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Sequential Approach for Management of Renewable Natural Resources with Applications to Forestry [1]

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Abstract

We propose a sequential approach for creation of models of management of renewable natural resources. Namely, a long-term model evaluates the basic trajectory of the system and gives restrictions from the ecological point of view, i.e. constraints for providing sustainable development. Next, a middle-term model gives local optimal planning values from the economical point of view within the previous restrictions. Afterwards, a short-term model gives further specialization of solutions for the next closer planning period. This approach can be interpreted as an example of the sequential goal pursuit. We describe the models in the forest management setting.

Key words: Renewable natural resources; sequential approach; system of models; forest management.

1 Introduction

Renewable natural resources management plays a significant role in the sustainable development strategy adopted by many world and regional organizations, and the great number of works are devoted to this subject; see e.g. [Academy of Finland, 2005], [Auty, 2003], [EEA, 2005], [Lofdhal, 2002], [Ugurlu and Aladag, 2009] and the references therein. The most important instances of renewable resources are air, water, soil, forests, and also fish and related products. The complexity of the problems here stems from the fact that adequate models should take into account wide variety of interconnected factors from different spheres such as environment, economy, and society, within long run periods. For this reason, the popular approach, which is based on utility (cost) evaluation of various non-economic factors and investigation of low-dimensional dynamic systems (see e.g. [Auty, 2003], [Beltratti et al., 1998], [Clark, 1990]), does not seem sufficient for real management. For instance, the very strong assumption that

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each generation has a common fixed utility function for the same n fixed goods (see [Chichilnisky, 2004]) is not very realistic and can not be improved even by the very weak assumptions on the properties of this function. Simulation can also describe only some selected partial features of the system due to a great number of possible interconnected scenarios. Nevertheless, the necessity of utilization of just mathematical models is obvious, because the human influence on environment within the sustainable development requires certain definite rules with clear evaluations of various factors.

In this paper, we propose a general approach to modeling of renewable natural resources based on a hierarchical system of mathematical models, involving long-, middle- and short-term ones, so that solutions of the previous model are used to formulate some constraints for the next one. The key statement is that the basic environmental issues must have priority over economic factors in long-term models, besides, it seems non-realistic to suppose that we can derive suitable evaluations of economic factors as well as give cost evaluation of environmental factors for such models. However, this priority is relative, i.e. serves only for creation of environmental restrictions for the basic trajectories of the system for a long run period, which further admit profit maximization goals.

Clearly, management of different kinds of renewable natural resources may have essential distinctions. For this reason, we describe our approach with applications to forestry as one of most important areas of such resources. It should be noted that there exist a number of mathematical models reflecting different aspects of forest management; see e.g. [Mendoza and Vanclay, 2008], [Amacher et al., 2009] and the references therein. Some of these models are based on an hierarchical approach, but involve either geographical (local, country, regional) or temporal levels, with taking into account all the kinds of factors, which may lead to rather complex models; see e.g. [Tittler et al., 2001], [Gunn, 2009], [Hiltunen et al., 2012]. Our system of models gives an opportunity to somewhat arrange essential features of forest management such as long rotation ages, many possible landowners, carbon sequestration bounds, uncertainty of demand and prices for long runs, profit maximization, etc.

2 The basic approach and long-term models

In creation of models of management of renewable natural resources we first make an arrangement of goals. In fact, such models must take into account many factors from different areas. However, we think that sustainable development is the principal goal. At the same time, one can not find suitable evaluations of economic factors for long-term models. For this reason, we should first create a set of basic trajectories of the system from the ecological point of view.

For instance, we take dynamic forest management models. Then the minimal and maximal productive ages denoted by l and L, respectively, are rather large (usually $L \approx 120$ and $l \approx 60$ for pine trees). In long-term models, the planning horizon T

should be greater than the rotation age, usually $T \approx 200$; see e.g. [Gong et al., 2013], [Gunn, 2009]. Clearly, any economic evaluation of the trajectory for such period will seem artificial. We now describe our basic dynamic long-term model. We find it more suitable to consider a discrete time; i.e., we divide the time horizon into stages (years) $t = 1, 2, \dots, T$. Next, we suppose that the total forest territory is bounded above by S and is divided into many stands, each stand containing only trees of the same age $i = 1, \dots, L$. For simplicity, we suppose that there are only one kind of trees, since the case of many species only increases dimensionality within the same model. We denote by u^t and w^t the harvest and planting territory (square) vectors (say, in hectares) within the t-th time stage, respectively, and by v^t the forest territory vector at the beginning of the t-th stage, so that u^t, w^t, and v^t are vectors in $\mathbb{R}^L$. We denote by l_0 the maximal planting age. Next, the natural forest dynamics can be described by a change mapping A, i.e. $v^{t+1} = A(v^t)$ in the absence of harvesting and planting areas. For simplicity, we suppose that A is linear, then the forest square changes during one stage are represented by a square matrix A of order L, i.e.

$$A = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \dots & \alpha_{L-1} & \alpha_L \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix},$$

where $\alpha_i \geq 0$, $i = 1, \dots, L$ are sowing coefficients from different ages (clearly $\alpha_i = 0$ for certain small ages); see e.g. [Uusivuori and Kuuluvainen, 2005], [Amacher et al., 2009]. The model has the following main *constraints*:

$$v^{t+1} = A(v^t - u^t) + w^t, \quad t = 1, 2, \dots, T-1; \tag{1}$$

$$\sum_{i=1}^{L} v_i^t \leq S, \ t = 1, 2, \dots, T; \quad v^T \in V; \tag{2}$$

$$v^t \geq 0, \quad t = 1, 2, \dots, T; \ u^t \geq 0, \ w^t \geq 0, \quad t = 1, 2, \dots, T-1; \tag{3}$$

$$u_i^t = 0, \ i = 1, \dots, l-1; \ w_i^t = 0, \ i = l_0 + 1, \dots, L; \quad t = 1, 2, \dots, T-1; \tag{4}$$

where the starting vector v^0 is known, and V denotes the set of feasible states for v^T. Among various additional conditions, we impose only lower bounds (i.e. minimal feasible volumes) Γ_t of carbon sequestration per stage t and insert the constraints

$$\langle \gamma, v^t \rangle \geq \Gamma_t, \quad t = 1, 2, \dots, T; \tag{5}$$

where γ_i denotes the carbon sequestration volume from one hectare of forest of age i per stage, for $i = 1, \dots, L$, then $\gamma = (\gamma_1, \dots, \gamma_L)^\top$.

There exist several ways to define the goal function. Let us define the coefficients $\mu_i \geq 0$, $i = l, \dots, L$ and η_i, $i = 1, \dots, l_0$, where μ_i is the yield of timber (in m^3) from one hectare of forest of age i and η_i is the yield from one hectare of forest of age i after attaining age l. Then, we can consider the following *optimization problem*:

$$\text{maximize} \quad \sum_{t=1}^{T-1} \left(\sum_{i=l}^{L} \mu_i u_i^t - \sum_{i=1}^{l_0} \eta_i w_i^t \right) \tag{6}$$

subject to (1)–(5). If V is a polyhedral set, (1)–(6), is a linear programming (LP for short) problem. To smooth possible uneven stage distribution values, we can replace (6) with the following:

$$\text{maximize} \quad \sum_{i=l}^{L} \min_{t=1,\dots,T-1} (\mu_i u_i^t) - \sum_{i=1}^{l_0} \max_{t=1,\dots,T-1} (\eta_i w_i^t). \tag{7}$$

Clearly, problem (1)–(5), (7) also reduces easily to an LP one, if V is a polyhedral set. However, solutions may imply only harvest of one (maximal) age trees. To avoid this property, we can replace μ_i and η_i by more general weights $\tilde{\mu}_i$ and $\tilde{\eta}_i$, where $\tilde{\mu}_i$ reflects quality of timber of age i, ease of harvest of forest of age i, and its yield, whereas $\tilde{\eta}_i$ reflects efforts of planting of one hectare of forest of age i and its yield after attaining age l. Then, functions (6) and (7) are replaced by

$$\text{maximize} \quad \sum_{t=1}^{T-1} \left(\sum_{i=l}^{L} \tilde{\mu}_i u_i^t - \sum_{i=1}^{l_0} \tilde{\eta}_i w_i^t \right) \tag{8}$$

and

$$\text{maximize} \quad \sum_{i=l}^{L} \min_{t=1,\dots,T-1} (\tilde{\mu}_i u_i^t) - \sum_{i=1}^{l_0} \max_{t=1,\dots,T-1} (\tilde{\eta}_i w_i^t); \tag{9}$$

respectively. In such a way, we solve the scalar optimization problems (8) (or (9)) subject to (1)–(5).

If we need a stationary strategy we can impose the additional requirements, say, set either $u^t = u$ and $w^t = w$ or $u^t = \sigma_t u^0$ and $w^t = \tau_t w^0$ where the predefined profiles u^0 and w^0 are fixed in conformity of some rotation age rule (see e.g. [Amacher et al., 2009]) and σ_t and τ_t are scalar variables. For instance, the first additional requirement for (8), (1)–(5) is then rewritten as follows:

$$\text{maximize} \quad \sum_{i=l}^{L} \tilde{\mu}_i u_i - \sum_{i=1}^{l_0} \tilde{\eta}_i w_i$$

subject to

$$v^{t+1} = A(v^t - u) + w, \quad t = 1, 2, \dots, T-1;$$
$$\sum_{i=1}^{L} v_i^t \leq S, \ t = 1, 2, \dots, T; \quad v^T \in V,$$

$$\langle \gamma, v^t \rangle \geq \Gamma_t, \quad t = 1, 2, \dots, T;$$
$$v^t \geq 0, \quad t = 1, 2, \dots, T; \ u \geq 0, \ w \geq 0;$$
$$u_i = 0, \ i = 1, \dots, l-1; \ w_i = 0, \ i = l_0 + 1, \dots, L.$$

The choice of the main problem and its possible further modifications will depend on peculiarities of the region under consideration.

Clearly, we can utilize various efficient methods to find solutions of the above problems, even in the case of high dimensionality; see e.g. [Boyd and Vandenberghe, 2004], [Minoux, 1989], [Polyak, 1983]. These solutions enable us to find the basic trajectory $\{\bar{v}^t\}$ and feasible bounds $\{\bar{u}^t\}$ and $\{\bar{w}^t\}$ for harvest and planting territory (square) vectors at each stage.

3 Middle-term models

The main difference of the middle-term models from the previous ones consists in the opportunity to utilize some values of economic factors for evaluation of the management strategy. That is, now our goal is to maximize the total net present profit subject to similar constraints. Besides, we must insert the bounds derived from the solution of a previous long-term problem. The presence of such bounds is the main difference from the known forest management models; see e.g. [Uusivuori and Kuuluvainen, 2005], [Gunn, 2009], [Amacher et al., 2009]. As a result, we specialize the basic trajectory of the system enhanced by economic factors for some planning horizon $T' \ll T$.

Let us consider the t-th time stage. We denote by π_i^t and d_i^t the price of one unit and demand (in m^3) of timber from the forest of age i, respectively, for $i = 1, \dots, L$. Next, c_i^1, c_i^2, and c_i^3 denote costs of maintenance, harvesting and deliverance, and planting of one hectare of forest of age i, respectively, for $i = 1, \dots, L$. Also, ν denotes the discount rate.

We suggest two approaches to utilization bounds from the long-term models. The first one utilizes only the set V' of feasible states for $v^{T'}$, which is nothing but some neighborhood of $\bar{v}^{T'}$. This model has the following *constraints*:

$$v^{t+1} = A(v^t - u^t) + w^t, \quad t = 1, 2, \dots, T' - 1; \tag{10}$$
$$\sum_{i=1}^{L} v_i^t \leq S, \ t = 1, 2, \dots, T'; \quad v^{T'} \in V'; \tag{11}$$
$$\langle \gamma, v^t \rangle \geq \Gamma_t, \ v^t \geq 0, \quad t = 1, 2, \dots, T'; \tag{12}$$
$$d_i^t / \mu_i \geq u_i^t \geq 0, \ i = l, \dots, L; \ w^t \geq 0, \quad t = 1, 2, \dots, T' - 1; \tag{13}$$
$$u_i^t = 0, \ i = 1, \dots, l-1; \ w_i^t = 0, \ i = l_0 + 1, \dots, L; \quad t = 1, 2, \dots, T' - 1; \tag{14}$$

with the known starting state vector v^0. Hence, we can define the following present *profit maximization problem*:

$$\text{maximize} \quad \sum_{t=1}^{T'-1} \nu^t \left\{ \sum_{i=l}^{L} \pi_i^t \mu_i u_i^t - \sum_{i=1}^{L} c_i^1 (v_i^t - u_i^t) - \sum_{i=l}^{L} c_i^2 u_i^t - \sum_{i=1}^{l_0} c_i^3 w_i^t \right\} \tag{15}$$

subject to (10)–(14).

The second approach utilizes the harvest and planting bounds $\{\bar{u}^t\}$ and $\{\bar{w}^t\}$. Then the model has simpler *constraints*:

$$v^{t+1} = A(v^t - u^t) + w^t, \quad t = 1, 2, \dots, T' - 1; \tag{16}$$

$$\sum_{i=1}^{L} v_i^t \le S, \ t = 1, 2, \dots, T'; \tag{17}$$

$$v^t \ge 0, \quad t = 1, 2, \dots, T'; \ \bar{z}^t \ge u^t, \ w^t \ge \bar{w}^t, \quad t = 1, 2, \dots, T - 1; \tag{18}$$

$$u_i^t = 0, \ i = 1, \dots, l - 1; \ w_i^t = 0, \ i = l_0 + 1, \dots, L; \quad t = 1, 2, \dots, T' - 1; \tag{19}$$

where $\bar{z}_i^t = \min\{\bar{u}_i^t, d_i^t/\mu_i\}$ for $i = 1, \dots, L$. Then the problem utilizes again the profit maximization goal from (15) subject to (16)–(19). Clearly, (15)–(19) is a LP problem, and so is (10)–(15) if V' is a polyhedral set. Then they admit many efficient methods to find their solutions. In such a way we obtain a specialization of the basic trajectory $\{\tilde{v}^t\}$ and new feasible bounds $\{\tilde{u}^t\}$ and $\{\tilde{w}^t\}$ for harvest and planting territory (square) vectors at each stage.

4 Short-term models

These models usually reflect many aspects of forest management in more detail; see e.g. [Gunn, 2009], [Amacher et al., 2009]. We now present two possible models within one year associated with stage t^* of the previous models. Both the models utilize the harvest and planting bounds $\bar{u}_i$ and $\bar{w}_i$ for $i = 1, \dots, L$ whose definition depends on the approach to utilization of the middle-term models of Section 3.

(i) The first approach relays upon problem (10)–(15), which yields solutions $\{\tilde{u}^t\}$ and $\{\tilde{w}^t\}$, and utilizes the set V' of feasible states for $v^{T'}$. Then we set $\bar{u}_i = \min\{\bar{u}_i^{t^*}, \tilde{u}_i^{t^*}\}$ and $\bar{w}_i = \max\{\bar{w}_i^{t^*}, \tilde{w}_i^{t^*}\}$ for $i = 1, \dots, L$, i.e. correct the bounds by taking into account the middle-term profit maximization strategy.

(ii) The second approach relays upon problem (15)–(19), which yields solutions $\{\tilde{u}^t\}$ and $\{\tilde{w}^t\}$ without some pre-defined feasible state set $v^{T'}$. Then we set $\bar{u}_i = \max\{\bar{u}_i^{t^*}, \tilde{u}_i^{t^*}\}$ and $\bar{w}_i = \min\{\bar{w}_i^{t^*}, \tilde{w}_i^{t^*}\}$ for $i = 1, \dots, L$, i.e. the correction is optional and solutions of problem (15)–(19) serve mainly for control of deviations between results of short-term and middle-term models forecasted to the stage under consideration. Large deviations may cause corrections of some coefficients in the middle-term model.

We describe models based on profit maximization for different organizations of the forestry sector. The *first short-term model* is destined for maximization of the total profit of traders with taking into account shipment costs. We suppose there exist m landowners of the forest territory who can cut trees and sell timber and n consumers (buyers). The problem is formulated as follows:

$$\begin{aligned} \text{maximize} \quad & \sum_{i=l}^{L} \left\{ \sum_{s=1}^{n} \varphi_s^i(y_{is}) - \sum_{k=1}^{m} \sum_{s=1}^{n} \sigma_{ks}^i x_{ks}^i \right\} \\ & - \sum_{k=1}^{m} \left\{ \sum_{i=1}^{L} c_{ik}^1 (v_{ik} - u_{ik}) + \sum_{i=l}^{L} c_{ik}^2 u_{ik} + \sum_{i=1}^{l_0} c_{ik}^3 w_{ik} \right\} \end{aligned} \tag{20}$$

subject to

$$\sum_{s=1}^{n} x_{ks}^i = \mu_i u_{ik}, \quad i = l, \dots, L,\ k = 1, \dots, m; \tag{21}$$

$$\sum_{k=1}^{m} x_{ks}^i = y_{is}, \quad i = l, \dots, L,\ s = 1, \dots, n; \tag{22}$$

$$\sum_{i=1}^{L} (1 + \alpha_i)(v_{ik} - u_{ik}) + \sum_{i=1}^{l_0} w_{ik} \le S_k, \quad k = 1, \dots, m; \tag{23}$$

$$\sum_{k=1}^{m} u_{ik} \le \bar{u}_i, \quad i = 1, \dots, L; \quad \sum_{k=1}^{m} w_{ik} \ge \bar{w}_i, \quad i = 1, \dots, l_0; \tag{24}$$

$$u_{ik} \le v_{ik},\ u_{ik} \ge 0,\ w_{ik} \ge 0,\ i = 1, \dots, L, k = 1, \dots, m; \tag{25}$$

$$y_{is} \ge 0,\ i = l, \dots, L, s = 1, \dots, n; \tag{26}$$

besides, we set $u_{ik} = 0$ for $i = 1, \dots, l-1$, $k = 1, \dots, m$ and $w_{ik} = 0$ for $i = l_0+1, \dots, L$, $k = 1, \dots, m$.

Here, in addition to the previous notation, v_{ik}, u_{ik}, and w_{ik} denote the starting, harvest and planting square of forest of age i for owner k, $i = 1, \dots, L$; S_k denotes the total square of forest for owner k, $k = 1, \dots, m$; $\alpha_i \ge 0$ is the sowing coefficient from age i and $\mu_i \ge 0$ is the yield from one hectare of forest of age i for $i = 1, \dots, L$. Next, y_{is} is the volume of timber of age i purchased by consumer s and $\varphi_s^i(y_{is})$ is the amount paid by consumer s for the volume y_{is} of timber of age i for $i = l, \dots, L$, $s = 1, \dots, n$. Also, c_{ik}^1, c_{ik}^2, and c_{ik}^3 denote costs of maintenance, harvesting, and planting (regeneration) for one hectare of forest of age i for owner k, respectively, for $i = 1, \dots, L$, $k = 1, \dots, m$; and σ_{ks}^i denotes the cost of shipping of one unit (m^3) of timber of age i from owner k to consumer s for all $i = l, \dots, L$, $k = 1, \dots, m$, and $s = 1, \dots, n$.

Note that (21) and (22) represent the usual balance equations, (23) gives the restriction for total square of forest for owner k, whereas (24) gives total restrictions (bounds) for harvesting and planting squares of forest of age i. If the functions φ_s^i are concave (affine), (20)–(26) is a convex (linear) programming problem with the feasible polyhedral set. That is, there exist many efficient methods to find its solution; see e.g. [Boyd and Vandenberghe, 2004], [Minoux, 1989], [Polyak, 1983].

The *second short-term model* is destined for maximization of the profit of each of m traders (owners of the forest) with taking into account shipment costs, but within a multi-commodity imperfect competition (oligopolistic) timber market; see e.g. [Okuguchi and Szidarovszky, 1990]. The consumer side is presented in less detailed form via the price functions π^i for $i = l, \dots, L$, where $\pi^i(y_i)$ denotes the consumer's price for the volume y_i of timber of age i. We denote by σ_{ik} the cost of shipping of one unit (m^3) of timber of age i from trader k to the market (consumer) for all $i = l, \dots, L$ and $k = 1, \dots, m$. Also, set

$$u_k = (u_{ik})_{i=l,\dots,L} \text{ and } w_k = (w_{ik})_{i=l,\dots,L} \text{ for } k = 1, \dots, m;$$

and

$$u = (u_k)_{k=1,\dots,m}, \; w = (w_k)_{k=1,\dots,m}.$$

The other notation is the same as above. Then the problem is formulated as a constrained m-person non-cooperative game, where the k-th player (trader) has the utility (profit) function:

$$\begin{aligned} f_k(u, w_k) = \; & \sum_{i=l}^{L} (\mu_i u_{ik}) \left\{ \pi_i \left(\mu_i \sum_{s=1}^{m} u_{is} \right) - \sigma_{ik} \right\} \\ & - \sum_{i=1}^{L} c^1_{ik}(v_{ik} - u_{ik}) - \sum_{i=l}^{L} c^2_{ik} u_{ik} - \sum_{i=1}^{l_0} c^3_{ik} w_{ik}. \end{aligned} \tag{27}$$

The control variables must satisfy the joint and independent constraints (23)–(25). For the sake of clarity, we rewrite them here again:

$$\sum_{i=1}^{L} (1 + \alpha_i)(v_{ik} - u_{ik}) + \sum_{i=1}^{l_0} w_{ik} \le S_k, \quad k = 1, \dots, m; \tag{28}$$

$$\sum_{k=1}^{m} u_{ik} \le \bar{u}_i, \quad i = 1, \dots, L; \quad \sum_{k=1}^{m} w_{ik} \ge \bar{w}_i, \quad i = 1, \dots, l_0; \tag{29}$$

$$u_{ik} \le v_{ik}, \; u_{ik} \ge 0, \; w_{ik} \ge 0, \; i = 1, \dots, L, k = 1, \dots, m; \tag{30}$$

besides, we set $u_{ik} = 0$ for $i = 1, \dots, l-1$, $k = 1, \dots, m$ and $w_{ik} = 0$ for $i = l_0+1, \dots, L$, $k = 1, \dots, m$.

Due to the binding constraints (29), this is not a pure Nash equilibrium problem, because the feasible set $X \subset \mathbb{R}^{mL} \times \mathbb{R}^{mL}$ defined by the constraints (28)–(30) is not a Cartesian product. Nevertheless, this set is a convex polyhedron and problem (27)–(30) reduces easily to a general equilibrium problem or a variational inequality under the natural additional assumptions such as concavity of each function f_k in (u_k, w_k); see e.g. [Rosen, 1965], [Zukhovitskii et al., 1971], [Konnov, 2001]. In turn, these problems admit many iterative solution methods. Following [Belen'kii and Volkonskii, 1974], Section 11, we describe here for illustration only an extension of the known fictitious play method, which generates a sequence of iterates $\{(u^l, w^l)\}$ starting from a point $(u^0, w^0) \in X$.

Choose a positive sequence $\{\theta_l\}$ such that

$$0 \le \theta_l \le 1, l = 0, 1, \dots, \lim_{l \to \infty} \theta_l = 0, \sum_{l=0}^{\infty} \theta_l = \infty.$$

Given a point (u^l, w^l), we first solve the problem:

$$\begin{aligned} \text{maximize} \quad \Phi_l(u, w) = \; & \sum_{k=1}^{m} \left\{ \sum_{i=l}^{L} (\mu_i u_{ik}) \left[\pi_i \left(\mu_i \sum_{s=1}^{m} u_{is}^l + \mu_i u_{ik} \right) - \sigma_{ik} \right] \right. \\ & \left. - \sum_{i=1}^{L} c_{ik}^1 (v_{ik} - u_{ik}) - \sum_{i=l}^{L} c_{ik}^2 u_{ik} - \sum_{i=1}^{l_0} c_{ik}^3 w_{ik} \right\} \end{aligned}$$

subject to (28)–(30) and denote its solution by $(\bar{u}^l, \bar{w}^l)$. The next iterate (u^{l+1}, w^{l+1}) is defined as follows:

$$u^{l+1} := \theta_l \bar{u}^l + (1 - \theta_l) u^l, \quad w^{l+1} := \theta_l \bar{w}^l + (1 - \theta_l) w^l.$$

It was proved in [Belen'kii and Volkonskii, 1974], Section 11, that the sequence of iterates $\{(u^l, w^l)\}$ will converge to an equilibrium point under rather general assumptions.

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