An inventory model for deteriorating items with fuzzy random planning horizon

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Abstract

An inventory model for deteriorating items over a fuzzy random planning horizon incorporating inflation and time value of money is studied. It is assumed that planning horizon follows exponential distribution with imprecise parameter. The model is formulated using cost minimization principle. As the fuzzy cost cannot be minimized directly, the possibilistic mean value of the fuzzy cost is found and then optimized. The model is illustrated with a numerical example.

Keywords: Fuzzy Random Variable; Planning Horizon; Possibilistic Mean Value.

1 Introduction

In the existing literature, inventory models are generally developed under the assumption of finite or infinite planning horizon. Extensive research work has been done in this direction and are available in the standard literatures [4, 22, 28, 30]. But there are many real life situations where these assumptions are not valid, e.g., for a seasonal product, though planning horizon is normally assumed as finite and crisp in nature, but, in every year it fluctuates depending upon the environmental effects and it is better to estimate this horizon as a stochastic parameter with some feasible distribution. In 1983, Gurnani [21] pointed out that an infinite planning horizon is of rare occurrence because the costs are likely to vary disproportionately and because of change in product specifications and design or its abandonment or substitution by another product due to rapid development of technology. Chung and Kim [11] also suggested that the assumption of the infinite planning horizon is not realistic and called for a new model which relaxes the assumption of the infinite planning horizon. Moon and Yun [27] developed an Economic Order Quantity(EOQ) model where the planning horizon is a random variable following exponential distribution. Moon and Lee [26] developed an EOQ model for an item with random lifetime of the product under inflation and time value of money. They assumed exponential and normal distribution for the life time of the item and proposed simulation approach for solution when expected value of the objective is difficult for calculation.

During the last few decades, the monetary situation of most of the countries, affluent or otherwise, has changed a lot due to large scale inflation and consequent sharp decline in the purchasing power of money. As a result, several efforts have been made by researchers to reformulate the optimal inventory management policies

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taking into account inflation, time value of money, etc. The initial attempt in this direction was made by Buzacott [7] in 1975. He dealt with an EOQ model with inflation subject to different types of pricing policies. In the subsequent year, Bierman *et al.* [5] showed that the inflation rate does not affect the optimal order quantity perse; rather, the difference between the inflation rate and the discount rate affects the optimum order quantity. Several authors then extended these works to make the more realistic inventory model under inflation. Datta and Pal [12] studied the effect of internal and external inflation in an inventory model with time dependent demand rate, Dey *et al.* [13] considered inflationary effect when lead-time is fuzzy. Maiti *et al.* [25] developed a two storage inventory model with random planning horizon. Roy *et al.* [29] developed a production inventory model with stock dependent demand incorporating learning and inflationary effect in a random planning horizon. Recently, Huang and Ahmed [24] developed a stochastic programming approach for planning horizons of infinite horizon capacity planning problems.

Inventory models with crisp, stochastic and fuzzy parameters were studied by several authors. These models have been developed by considering the parameters as crisp and the remaining as either stochastic [22, 28] or fuzzy in nature [18, 2, 3] etc. Models, with inventory paremeters that are fuzzy random in nature have not been studied much, as yet. Dutta *et al.* [17] were the first to incorporate demand as fuzzy random variable in a simple newsboy problem with fuzzy random demand. Later on, a fuzzy mixture inventory model involving fuzzy random lead time demand has been developed by Chang *et al.* [10]. Bag *et al.* [1] developed a production inventory model with fuzzy random demand and with flexibility and reliability consideration. Dey and Chakraborty [14] developed fuzzy periodic review system with fuzzy random variable demand. Dey and Chakraborty [15] developed a fuzzy random continuous review inventory model. But, planning horizon as a fuzzy random variable is yet to be considered.

When we estimate a parameter θ of a distribution from statistical data we either find point estimation or find interval estimation of the parameter. We never expect that the computed point estimation of θ from a set of statistical data will be equal to the exact value of the parameter θ. So we often find $(1-\beta)100\%$ confidence interval of the parameter, where choice of β depends on the estimator. Denoting these confidence intervals as $[\theta_1(\beta), \theta_2(\beta)]$, Buckley [6] pointed out that if all these confidence intervals (for $0 < \beta \leq 1$) are placed one on top of the other, one can get a triangular shaped fuzzy number as an estimated value of the parameter θ, with α-cuts $[\theta_1(\alpha), \theta_2(\alpha)]$ for $0 < \alpha \leq 1$. So, when some parameters of an inventory problem are random in nature it is better to estimate the parameters of the corresponding distribution as fuzzy rather crisp (as they are estimated from past data or from experts opinion). As a result, it is better to estimate the planning horizon of a seasonal product as fuzzy random rather random. Though some models have been developed under the assumption of random planning horizon [27] none has considered this horizon as fuzzy random.

In real world problem, deterioration is also a natural phenomenon. There are some physical goods which deteriorate with the progress of time during their normal storage. In this area, a lot of research work have been published by several researchers [20, 19, 9].

Here, an inventory model for deteriorating items is developed considering the effect of inflation and time value of money on different inventory costs. It is assumed that planning horizon is fuzzy random in nature and follows exponential distribution with imprecise parameter. The model is formulated to minimize the total expected cost from the system for the planning horizon. Alpha cut of the expected cost is obtained following Buckley [6] and then the possibilistic mean value of the fuzzy cost [8] is found and then optimized using LINGO software. The model is illustrated with a numerical example. The outline of this paper is as follows. Section 2 contains discussion on the basics of fuzzy set theory connecting to this work. Section 3 contains relevant assumption and notations connected to the model. Section 4 presents the mathematical formulation and analysis of the proposed inventory model with fuzzy random planning horizon. Section 5 is the illustration with numerical example. Final Section 6 contains the concluding remarks.

2 Mathematical Preliminaries

2.1 Fuzzy Number

[16]

A fuzzy number $\widetilde{A}$ is a fuzzy set on the real line, $\Re$, if its membership function μ has the following properties:

(i) $\mu_{\widetilde{A}}(x)$ is upper semi continuous,

(ii) $\mu_{\widetilde{A}}(x)$ = 0, outside some interval [a_1, a_4],

(iii) there exist real numbers a_2 and a_3, $a_1 \leq a_2 \leq a_3 \leq a_4$ such that $\mu_{\widetilde{A}}(x)$ is increasing on [a_1, a_2], decreasing on [a_3, a_4], and $\mu_{\widetilde{A}}(x)$ = 1 for each x in [a_2, a_3].

2.2 Triangular Fuzzy Number (TFN)

[16]

A TFN $\widetilde{A}$ is specified by the triplet (a_1, a_2, a_3) where $a_1 < a_2 < a_3$ and is defined by its continuous membership function $\mu_{\tilde{A}}(x)$: $X \rightarrow$ [0, 1] as follows:

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & \text{if } \; a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{if } \; a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

2.3 α-cuts

[16]

α-cuts of a fuzzy set $\tilde{A}$ in X is a crisp subset of X denoted by $A(\alpha)$ and is defined by $A(\alpha)$ = { $x \in X$ / $\mu_{\tilde{A}}(x) \geq \alpha$ }, $\forall$ $\alpha \in$ [0, 1]. An α-cut of $\tilde{A}$=(a_1, a_2, a_3) can be expressed by the following interval $A(\alpha)$=[a_1+(a_2-a_1)α, a_3-(a_3-a_2)α], $\alpha \in$[0, 1]. Let F be the set of all fuzzy numbers. Then for any $\tilde{A}$, $\tilde{B} \in F$ and for any $\lambda \in R$, ($\tilde{A}$+$\tilde{B}$)(α)=$A(\alpha)$+$B(\alpha)$, ($\tilde{A}\tilde{B}$)(α)=$A(\alpha)B(\alpha)$, (λ $\tilde{A}$)(α)=λ $A(\alpha)$.

2.4 Fuzzy Random Variable[6]

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite set and P be a probability function defined on all subsets of X with $P\{x_i\} = a_i$, $1 \leq i \leq n$, $0 < a_i < 1$ for all i and $\sum_{i=1}^{n} a_i = 1$. X together with P is a discrete probability distribution. In practice all the a_i values must known exactly. Many times these values are estimated (as discussed in introduction), or they are provided by experts. We now assume that some of these a_i values are uncertain and we model this uncertainty using fuzzy numbers, i.e., we assume a_i as fuzzy number $\tilde{a}_i$. Then X together with $\tilde{a}_i$ values is a discrete (finite) fuzzy probability distribution and corresponding random variable is called discrete fuzzy random variable. In this case we write $\tilde{P}$ for fuzzy P and we have $\tilde{P}\{x_i\} = \tilde{a}_i$, $1 \leq i \leq n$,

$0 < \tilde{a}_i < 1, 1 \leq i \leq n.$
According to this definition if $A = \{x_1, x_2, ..., x_k\} \subset X$, then

$$\tilde{P}(A)[\alpha] = \left\{ \sum_{i=1}^{k} a_i | S \right\}$$

for $0 \leq \alpha \leq 1$, where S stands for the statement $\{a_i \in \tilde{a}_i[\alpha], 1 \leq i \leq n$ and $\sum\limits_{i=1}^{n} a_i = 1\}$.
Let X be a random variable with probability density function $f(x;\theta)$, where $\theta = (\theta_1, \theta_2, ..., \theta_n)$ is a vector of parameters. Then we know that probability of the event $X \in A$ is denoted by $P(X \in A)$ and defined as

$$P(X \in A) = \int_A f(x;\theta)dx$$

When θ is fuzzy number $\tilde{\theta}$ then X is a fuzzy random variable with probability density function $f(x;\tilde{\theta})$ and in that case probability of the event $X \in A$ is denoted by $\tilde{P}(X \in A)$ and defined as

$$\tilde{P}(X \in A)[\alpha] = \left\{ \int_A f(x;\tilde{\theta})dx | S \right\}$$

where S stands for the statement $\{\theta_i \in \tilde{\theta}_i[\alpha], 1 \leq i \leq n\}$.

2.5 Possibilistic Mean Value of a Fuzzy Number

[8]

The interval valued possibilistic mean is defined as $M(\widetilde{A})$=[$M_*(\widetilde{A})$, $M^*(\widetilde{A})$]. Where $M_*(\widetilde{A})$ and $M^*(\widetilde{A})$ are the lower and upper possibilistic mean values of $\widetilde{A}$ and are respectively defined by

$$M_*(\widetilde{A}) = \frac{\int_0^1 \alpha A_L(\alpha)d\alpha}{\int_0^1 \alpha d\alpha}, M^*(\widetilde{A}) = \frac{\int_0^1 \alpha A_R(\alpha)d\alpha}{\int_0^1 \alpha d\alpha}.$$

The possibilistic mean values of $\widetilde{A}$ is defined as

$$\overline{M}(\widetilde{A}) = \frac{M_*(\widetilde{A}) + M^*(\widetilde{A})}{2}$$

In other words, one can write

$$\overline{M}(\widetilde{A}) = \int_0^1 \alpha(A_L(\alpha) + A_R(\alpha))d\alpha$$

3 Notations and Assumptions

The following notations and assumptions are used in developing the model.

3.1 Notations

(i) $q(t)$=the inventory level at time t

(ii) H =the planning horizon,which is fuzzy random in nature and h is the real variable corresponding to H

(iii) D = demand of the item per unit time

(iv) N = the number of fully accommodated cycles to be made during the time horizon

(v) T =the length of a cycle

(vi) Q = the order quantity

(vii) β = the deterioration rate

$(viii)$ t_l = the last cycle length. So t_l=h-NT

(ix) c_p = the purchase cost per unit quantity

(x) A = the ordering cost

(xi) h = the holding costs per unit quantity per unit time

(xii) i and r are the inflation and discount rate respectively and R=r-i

3.2 Assumptions

(i) Inventory system involves only one deteriorating item

(ii) Planning horizon H is stochastically governed by exponential distribution with fuzzy parameter $\tilde{\lambda}$, its density function $f(h) = \tilde{\lambda}e^{-\tilde{\lambda}h}$, where h is the real variable corresponding to H. Here it is assumed that $\tilde{\lambda}$ is a triangular fuzzy number with components (a, b, c)

(iii) Ordering cost A linearly depends on order quantity, i.e., of the form A=c_o+c_1Q where c_o and c_1 are constants

(iv) Time horizon (h) ends during the $(N+1)^{th}$ cycle and remaining items in hand at the end of the cycle is assumed as lost

(v) Here T is the only decision variable

4 Mathematical Formulation and Analysis

In the development of the model, we assume that at the beginning of every cycle, company purchases an amount Q units of the item. It is also assumed that $r > i$, i.e., the interest rate is larger than the inflation rate, which is a practical assumption.
So instantaneous state $q(t)$ of the item during j^{th} cycle, $(j-1)T \leq t \leq jT$ (j=1, 2, 3,...,N) is given by

$$\frac{dq(t)}{dt} = -D - \beta q \quad \text{for} \quad (j-1)T \leq t \leq jT \tag{1}$$

with boundary conditions $q((j-1)T) = Q$, $q(jT) = 0$.
Solving we get

$$\begin{aligned} q(t) &= -\frac{D}{\beta} + (Q + \frac{D}{\beta})e^{\beta\{(j-1)T-t\}} \quad \text{for} \quad (j-1)T \le t \le jT \\ Q &= \frac{D}{\beta}(e^{\beta T} - 1) \\ \text{Hence } q(t) &= \frac{D}{\beta}\left(e^{\beta(jT-t)} - 1\right) \quad \text{for} \quad (j-1)T \le t \le jT \end{aligned}$$

Present value of holding cost during j^{th} cycle, H_j, is given by

$$\begin{aligned} H_j &= h\int\limits_{(j-1)T}^{jT} \frac{D}{\beta}\left(e^{\beta(jT-t)} - 1\right)e^{-Rt}dt \\ &= h\left[\frac{De^{\beta T}}{\beta(\beta+R)} - \frac{D}{\beta R} + \frac{De^{-RT}}{R(\beta+R)}\right]e^{-(j-1)RT} \end{aligned}$$

The present value of total ordering, purchase costs up to the beginning of $(N+1)^{th}$ cycle is

$$\begin{aligned} C_1 &= \sum_{i=0}^{N}\left[(c_o + c_1 Q) + c_p Q\right]e^{-iRT} \\ &= \left[c_o + \frac{(c_1+c_p)D(e^{\beta T}-1)}{\beta}\right]\sum_{i=0}^{N} e^{-iRT} \\ &= \left[c_o + \frac{(c_1+c_p)D(e^{\beta T}-1)}{\beta}\right]\left[\frac{1-e^{-R(N+1)T}}{1-e^{-RT}}\right] \end{aligned}$$

The present value of total holding cost up to the beginning of $(N+1)^{th}$ cycle is

$$\begin{aligned} C_2 &= \sum_{j=1}^{N} H_j \\ &= hD\left[\frac{e^{\beta T}}{\beta(\beta+R)} - \frac{1}{\beta R} + \frac{e^{-RT}}{R(\beta+R)}\right]\sum_{i=0}^{N-1} e^{-iRT} \\ &= hD\left[\frac{e^{\beta T}}{\beta(\beta+R)} - \frac{1}{\beta R} + \frac{e^{-RT}}{R(\beta+R)}\right]\left[\frac{1-e^{-RNT}}{1-e^{-RT}}\right] \end{aligned}$$

Present value of inventory holding cost during last cycle, i.e., $(N+1)^{th}$ cycle is given by

$$\begin{aligned} C_3 &= h\int\limits_{NT}^{h}\left[(Q+\frac{D}{\beta})e^{-\beta(t-NT)} - \frac{D}{\beta}\right]e^{-Rt}dt \\ &= \frac{hD}{\beta}\left[\frac{-e^{\beta T}e^{-(R+\beta)h_1}e^{\beta NT}}{(R+\beta)} + \frac{e^{-Rh_1}}{R} + \frac{\{R(e^{\beta T}-1)-\beta\}}{R(\beta+R)}e^{-NRT}\right] \end{aligned}$$

So, expected value of total ordering and purchase cost is

$$\widetilde{C}_4 = \sum_{N=0}^{\infty} \int\limits_{NT}^{(N+1)T} \left[c_o + \frac{(c_1+c_p)D(e^{\beta T}-1)}{\beta}\right]\left[\frac{1-e^{-R(N+1)T}}{1-e^{-RT}}\right]\widetilde{\lambda}e^{-\widetilde{\lambda}h_1}dh_1$$

Then according to Dubois and Prade [16] α-cut of $\widetilde{C}_4$ is

$$\begin{aligned}
C_4(\alpha) &= \alpha\text{-cut of}\left[\sum_{N=0}^{\infty} \int\limits_{NT}^{(N+1)T} \left[c_o + \frac{(c_1+c_p)D(e^{\beta T}-1)}{\beta}\right]\left[\frac{1-e^{-R(N+1)T}}{1-e^{-RT}}\right]\right. \\
&\quad \left.\widetilde{\lambda}e^{-\widetilde{\lambda}h_1}dh_1\right] \\
&= \left[c_o + \frac{(c_1+c_p)D(e^{\beta T}-1)}{\beta}\right]\left[\frac{1}{1-e^{-RT}}\right]\sum_{N=0}^{\infty}\left[\left(1-e^{-R(N+1)T}\right)\times\right. \\
&\quad \left.\int\limits_{NT}^{(N+1)T} (\alpha\text{ -cut of }(\widetilde{\lambda}e^{-\widetilde{\lambda}h_1}))dh\right] \\
&= \left[c_o + \frac{(c_1+c_p)D(e^{\beta T}-1)}{\beta}\right]\left[\frac{1}{1-e^{-RT}}\right]\sum_{N=0}^{\infty}\left[\left(1-e^{-R(N+1)T}\right)\times\right. \\
&\quad \left.\int\limits_{NT}^{(N+1)T} \left[\{a+(b-a)\alpha\}e^{-\{c-(c-b)\alpha\}}h, \{c-(c-b)\alpha\}e^{-\{a+(b-a)\alpha\}}h_1\right]dh_1\right] \\
&= \left[\left(\frac{e_1}{1-e^{-\{R+c-(c-b)\alpha\}T}}\right)\left(\frac{a+(b-a)\alpha}{c-(c-b)\alpha}\right), \left(\frac{e_1}{1-e^{-\{R+a+(b-a)\alpha\}T}}\right)\right. \\
&\quad \left.\left(\frac{c-(c-b)\alpha}{a+(b-a)\alpha}\right)\right] \\
&\quad \text{Where } e_1 = \left[c_o + \frac{(c_1+c_p)D(e^{\beta T}-1)}{\beta}\right]
\end{aligned}$$

The expected holding cost up to the beginning of $(N+1)^{th}$ cycle is

$$\widetilde{C}_5 = \sum_{N=0}^{\infty} \int\limits_{NT}^{(N+1)T} c_h D\left[\frac{e^{\beta T}}{\beta(\beta+R)} - \frac{1}{\beta R} + \frac{e^{-RT}}{R(\beta+R)}\right]\left[\frac{1-e^{-RNT}}{1-e^{-RT}}\right]\widetilde{\lambda}e^{-\widetilde{\lambda}h_1}dh_1$$

So α-cut of $\widetilde{C}_5$ is

$$\begin{aligned}
C_5(\alpha) &= \alpha\text{-cut of } \Bigg[\sum_{N=0}^{\infty} \int_{NT}^{(N+1)T} hD\Big[\frac{e^{\beta T}}{\beta(\beta+R)} - \frac{1}{\beta R} + \frac{e^{-RT}}{R(\beta+R)}\Big] \\
&\quad \Big[\frac{1-e^{-RNT}}{1-e^{-RT}}\Big]\widetilde{\lambda} e^{-\widetilde{\lambda} h_1} dh_1\Bigg] \\
&= \Bigg[\Bigg(\frac{e_2 e^{-\{c-(c-b)\alpha\}T}}{1-e^{-\{R+c-(c-b)\alpha\}T}}\Bigg)\Bigg(\frac{a+(b-a)\alpha}{c-(c-b)\alpha}\Bigg), \Bigg(\frac{e_2 e^{-\{a+(b-a)\alpha\}T}}{1-e^{-\{R+a+(b-a)\alpha\}T}}\Bigg) \\
&\quad \Bigg(\frac{c-(c-b)\alpha}{a+(b-a)\alpha}\Bigg)\Bigg], \\
&\quad \text{where } e_2 = hD\Big[\frac{e^{\beta T}}{\beta(\beta+R)} - \frac{1}{\beta R} + \frac{e^{-RT}}{R(\beta+R)}\Big]
\end{aligned}$$

The expected holding cost during $(N+1)^{th}$ cycle is

$$\begin{aligned}
\widetilde{C}_6 &= \sum_{N=0}^{\infty} \int_{NT}^{(N+1)T} \frac{hD}{\beta}\Big[\frac{-e^{\beta T} e^{-(R+\beta)h_1} e^{\beta NT}}{(R+\beta)} + \frac{e^{-Rh_1}}{R} \\
&\quad + \frac{[R(e^{\beta T}-1)-\beta]}{R(\beta+R)} e^{-NRT}\Big]\widetilde{\lambda} e^{-\widetilde{\lambda} h_1} dh_1
\end{aligned}$$

So α-cut of $\widetilde{C}_6$ is

$$\begin{aligned}
C_6(\alpha) &= \alpha\text{-cut of } \Bigg[\sum_{N=0}^{\infty} \int_{NT}^{(N+1)T} \frac{hD}{\beta}\Big[\frac{-e^{\beta T} e^{-(R+\beta)h_1} e^{\beta NT}}{(R+\beta)} + \frac{e^{-Rh_1}}{R} + \\
&\quad \frac{\{R(e^{\beta T}-1)-\beta\}}{R(\beta+R)} e^{-NRT}\Big]\widetilde{\lambda} e^{-\widetilde{\lambda} h_1} dh_1\Bigg] \\
&= \Bigg[-e_3\Bigg(\frac{c-(c-b)\alpha}{\beta+R+a+(b-a)\alpha}\Bigg)\Bigg(\frac{1-e^{-\{\beta+R+a+(b-a)\alpha\}T}}{1-e^{-\{R+a+(b-a)\alpha\}T}}\Bigg) \\
&\quad +e_4\Bigg(\frac{a+(b-a)\alpha}{R+c-(c-b)\alpha}\Bigg) + e_5\Bigg(\frac{a+(b-a)\alpha}{c-(c-b)\alpha}\Bigg)\Bigg(\frac{1-e^{-\{c-(c-b)\alpha\}T}}{1-e^{-\{R+c-(c-b)\alpha\}T}}\Bigg), \\
&\quad -e_3\Bigg(\frac{a+(b-a)\alpha}{\beta+R+c-(c-b)\alpha}\Bigg)\Bigg(\frac{1-e^{-\{\beta+R+c-(c-b)\alpha\}T}}{1-e^{-\{R+c-(c-b)\alpha\}T}}\Bigg) \\
&\quad +e_4\Bigg(\frac{c-(c-b)\alpha}{R+a+(b-a)\alpha}\Bigg) + e_5\Bigg(\frac{c-(c-b)\alpha}{a+(b-a)\alpha}\Bigg)\Bigg(\frac{1-e^{-\{a+(b-a)\alpha\}T}}{1-e^{-\{R+a+(b-a)\alpha\}T}}\Bigg)\Bigg] \\
&\quad \text{Where } e_3 = \frac{hDe^{\beta T}}{\beta(\beta+R)}, e_4 = \frac{hD}{\beta R}, e_5 = \frac{h[DR(e^{\beta T}-1)-D\beta]}{\beta R(\beta+R)}
\end{aligned}$$

Therefore the total expected cost ($\widetilde{EC}$) is given by

$$\widetilde{EC} = \widetilde{C}_4 + \widetilde{C}_5 + \widetilde{C}_6$$

So α-cut of $\widetilde{EC}$ is given by

$$\begin{aligned}
EC(\alpha) &= (\widetilde{C}_4+\widetilde{C}_5+\widetilde{C}_6)(\alpha) = C_4(\alpha)+C_5(\alpha)+C_6(\alpha)\\
&= \Bigg[\left(\frac{1}{1-e^{-\{R+c-(c-b)\alpha\}T}}\right)\left(\frac{a+(b-a)\alpha}{c-(c-b)\alpha}\right)\{(e_1+e_5)\\
&\quad +(e_2-e_5)e^{-[c-(c-b)\alpha]T}\} - e_3\left(\frac{c-(c-b)\alpha}{\beta+R+a+(b-a)\alpha}\right)\\
&\quad \left(\frac{1-e^{-\{\beta+R+a+(b-a)\alpha\}T}}{1-e^{-\{R+a+(b-a)\alpha\}T}}\right)+e_4\left(\frac{a+(b-a)\alpha}{R+c-(c-b)\alpha}\right),\\
&\quad \left(\frac{1}{1-e^{-\{R+a+(b-a)\alpha\}T}}\right)\left(\frac{c-(c-b)\alpha}{a+(b-a)\alpha}\right)[(e_1+e_5)\\
&\quad +(e_2-e_5)e^{-\{a+(b-a)\alpha\}T}] - e_3\left(\frac{a+(b-a)\alpha}{\beta+R+c-(c-b)\alpha}\right)\\
&\quad \left(\frac{1-e^{-\{\beta+R+c-(c-b)\alpha\}T}}{1-e^{-\{R+c-(c-b)\alpha\}T}}\right)+e_4\left(\frac{c-(c-b)\alpha}{R+a+(b-a)\alpha}\right)\Bigg]
\end{aligned}$$

Let $[ECL_\alpha, ECR_\alpha]$ be the alpha cut of expected total cost ($\widetilde{EC}$). Then according to Buckley [6]

$$\begin{aligned}
ECL_\alpha &= \Bigg[\left(\frac{1}{1-e^{-\{R+c-(c-b)\alpha\}T}}\right)\left(\frac{a+(b-a)\alpha}{c-(c-b)\alpha}\right)\{(e_1+e_5)+(e_2-e_5)e^{-\{c-(c-b)\alpha\}T}\}\\
&\quad -e_3\left(\frac{c-(c-b)\alpha}{\beta+R+a+(b-a)\alpha}\right)\left(\frac{1-e^{-\{\beta+R+a+(b-a)\alpha\}T}}{1-e^{-\{R+a+(b-a)\alpha\}T}}\right)\\
&\quad +e_4\left(\frac{a+(b-a)\alpha}{R+c-(c-b)\alpha}\right)\Bigg]\\
ECR_\alpha &= \Bigg[\left(\frac{1}{1-e^{-\{R+a+(b-a)\alpha\}T}}\right)\left(\frac{c-(c-b)\alpha}{a+(b-a)\alpha}\right)\{(e_1+e_5)+(e_2-e_5)e^{-\{a+(b-a)\alpha\}T}\}\\
&\quad -e_3\left(\frac{a+(b-a)\alpha}{\beta+R+c-(c-b)\alpha}\right)\left(\frac{1-e^{-\{\beta+R+c-(c-b)\alpha\}T}}{1-e^{-\{R+c-(c-b)\alpha\}T}}\right)\\
&\quad +e_4\left(\frac{c-(c-b)\alpha}{R+a+(b-a)\alpha}\right)\Bigg]
\end{aligned}$$

So according to Carlson and Fullar [8], the possibilistic mean value of $\widetilde{EC}$ is

$$\begin{aligned}
\overline{M}(\widetilde{EC}) &= \int_0^1 \alpha(ECL_\alpha+ECR_\alpha)d\alpha\\
&= \{e_1e_6+e_2e_7+(e_6-e_7)e_5+(e_4-e_3)e_8\}
\end{aligned}$$

Where the expressions of e_1, e_2, e_3, e_4 and e_5 are given above and

$$\begin{aligned}
e_6 \quad = \quad & \left(\frac{a(c-b)(R+c)+(b-a)(R+c)^2}{RT(c-b)^3}\right)\log\left(\frac{c(R+b)}{b(R+c)}\right) \\
& -\left(\frac{2(R+c)(b-a)+a(c-b)}{T(c-b)^3}\right)\log\left(\frac{c}{b}\right)+\frac{(b-a)}{T(c-b)^3}\left\{(c-b)+R\log\left(\frac{c}{b}\right)\right\} \\
& -\left(\frac{c(b-a)(R+a)+(c-b)(R+a)^2}{RT(b-a)^3}\right)\log\left(\frac{b(R+a)}{a(R+b)}\right) \\
& +\left(\frac{2(R+a)(c-b)+c(b-a)}{T(b-a)^3}\right)\log\left(\frac{b}{a}\right)+\frac{(c-b)}{T(b-a)^3}\left\{(b-a)+R\log\left(\frac{b}{a}\right)\right\}, \\
e_7 \quad = \quad & \left(\frac{(R+c)(1+RT)a(c-b)+(R+c)^2(1+RT)(b-a)}{RT(c-b)^3}\right)\log\left(\frac{c(R+b)}{b(R+c)}\right) \\
& -\left(\frac{(R+c)(1+RT)(b-a)+\{(R+c)T+(1+RT)\}a(c-b)}{T(c-b)^3}\right)\log\left(\frac{c}{b}\right) \\
& +\left(\frac{\{(R+c)T+(1+RT)\}(b-a)+a(c-b)T}{T(c-b)^3}\right)\left((c-b)+R\log\left(\frac{c}{b}\right)\right) \\
& -\left(\frac{(b-a)}{(c-b)^3}\right)\left(2R(c-b)+\frac{c^2-b^2}{2}+R^2\log\left(\frac{c}{b}\right)\right)
\end{aligned} \tag{2}$$

$$\begin{aligned}
& -\left(\frac{(R+a)(1+RT)c(b-a)+(R+a)^2(1+RT)(c-b)}{RT(b-a)^3}\right)\log\left(\frac{b(R+a)}{a(R+b)}\right) \\
& +\left(\frac{(R+a)(1+RT)(c-b)+\{(R+a)T+(1+RT)\}c(b-a)}{T(b-a)^3}\right)\log\left(\frac{b}{a}\right) \\
& -\left(\frac{\{(R+a)T+(1+RT)\}(c-b)+c(b-a)T}{T(b-a)^3}\right)\left((b-a)+R\log\left(\frac{b}{a}\right)\right) \\
& +\left(\frac{(c-b)}{(b-a)^3}\right)\left(2R(b-a)+\frac{b^2-a^2}{2}+R^2\log\left(\frac{b}{a}\right)\right), \\
e_8 \quad = \quad & \left(\frac{1}{(c-b)^3}\right)\left((R+c)a(c-b)+(b-a)(R+c)^2\right)\log\left(\frac{R+c}{R+b}\right) \\
& -\left(\frac{1}{(c-b)^2}\right)\left((R+c)(b-a)+a(c-b)+(b-a)(R+c)\right) \\
& +\frac{(b-a)(2R+b+c)}{2(c-b)^2}-\left(\frac{1}{(b-a)^3}\right)\left((R+a)c(b-a)+(c-b)(R+a)^2\right) \\
& \times\log\left(\frac{R+b}{R+a}\right)+\left(\frac{1}{(b-a)^2}\right)\left((R+a)(c-b)+c(b-a)+(c-b)(R+a)\right) \\
& -\frac{(c-b)(2R+a+b)}{2(b-a)^2}.
\end{aligned}$$

Finally, we minimize the possibilistic mean value ($\overline{M}(\widetilde{EC})$) of the expected fuzzy cost using LINGO software following generalized reduced gradient technique.

5 Numerical Illustration

A retailer purchase a finished goods at the cost of 5 per unit quantity. Ordering cost is in the linear form (c_0+c_1Q) where c_0(minimum ordering cost) and c_1 are given by c_0=25, c_1=0.25 and Q is the order quantity. The inventory holding cost is 0.5 per unit quantity. Then the retailer sells these units in the market. The demand of the item is 500 units. Suppose 1% of the items is deteriorate. The planning horizon follows exponential distribution with parameter as "around 0.5". The inflation rate is 0.2 and discount rate is 0.3 and other input parameters are as follows:
h=1.5, c_p=5, D=500, d=0.3, i=0.2, R=d-i=0.1, $\widetilde{\lambda}$=(a, b, c)=(0.40, 0.5, 0.6), $\beta = .01$ in their respective units.
Hence problem is to determine the optimal time period (T^*), so that the total expected cost from the system for the planning horizon is minimum.
For the above parametric values, minimum solution is obtained using LINGO software. The solution is T=3.29821, $\overline{M}(\widetilde{EC})$= 4512. In other words, the minimum cost of the businessman is $\overline{M}(\widetilde{EC})$=4512.

5.1 Conclusion

For the first time an inventory model is developed in a fuzzy random planning horizon considering the effect of inflation and time value of money on different inventory costs. The reason for adaptation of this model is threefold:

- Firstly, it is very difficult to define time horizon of an inventory problem precisely. Specially for seasonal goods, which are normally stochastic in nature, which render stochastic planning horizon for the model.
- Secondly, error occurs in estimating parameters of the distribution of the planning horizon as crisp number. Here it is estimated as fuzzy rather than crisp.
- Thirdly, to introduce a general solution methodology which helps one to extend this amalgamation for other inventory models.

Such a realistic situation has been considered in this model. So, from the economical point of view, the proposed model will be useful to the business houses in the present context as it gives a better inventory control system.

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