R(x) = \frac{\sum_{i=1}^{n} a_{R_i} \prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l} b_{L_i} \prod_{j=1}^{n} x_j^{q_j}}. \tag{11}$$

The center of the objective function

$$Z_C(x) = \frac{1}{2}\Big[Z_L(x) + Z_R(x)\Big]. \tag{12}$$

Thus the problem (9) is transformed in to

$$\text{Minimize}\Big\{Z_C(x), Z_R(x); \quad x \in S\Big\} \tag{13}$$

subject to the non-negativity constraints of the problem, where Z_C, Z_R are defined by (11) and (12).

Nearest interval approximation

According to Gregorzewski [27] we determine the interval approximation of a fuzzy number as: Let $\widetilde{A} = (a_1, a_2, a_3)$ be an arbitrary triangular fuzzy number with a α - cuts $[A_L(\alpha), A_R(\alpha)]$ and with the following membership function

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}; & a_1 \le x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}; & a_2 < x < a_3 \\ 0; & \text{otherwise.} \end{cases}$$

Then by nearest interval approximation method, the lower limit C_L and upper limit C_R of the interval are

$$\begin{aligned} C_L &= \int_0^1 A_L(\alpha) d\alpha = \int_0^1 [a_1 + (a_2 - a_1)\alpha] d\alpha = \frac{a_1 + a_2}{2}, \\ C_R &= \int_0^1 A_R(\alpha) d\alpha = \int_0^1 [a_3 - (a_3 - a_2)\alpha] d\alpha = \frac{a_2 + a_3}{2}. \end{aligned}$$

Therefore, the interval number considering $\widetilde{A}$ as triangular fuzzy number is $\Big[\frac{a_1 + a_2}{2}, \frac{a_2 + a_3}{2}\Big]$.

3 Model Formulation

In this model, the scrap and re-workable items in the total lot is taken into account. The purpose of this EOQ model is to find out the optimum order quantity of inventory items by minimizing the total average cost. We discuss the model using the following notations and assumptions throughout the paper.

Notations : For the sake of clarity, the following notations are used throughout the paper.

y : Order size.
D : Demand rate.
c : Unit variable cost.
k : Fixed cost for placing an order.
P_S : Percentage of scrap items.
P_R : Percentage of re-workable items.
P : Percentage of scrape and re-workable items.
t_1 : Inspection period.
t_2 : Rework period.
t_3 : Remaining period to consume the entire inventory after receiving reworked items.
Z_1 : Inventory level after the inspection period.
Z_2 : Inventory level after the selling of the scrap items and return reworked items.
Z_3 : Inventory level just before receiving the reworked items.
Z_4 : Inventory level just after receiving the reworked items.
a : Unit selling price of items of good quality.
b : Unit selling price of items of scrap items.
x : Inspection rate.
L : Rework rate.
h : Unit holding cost.
d : Unit inspection cost.
R : Unit rework cost.
T : Cycle length.

Assumptions : The mathematical model is developed on the basis of the following assumptions:

(i) Shortages are not allowed.

(ii) The inspection and rework process is error free.

(iii) The quantity of good items is sufficient to satisfy the demand during the period of inspection.

Fig.1 represents the model where the lot size y is received with purchasing cost c per unit and

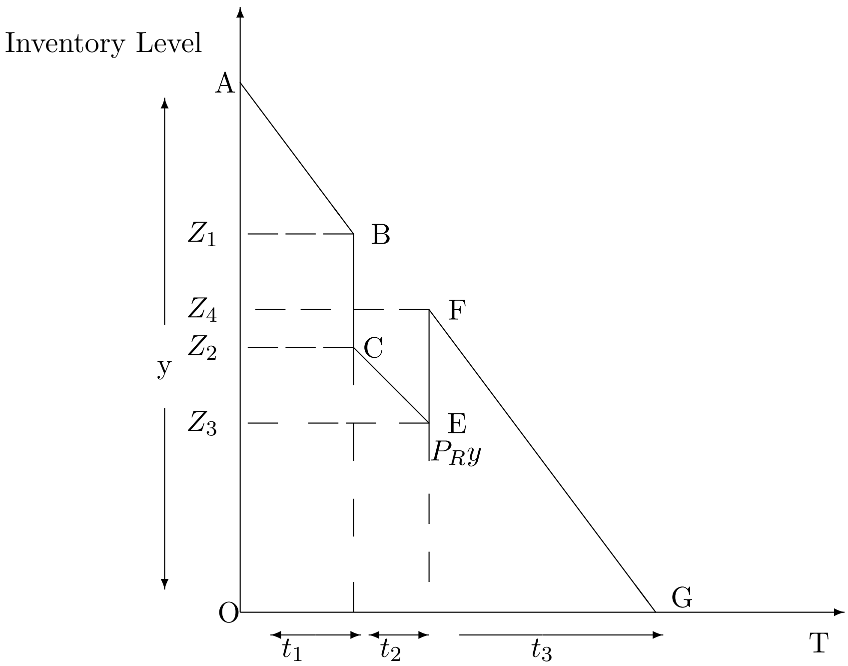

Figure 1: Inventory level over time

the ordering cost k per lot. the total lot goes through a 100 % inspection at a constant rate x. The inspection process takes place during time t_1. After inspection, it is found that there is a fraction of scrap (P_S) and re-workable (P_R) items. The re-workable items are re-worked at a cost R per unit. Re-work rate is L and the re-work process is done during time t_2. t_3 is the remaining period to consume the entire inventory after receiving reworked items. The total cycle length is T. To find the optimal order quantity, we minimize the total average cost.

According to the 1st assumption, to avoid shortage, the good items are at least equal to demand during inspection time. So,

$$\Big(1 - P_S - P_R\Big)y \geq Dt_1.$$

Obviously, $t_1 = \dfrac{y}{x}$. The inventory level just before the end of the inspection process is,

$$Z_1 = \left(1 - \frac{D}{x}\right) y.$$

t_2 is the time between the re-workable items are sent and they are received back given by $t_2 = \dfrac{1}{L} y P_R$. The inventory level after the removal of the scrap items and the return of the reworked items is

$$Z_2 = (1 - \frac{D}{x})y - Py = \left(1 - P - \frac{D}{x}\right) y.$$

Inventory level just before receiving the reworked items is

$$Z_3 = \left(1 - P - \frac{D}{x} - \frac{DP_R}{L}\right) y.$$

Inventory level when the reworked items are put in the inventory is

$$Z_4 = \left(1 - P_S - \frac{D}{x} - \frac{DP_R}{L}\right) y.$$

Finally, the time to consume Z_4 is $t_3 = \dfrac{Z_4}{D}$. The total cycle period is,

$$T = t_1 + t_2 + t_3 = \frac{(1 - P_S)}{D} y.$$

Obtaining the total cost $TC(y)$ (see the Appendix), the total average cost $C(y) = \frac{TC(y)}{T}$ is given by,

$$C(y) = \frac{kD}{(1 - P_S)y} + hy\left[\frac{(1 - P_S)}{2} + \frac{P_S D}{x(1 - P_S)} - \frac{DP_R^2}{L(1 - P_S)}\right]. \tag{14}$$

To maximize the order quantity, we have to minimize the total average cost. Using calculus method we optimize $C(y)$ and get the optimum values.

Up to this stage, we are assuming that the demand, ordering cost, holding cost etc. as real numbers i.e. of fixed value. But in real life business situations all these components are not always fixed, rather these are different in different situations. For example, the demand of a certain item vary due to various reasons. The carrying cost or the cost for placing an order may fluctuate due to changes in transportation charge, mailing charge, telephonic charge etc. Holding cost may change for preserving an item in winter, summer or rainy season. To overcome these ambiguities we approach with intuitionistic fuzzy variables, where demand and other cost components are considered as triangular fuzzy numbers.

Let us assume the fuzzy demand $\widetilde{D} = (D - \triangle D_1, D, D + \triangle D_2)$, fuzzy holding cost $\widetilde{h} = (h - \triangle h_1, h, h + \triangle h_2)$, fuzzy ordering cost $\widetilde{k} = (k - \triangle k_1, k, k + \triangle k_2)$. Replacing the real valued variables D, h, k by the triangular fuzzy variables $\widetilde{D}, \widetilde{h}, \widetilde{k}$ in Eq. (14) we get,

$$\widetilde{C}(y) = \frac{\widetilde{k}\widetilde{D}}{(1 - P_S)y} + \widetilde{h}y\left[\frac{(1 - P_S)}{2} + \frac{P_S \widetilde{D}}{x(1 - P_S)} - \frac{P_R^2 \widetilde{D}}{L(1 - P_S)}\right]. \tag{15}$$

where, $y \geq 0$. Now we represent the fuzzy EOQ model to a deterministic form so that it can be easily tackled. Following Grzegorzewski [27], the fuzzy numbers are transformed into interval numbers as

$\widetilde{D} = (D - \triangle D_1, D, D + \triangle D_2) \equiv [D_L, D_R]$,
$\widetilde{h} = (h - \triangle h_1, h, h + \triangle h_2) \equiv [h_L, h_R]$,

$\widetilde{k} = (k - \triangle k_1, k, h + \triangle k_2) \equiv [k_L, k_R]$.

Using the above expression (15) becomes

$$\widetilde{C}(y) = [f_L, f_R], \tag{16}$$

where,

$$f_L = \frac{k_L D_L}{(1-P_S)y} + h_L y\left[\frac{(1-P_S)}{2} + \frac{P_S D_L}{x(1-P_S)} - \frac{P_R^2 D_R}{L(1-P_S)}\right]. \tag{17}$$

and

$$f_R = \frac{k_R D_R}{(1-P_S)y} + h_R y\left[\frac{(1-P_S)}{2} + \frac{P_S D_R}{x(1-P_S)} - \frac{P_R^2 D_L}{L(1-P_S)}\right]. \tag{18}$$

The composition rules of intervals are used in these equations.

Hence the proposed model can be stated as

$$\text{Minimize}\{f_L(y), f_R(y)\}. \tag{19}$$

Generally, the multi-objective optimization problem (19), in case of minimization problem, can be formulated in a conservative sense from (10) as

$$\begin{gathered}\text{Minimize}\{f_C(y), f_R(y)\}. \\ \text{Subject to } 0 \le y, \text{ where, } \ f_C = \frac{f_L + f_R}{2}.\end{gathered} \tag{20}$$

Here the interval valued problem (19) is represented as

$$\begin{gathered}\text{Minimize } \{f_L(y), f_C(y), f_R(y)\} \\ \text{Subject to } \ y \ge 0.\end{gathered} \tag{21}$$

The expression (21) gives a better approximation than those obtained from (19). Moreover in this case the decision maker (DM) has the freedom to choose any one of the three functions f_L, f_C and f_R for minimization.

IF programming technique for solution

To solve multi-objective minimization problem given by (21), we have used the following IF programming technique.

For each of the objective functions $f_L(y), f_C(y), f_R(y)$, we first find the lower bounds L_L, L_C, L_R (best values) and the upper bounds U_L, U_C, U_R (worst values), where L_L, L_C, L_R are the aspired level achievement and U_L, U_C, U_R are the highest acceptable level achievement for the objectives $f_L(y), f_C(y), f_R(y)$ respectively and $d_k = U_k - L_k$ is the degradation allowance, or leeway, for objective $f_k(y), k = L, C, R$. Once the aspiration levels and degradation allowance for each of the objective function has been specified, we formed a fuzzy model and then transform the fuzzy model into a crisp model. The steps of intuitionistic fuzzy programming technique is given below.

Step 1: Solve the multi-objective cost function as a single objective cost function using one objective at a time and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From step 2, we find for each objective, the best L_k and worst U_k values corresponding to the set of solutions. The initial fuzzy model of (15) can then be stated as, in terms of the aspiration levels for each objective, as follows : Find y satisfying $f_k \widetilde{<} L_k$, $k = L, C, R$ subject to the non negativity conditions.

Step 4 : Define membership function $(\mu_{f_k}; k = L, C, R,)$ and a non membership function $(\nu_{f_k}; k = L, C, R,)$ for each objective function. An exponential membership function is defined by

$$\mu_{f_k} = \begin{cases} 1, & \text{if } f_k \leq L_k \\ \dfrac{e^{-w\left(\dfrac{f_k - L_k}{U_k - L_k}\right)} - e^{-w}}{1 - e^{-w}}, & \text{if } L_k \leq f_k \leq U_k \\ 0, & \text{if } f_k \geq U_k. \end{cases} \tag{22}$$

A quadratic non-membership function is defined by

$$\nu_{f_k} = \begin{cases} 0, & \text{if } f_k \leq L_k \\ \left(\dfrac{f_k - L_k}{U_k - L_k}\right)^2, & \text{if } L_k \leq f_k \leq U_k \\ 1, & \text{if } f_k \geq U_k. \end{cases} \tag{23}$$

μ_{f_k} is strictly monotonic decreasing function with property $\mu_f(L_k) = 1, \mu_f(U_k) = 0$, where as ν_{f_k} is a parabolic functions with property $\nu_f(L_k) = 0$ and $\nu_f(U_k) = 1$. These two functions are continuous within $[L_k, U_k]$. Therefore, quite naturally the functions meet at a point somewhere in $[L_k, U_k]$.

Step 5 : After determining the exponential membership and quadratic non-membership function defined in (22) and (23) for each objective functions following [12], [33] the problem (21) can be formulated an equivalent crisp model on the basis of definition 2 of this paper as

$$\begin{aligned} &\max \alpha, \quad \min \beta \\ &\alpha \leq \mu_{f_k}(x); \qquad k = L, C, R \\ &\beta \geq \nu_{f_k}(x); \qquad k = L, C, R \\ &\alpha \geq \beta; \qquad \text{and } \alpha + \beta \leq 1; \alpha, \beta \geq 0 \end{aligned}$$

where α denotes the minimal acceptable degree of objective(s) and constraints and β denotes the maximal degree of rejection of objective(s) and constraints. The IFO model can be changed into the following crisp (non-fuzzy) optimization model as :

$$\left.\begin{array}{ll} \max(\alpha-\beta) & \\ \alpha \le \mu_{f_k}(x); & k=L,C,R \\ \beta \ge \nu_{f_k}(x); & k=L,C,R \\ \alpha \ge \beta; & \text{and } \alpha+\beta\le 1; \alpha,\beta\ge 0 \end{array}\right\} \tag{24}$$

which can be written in the form

$$\left.\begin{array}{ll} \max(\alpha-\beta) & \\ \alpha \le \frac{e^{-w\left(\frac{f_L-L_L}{U_L-L_L}\right)}-e^{-w}}{1-e^{-w}}; & \beta \ge \left(\frac{f_L-L_L}{U_L-L_L}\right)^2 \\ \alpha \le \frac{e^{-w\left(\frac{f_C-L_C}{U_C-L_C}\right)}-e^{-w}}{1-e^{-w}}; & \beta \ge \left(\frac{f_C-L_C}{U_C-L_C}\right)^2 \\ \alpha \le \frac{e^{-w\left(\frac{f_R-L_R}{U_R-L_R}\right)}-e^{-w}}{1-e^{-w}}; & \beta \ge \left(\frac{f_R-L_R}{U_R-L_R}\right)^2 \\ \alpha\ge\beta, \alpha+\beta\le 1; & \text{and } \alpha,\beta\ge 0, y\ge 0 \end{array}\right\} \tag{25}$$

Step 6 : Now the above problem can be solved by a non-linear optimization technique and optimal solution of α, (say α^*) and β, (say β^*) are obtained.

Step 7 : Now after obtaining α^* and β^*, the DM selects the most important objective function from among the objective functions f_L, f_C and f_R. Here f_R is selected as DM would like to minimize his/her worst case. Then the problem becomes (for $\alpha=\alpha^*$ and $\beta=\beta^*$)

$$\left.\begin{array}{c} \min f_R \\ f_L\le m_L, \quad f_C\le m_C, \quad f_R\le m_R; \\ f_L\ge n_L, \quad f_C\ge n_C, \quad f_R\ge n_R; \\ y\ge 0; \quad \alpha\ge\beta; \\ \text{and} \quad \alpha+\beta\le 1; \quad \alpha,\beta\ge 0 \end{array}\right\} \tag{26}$$

where

$$\left.\begin{array}{c} m_L = L_L - \frac{U_L-L_L}{w}\left[\ln\{\alpha^*(1-e^{-w})\}+e^{-w}\right], \quad n_L = L_L+\sqrt{\beta^*}(U_L-L_L), \\ m_C = L_C - \frac{U_C-L_C}{w}\left[\ln\{\alpha^*(1-e^{-w})\}+e^{-w}\right], \quad n_C = L_C+\sqrt{\beta^*}(U_C-L_C), \\ m_R = L_R - \frac{U_R-L_R}{w}\left[\ln\{\alpha^*(1-e^{-w})\}+e^{-w}\right], \quad n_R = L_R+\sqrt{\beta^*}(U_R-L_R). \\ y\ge 0; \quad \alpha\ge\beta; \\ \text{and} \quad \alpha+\beta\le 1; \quad \alpha,\beta\ge 0. \end{array}\right\} \tag{27}$$

Step 8 : Pareto-optimal solution

Now after deriving the optimum decision variables, Pareto-optimality test is performed according to [13], let the decision vector y^* and the optimum values $f_L^* = f_L(y^*), f_C^* = f_C(y^*)$ and $f_R^* = f_R(y^*)$ are obtained from (26). With these values, the following problem is solving using a non-linear optimization technique

$$\left.\begin{array}{c} \min V = (\epsilon_L + \epsilon_C + \epsilon_R) \\ \text{subject to} \quad f_L + \epsilon_L = f_L^*, \quad f_C + \epsilon_C = f_C^*, \quad f_R + \epsilon_R = f_R^*; \\ \epsilon_L, \epsilon_C, \epsilon_R \geq 0, \quad y \geq 0, \quad \alpha \geq \beta \\ \text{and} \quad \alpha + \beta \leq 1; \quad \alpha, \beta \geq 0. \end{array}\right\} \tag{28}$$

The optimal solution of (28), say $y^{**}, f_L^{**}, f_C^{**}$ and f_R^{**} are called strong Pareto Optimal solution provided V is very small otherwise it is called weak Pareto solution.

4 Numerical Example

In this section, the above mentioned algorithm is illustrated by a numerical example.

Here we consider a fan dealing business where the parameters demand, ordering cost and holding cost are considered as triangular fuzzy numbers (TFN). After that, the fuzzy numbers are transformed into interval numbers using nearest interval approximation following [27]. The demand of fans per year is $\widetilde{D} = (40000, 50000, 60000)$. For each order the dealer pays cost $\widetilde{k} = \$(90, 100, 110)$. Purchasing cost of each fan is $c = \$25$. The annual holding cost of the item is $\widetilde{h} = \$(3.5, 4.5, 5.5)/unit$. After placing the order the items are checked at a constant rate, $x = 1unit/min = 175200unit/year$ and the cost for this inspection process is $d = \$0.5/unit$. Among the total lot fraction of scrap and re-workable items are $P_S = 0.125$ and $P_R = 0.05$ respectively.

Following [27], the fuzzy numbers $\widetilde{D}, \widetilde{h}$, and $\widetilde{k}$ are transformed into interval numbers as,

$$\left.\begin{array}{c} \widetilde{D} = [D_L, D_R] = [45000, 55000], \quad \text{since } \triangle_{\text{D1}} = \triangle_{\text{D2}} = 5000 \\ \widetilde{h} = [h_L, h_R] = \$[4, 5], \quad \triangle_{h1} = \triangle_{h2} = 0.5 \\ \widetilde{k} = [k_L, k_R] = \$[95, 105], \quad \triangle_{k1} = \triangle_{k2} = 5 \end{array}\right\} \tag{29}$$

Individual minimum and maximum of objective functions f_L, f_C, f_R are given in Table 1

Table 1: Individual minimum and maximum of objective functions

Objective functions	optimize f_L	optimize f_C	optimize f_R
f_L	$f_L' = 6065.300087$	$f_L'' = 6066.151774$	$f_L''' = 6067.945785$
f_C	$f_C' = 7011.893018$	$f_C'' = 7010.908545$	$f_C''' = 7011.480856$
f_R	$f_R' = 7958.486595$	$f_R'' = 7955.66597$	$f_R''' = 7955.01658$

Now we calculate
$L_L = \min(f'_L, f''_L, f'''_L) = 6065.300087$, $U_L = \max(f'_L, f''_L, f'''_L) = 6067.945785$,
$L_C = \min(f'_C, f''_C, f'''_C) = 7010.908545$, $U_C = \max(f'_C, f''_C, f'''_C) = 7011.893018$,
$L_R = \min(f'_R, f''_R, f'''_R) = 7955.01658$, $U_R = \max(f'_R, f''_R, f'''_R) = 7958.486595$.
Using the equation (25), we formulate the following problem as :

$$\left.\begin{array}{c}\max z = \alpha - \beta; \\ (1-e^{-w})\alpha \leq -e^{-w} + e^{-w\left(\frac{1846663.635}{y}+0.7115022y-2292.51414\right)}; \\ (1-e^{-w})\alpha \leq -e^{-w} + e^{-w\left(\frac{5833432.855}{y}+2.1734857y-7121.483824\right)}; \\ (1-e^{-w})\alpha \leq -e^{-w} + e^{-w\left(\frac{1902009.069}{y}+0.6907914y-2292.502073\right)}; \\ 6.9997179\beta y^2 \geq (4885714.286 + 1.88242y^2 - 6065.300087y)^2; \\ 0.969187\beta y^2 \geq (5742857.143 + 2.139738y^2 - 7010.908545y)^2; \\ 12.0410041\beta y^2 \geq (6600000 + 2.3970564y^2 - 7955.01658y)^2; \\ y \geq 0; \quad \alpha \geq \beta; \\ \text{and } \alpha + \beta \leq 1; \quad \alpha, \beta \geq 0 \end{array}\right\} \quad (30)$$

4.1 Results and Discussions

The solutions obtained from Eq.(30) is given in Table 2 – 4

Table 2: Optimal values of α and β

w	Maximum α	Minimum β
0.01	0.7490	0.0625

Table 3: Optimal results when f_C is chosen as the most important objective functions.

f_L^*	f_C^*	f_R^*	y^*
6605.511	7011.157	7956.8025	1624.540

Table 4: Pareto-Optimal results.

V	f_L^{**}	f_C^{**}	f_R^{**}	y^{**}
0.0004	6065.511	7011.1566	7956.8019	1624.540

In Table 4, the value of V is quite small and hence, the optimal results in Table 3 are strong Pareto-Optimal solution and can be accepted.

Now we derive the Pareto-Optimal results considering the components as fuzzy numbers i.e. only the exponential membership function is considered here.

Table 5: Optimal value of α

w	Maximum α
0.01	0.7406

Table 6: Optimal results when f_L is chosen as the most important objective functions.

f_L^*	f_C^*	f_R^*	y^*
6605.9616	7010.9224	7955.8839	1635.008

Table 7: Pareto-Optimal results.

V	f_L^{**}	f_C^{**}	f_R^{**}	y^{**}
0.0004	6065.3000	7011.8930	7958.4865	1611.032

Comparing results of fuzzy and intuitionistic fuzzy optimization we can see that intuitionistic fuzzy gives much better result than that of fuzzy.

5 Conclusion

This paper proposes a solution procedure for inventory model, where the parameters such as demand, holding cost and ordering cost are fuzzy numbers. In this model the total lot has a certain percentage of scrap and re-workable items. The scrap items are sold at a salvage cost in a secondary market whereas, the other items are sold at same price after making over the defects of the items. The concept of optimization in an intuitionistic fuzzy environment is introduced in this paper where, a degree of rejection is taken into account not only the acceptance.

There are various types of membership and non-membership functions in intuitionistic fuzzy optimization such as, (a) linear, (b) piecewise linear, (c) exponential, (d) hyperbolic, (e) logistic, (f) parabolic, (g) S-shaped etc. In most of the cases, linear membership and non-membership functions are used it is defined by fixing two points : (1) upper level of acceptance and (2) lower level of rejectability. The non-linear membership and non-membership functions provides a better approximation when the decision maker deals with intuitionistic fuzzy environment for describing the degree of preference or rejection. In this paper, the exponential membership and quadratic non-membership function is considered.

At first, expression for the total cost is developed containing fuzzy parameters. Then each fuzzy quantity is approximated by interval number. After that the problem of minimizing the cost function is transformed into a multi-objective inventory problem, where the objective functions are left limit, right limit and the center of the interval function. The intuitionistic fuzzy optimization technique is then used to found out the optimal result. The advantage of this method is, the decision maker can easily minimize the worse or maximize the better case. Also, he/she can get a strong Pareto-Optimal result by changing the membership and non-membership functions according to his/her choice.

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Appendix

The total cost comprises of four costs namely, procurement cost, rework cost, inspection cost and holding cost. Procurement cost $PC(y) = k + cy$; Rework cost $RC(y) = RP_R y$; Inspection cost $IC(y) = dy$ and the inventory holding cost is

$$\begin{aligned} HC(y) &= h \times \text{area } OABCEFG \\ &= h\left[\frac{(1-P_S)yT}{2} + \frac{y^2 P_S}{x} - \frac{y^2 P_R^2}{L}\right]. \end{aligned}$$

Therefore the total cost

$$\begin{aligned} TC(y) &= PC(y) + RC(y) + IC(y) + HC(y) \\ &= k + cy + RP_R y + dy + h\left[\frac{(1-P_S)yT}{2} + \frac{y^2 P_S}{x} - \frac{y^2 P_R^2}{L}\right]. \end{aligned}$$

Replacing T by $\frac{(1-P_S)y}{D}$, we get the cost function with variable y. Thus the total cost is,

$$C(y) = \frac{kD}{(1-P_S)y} + hy\left[\frac{(1-P_S)}{2} + \frac{P_S D}{x(1-P_S)} - \frac{DP_R^2}{L(1-P_S)}\right].$$