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An Integrated Supply Chain Model under Fuzzy Chance Constraints

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Abstract

In this paper, we passing the value of vertical information sharing in terms of inventory replenishment/requirement from the customer(s)$\rightarrow$ retailers $\rightarrow$ producer $\rightarrow$ supplier(s). The constant imprecise fuzzy demands of the goods are made to the retailers by the customers. These goods are produced (along with a defectiveness, which decreases due to learning effects) from the raw materials in the producers production center with a constant production rate(unknown). Producer stores these raw materials in a warehouse by purchasing these from a suppliers and the suppliers collect these raw materials from open market / nature at a constant collection rate (unknown). The whole system is considered for a finite time period with fuzzy demand for finished products and fuzzy inventory costs. Here shortages are allowed and fully backlogged. The fuzzy chance constraints on the available space of the producer and transportation costs for both producer, retailers are defuzzified using necessity approach. Besides, this research tries to looking for a period of inventory management system maintenance so that it can found the minimum cost incurred or which still tolerable for tracking inventory management system itself. Results indicate the efficiency of proposed approach in performance measurement. This paper attempts to provide the reader a complete picture of supply chain management through a systematic literature review.

Keywords: Supply Chain Model, Fuzzy Chance Constraint, Learning-effect.

AMS Subject Classifications: 90B05, 90B50.

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1 Introduction

A supply chain model (SCM) is a network of supplier, producer, distributor and customer which synchronizes a series of inter-related business process in order to: (i) optimal procurement of raw materials from nature; (ii) transportation of raw-materials into warehouse; (iii) production of the goods in the production center; (iv) distribution of these finished goods to retailer for sale to the customers. With a recent paradigm shift to the supply chain (SC), the ultimate success of a firm may depend on its ability to link supply chain members seamlessly.

One of the earliest efforts to create an integrated supply chain model dates back to Cachon and Zipkin [1], Cohen and Lee [4], Nair et.al. [14]. They developed a production, distribution and inventory (PDI) planning system that integrated three supply chain segments comprised of supply, storage / location and customer demand planning. The core of the PDI system was a network model and diagram that increased the decision maker's insights into supply chain connectivity. The model, however was confined to a single-period and single-objective problem. Viswanathan and Piplani [20] concerned an integrated inventory model through common replenishment in the SC. Hill. et.al. [7] discussed the SCM with lost sale. Recently Sarmah et.al. [18]designed a coordination of a single-manufacturer/multi-buyer supply chain. All the above SCMs are considered with constant, known demand and production rates.

Gradually the fuzzy demand over a finite planning horizon has attracted the attention of researchers (cf. Xie et,al. [22] and others). This type of demand is observed in the case of fashionable goods, daily emerging products, etc. Moreover, the most of the product goods are breakable. Here the decrease of breakability represents by the transmission of learning justified through the experience gain in planning, organization and the familarity of the workers with their tasks. Keachie and Fontana [9] first introduced the learning effect for a decision making problem in inventory control system. Jaber and Bonney [8]showed the learning effect of lot sizes in an economic manufacturing quantity model.

After the development of fuzzy set theory by Zadeh [23], it has been extensively used in different field of science and technology to model complex decision making problems. Since Zimmermann [24, 25] first introduced fuzzy set theory into the ordinary linear programming (LP) and multi-objective linear programming (MOLP) problems, several fuzzy mathematical programming and techniques have developed by researchers to solve fuzzy production and/or distribution planing problems (cf. Liang [11], Maiti and Maiti [13], Liu and Iwamura [12], Santoso, Ahmed, Goetschalckx and Shapiro[19]). Moreover, Petrovic [16] developed a heuristic based on fuzzy sets theory to determine the order quantities for a supply chain in the presence of uncertainties associated in the presence of uncertainties associated with customer demand, deliveries. Piedro et.al. [15] designed a supply chain scheduling model as a multi-products, multi-stages and multi-periods mixed integer nonlinear programming problem with uncertain market demand, to satisfy conflict objectives. Wang and Shu [21] presented a fuzzy supply chain model by combining possibility theory and genetic algorithm approach to provide an alternative framework to handle supply chain uncertainties and to determine inventory strategies. Xie, Petrovic and Bumham [22] designed a two-level hierarchical method to inventory management and control in serial supply chains, in which the supply chain operated under imprecise customer demand and was modelled by fuzzy sets.

In this paper, we consider a supply-chain (SC) production-inventory control system consisting of a single supplier, single producer (having a warehouse and a production center) and retailer. The imprecise demands of the goods are made to the retailer by the customers. These goods are produced (along with a defectiveness) from a raw material in the producer's production center with controllable production rate. Producer store these raw materials in a warehouse purchasing these from a supplier and the supplier collects these raw materials from open market / nature at a constant collection rate. The SC starts with the collection of raw materials, then storage and production and ends with the distribution of finished goods to the retailer and sale of those units by the retailer to the customers. The whole system is considered for a finite time period with fuzzy demand for finished products and fuzzy inventory costs. Here shortages are fully backlogged. There are fuzzy chance constraints on the transportation costs for both producer and retailer and also a space constraint is considered. Then for the integrated case the model is formulated as fuzzy chance constraint programming problem where constraint is satisfied with some predefined degree of necessity. As optimization of fuzzy objective is not well defined a necessity based return of the objective is optimized under the constraint. Then the model is transferred to a crisp one using fuzzy extension principle and solved using LINGO software. For non-integrated case the model is solved applying an appropriate interactive fuzzy decision making method (IFDM) for multi-objective is applied to solve the model. The fuzzy model provides the decision maker with alternative decision plans for different degrees of satisfaction. This proposal is tested by using data from a real supply chain. Results indicate the efficiency of proposed approach in performance measurement.

2 Prerequisite Mathematics:

Any fuzzy subset $\tilde{A}$ of $\Re$(where $\Re$ represents the set of real numbers) with membership function $\mu_{\tilde{A}} : \Re \longrightarrow [0,1]$ is called a fuzzy number. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers with membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ respectively. Then taking degree of uncertainty as the semantics of fuzzy number, according to Liu and Iwamura [12]:

$$\text{Pos } (A \star \tilde{B}) = \sup\{\min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)), x, y \in \Re, x \star y\} \tag{1}$$

where the abbreviation Pos represent possibility and $\star$ is any one of the relations $>, <, =, \leq, \geq$. Analogously if $\tilde{B}$ is a crisp number, say b, then

$$\text{Pos } (\tilde{A} \star b) = \sup\{\mu_{\tilde{A}}(x), x \in \Re, x \star b\} \tag{2}$$

On the other hand necessity measure of an event $\tilde{A}\star\tilde{B}$ is a dual of possibility measure. The grade of necessity of an event is the grade of impossibility of the opposite event and is defined as:

$$\text{Nes } (\tilde{A} \star \tilde{B}) = 1 - \text{Pos } (\overline{\tilde{A} \star \tilde{B}}) \tag{3}$$

where the abbreviation Nes represents necessity measure and $\overline{\tilde{A} \star \tilde{B}}$ represents complement of the event $\tilde{A} \star \tilde{B}$. If $\tilde{A}, \tilde{B} \in \Re$ and $\tilde{C} = f(\tilde{A}, \tilde{B})$ where $f : \Re \times \Re \rightarrow \Re$ be a binary operation then membership function $\mu_{\tilde{C}}$ of $\tilde{C}$ can be obtained using **Fuzzy Extension Principle** [23] as

$$\mu_{\tilde{C}}(z) = \sup \{\min (\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)), x, y \in \Re, \text{ and } z = f(x, y), \forall z \in \Re\} \tag{4}$$

According to this principle if $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers (TFNs) with positive components then $\tilde{A}+\tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)$ is a TFN. Furthermore if $a_2 - a_1,\ a_3 - a_2,\ b_2 - b_1,\ b_3 - b_2$ are small then $\tilde{A}.\tilde{B} = (a_1.b_1, a_2.b_2, a_3.b_3)$ is approximately a TFN.

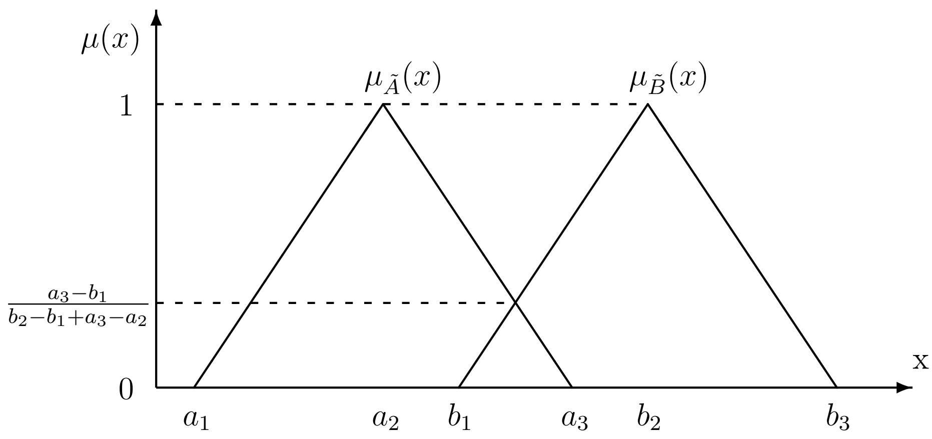

Fig-1: Comparison of two TFNs $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$

Lemma-1: If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be TFNs with $0 < a_1$ and $0 < b_1$ then Nes$(\tilde{b} > \tilde{a}) \geq \alpha$ iff $\dfrac{a_3 - b_1}{b_2 - b_1 + a_3 - a_2} \leq 1 - \alpha$.

Proof: Now for two fuzzy numbers $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$,

(i) if $a_2 \geq b_2$, then the relation $\tilde{b} \leq \tilde{a}$ is obviously true,

(ii) if $a_3 \leq b_1$, then the above relation is always false,

(iii) otherwise, the relation have a chance, calculated from Definition-1, as

$$\sup\{\min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)), x, y \in \Re, x \geq y\} = \xi$$

Therefore, is clear that

$$Pos(\tilde{b} \leq \tilde{a}) = \begin{cases} 1 & \text{for } a_2 \geq b_2 \\ \dfrac{a_3 - b_1}{b_2 - b_1 + a_3 - a_2} & \text{for } a_2 \leq b_2 \text{ and } a_3 \geq b_1 \\ 0 & \text{otherwise} \end{cases}$$

Again, from the relation (3) between Possibility and Necessity, We have

Nes$(\tilde{b} > \tilde{a}) \geq \alpha$

$\Leftrightarrow \{1 - Pos(\tilde{b} \leq \tilde{a})\} \geq \alpha$

$\Leftrightarrow Pos(\tilde{b} \leq \tilde{a}) \leq 1 - \alpha$.

Hence, for $0 \leq \alpha \leq 1$, $Nes(\tilde{b} \leq \tilde{a}) \geq \alpha$, iff $\dfrac{a_3 - b_1}{b_2 - b_1 + a_3 - a_2} \leq 1 - \alpha$, and hence the result follows.

Lemma-2: If $\tilde{a} = (a_1, a_2, a_3)$ be a TFN with $0 < a_1$ and b be a crisp number then Nes$(b > \tilde{a}) \geq \alpha$ iff $\dfrac{a_3 - b}{a_3 - a_2} \leq 1 - \alpha$.

Proof: Proof follows from Lemma-1(Put $b_1 = b_2 = b_3 = b$ in Lemma-1).

3 Assumptions and Notations:

The following assumptions and notations are used in developing the proposed SCM.

(i) The model is developed for a finite time horizon.

(ii) Multiple suppliers, single producer and multiple retailers are considered.

(iii) Collection rate of raw material, production rate of the produced goods are constant.

(iv) Demand rate of finished goods met by the retailer is imprecise in nature.

(v) The holding cost, set-up cost, purchasing cost by retailer, total warehouse space, total transportation cost to transport raw materials from suppliers to production warehouse, total transportation cost to transport the produced goods from producer to retailer are taken as fuzzy in nature.

(vi) One type of raw material and finished product are considered.

(vii) Producer possesses two systems- a warehouse and a production center.

(viii) Shortages of goods are allowed and fully backlogged.

(ix) Multiple lot-size deliveries per order are considered instead of a single delivery per order.

(x) Lot size is the same for each delivery.

(xi) Space constraints to the producer is allowed.

(xii) There is limited transportation cost.

The following notations are used for the proposed SCM.
For supplier

(i) l = supplier's index, where l=1,2,...,L.

(ii) $q_{sl}(t)$ = inventory level at time t .

(iii) C_l = collection rate of the supplier(a decision variable).

(iv) Q_{sl} =maximum inventory at each interval.

(v) $\widetilde{h_{sl}}$ = fuzzy holding cost per unit quantity per unit time.

(vi) $\widetilde{H_s}$ = total holding cost which is fuzzy in nature.

(vii) p_{sl} = per unit purchasing cost of goods(constant).

(viii) $\widetilde{A}_s$ = fuzzy ordering cost per unit quantity.

(ix) $\widetilde{TC}_s$ = supplier's raw material cost which is fuzzy in nature.

For producer's warehouse

(i) $q_{PW}(t)$ = inventory level at time t .

(ii) U = production rate of the finished goods (a decision variable).

(iii) Q_{PW} =maximum inventory at each interval.

(iv) $\widetilde{h}_{PW}$ = fuzzy holding cost per unit quantity per unit time.

(v) $\widetilde{H}_{PW}$ = total holding cost which is fuzzy in nature.

(vi) p_{PW} = per unit production cost of goods(constant).

(vii) $\widetilde{A}_{PW}$ = fuzzy ordering cost per unit quantity.

(viii) $\widetilde{TC}_{PW}$ = fuzzy total cost of raw materials for producer's warehouse.

For the producer

(i) $q_P(t)$ = inventory level at time t .

(ii) λ = defective rate of production, $\lambda = \lambda^{-iT_1}$.

(iii) Q_P =maximum inventory of produced goods at each interval.

(iv) $\widetilde{h}_P$ = fuzzy holding cost per unit quantity per unit time.

(v) $\widetilde{H}_P$ = total holding cost which is fuzzy in nature.

(vi) p_P = per unit production cost of goods(constant).

(vii) $\widetilde{A}_P$ = fuzzy ordering cost per unit quantity.

(viii) $\widetilde{TC}_P$ = fuzzy total cost for producer's finished goods.

For the retailer

(i) k = retailer's index, where k=1,2,...,K.

(ii) $q_{Rk}(t)$ = inventory level at time t .

(iii) $\widetilde{D}_k$ = demand rate of the produced goods which is fuzzy in nature.

(iv) Q_{Rk} =maximum inventory at each interval.

(v) $\widetilde{h}_{Rk}$ = fuzzy holding cost per unit quantity per unit time.

(vi) $\widetilde{H}_R$ = total holding cost which is fuzzy in nature.

(vii) $\widetilde{p}_{Rk}$ = per unit purchasing cost of goods which is fuzzy in nature.

(viii) $\widetilde{A}_R$ = fuzzy ordering cost per unit quantity.

(ix) $\widetilde{C}_{3k}$ = per unit shortage cost which is fuzzy in nature.

(x) $\widetilde{S}_{Rk}$ = total amount of shortage which is fuzzy in nature.

(xi) $\widetilde{TC}_R$ = fuzzy total cost for the retailer.

Common notations

(i) T = length of order cycle.

(ii) n = number of deliveries per order cycle T(a decision variable).

(iii) t = delivery cycle.

(iv) T_1 = length of time of each of the n equal sub intervals of order cycle(T)(a decision variable).

(v) T_R = total shortage period.

(vi) $\widetilde{W}$ = fuzzy total space to keep the raw materials in the warehouse to keep the finished goods.

(vii) $\widetilde{T}_{11}$ = fuzzy total transportation cost to transport the raw materials from supplier's to production warehouse.

(viii) $\widetilde{T}_{21}$ = fuzzy total transportation cost to transport the produced goods from producer to retailer.

4 Mathematical Formulation of the Supply Chain:

This paper develops a supply-chain system which consists multiple suppliers, single producer and multiple retailers. The suppliers are collect the raw material at a constant collection rate, this raw material is purchased by producer and then transported and stored in his / her warehouse, from which raw material is used for production and finished goods are produced at a production rate which is taken as control variable. Then the goods are purchased by retailers, who sells these goods in a market with imprecise demand. The system is considered over a finite time horizon and hence several cycles of procurement, production, etc are repeated within the said time period. There are some resource constraints for the producer and retailer on purchasing the raw materials and finished goods respectively. For the retailer, the model is developed with shortages which are fully-backlogged. The purpose of this study is to find the optimal collection rate, optimal production rate, the number of cycles to each partner and length of time of each of the n equal sub intervals of order cycle so that total or individual costs are minimum.

4.1 Inventory model of supplier's raw material :

In this model supplier collect raw material from nature and satisfies the producers warehouse. Therefore supplier's raw material inventory quantity $q_{sl}(t)$ at any time t can be expressed as

$$\frac{dq_{sl}}{dt} = C_l, \qquad iT_1 \leq t \leq (i+1)T_1, \quad i = 0, 1, 2, ..., n-2. \tag{5}$$

Now from the help of boundary condition $q_{sl}(iT_1) = 0$ the inventory at any time t, $q_{sl}(t)$, is given by:

$$q_{sl}(t) = C_l(t - iT_1). \tag{6}$$

and using the boundary condition $q_s((i+1)T_1) = Q_s$ we get

$$Q_s = \sum_{l=1}^{L} Q_{sl} = C_l T_1. \tag{7}$$

Holding cost of raw material is

$$\begin{aligned} &= \widetilde{h_{sl}} \int_{iT_1}^{(i+1)T_1} q_{sl}(t)dt \\ &= \frac{\widetilde{h_{sl}} C_l T_1^2}{2} \end{aligned} \tag{8}$$

So total holding cost of raw material is

$$\begin{aligned} H_s &= \sum_{l=1}^{L}\sum_{i=0}^{n-2} \frac{\widetilde{h_{sl}} C_l T_1^2}{2} \\ &= (n-2)\sum_{l=1}^{L} \frac{\widetilde{h_{sl}} C_l T_1^2}{2} \end{aligned} \tag{9}$$

Total collection cost of raw material is

$$\begin{aligned} C_s &= \sum_{l=1}^{L}\sum_{i=0}^{n-2} p_{sl} C_l T_1 \\ &= (n-2)\sum_{l=1}^{L} p_{sl} C_l T_1 \end{aligned} \tag{10}$$

The total raw material cost for the supplier is the sum of the set up cost, collection cost and holding cost as follows:

$$\widetilde{TC_s} = \sum_{l=1}^{L} \widetilde{A_{sl}} + (n-2)\sum_{l=1}^{L} p_{sl} C_l T_1 + (n-2)\sum_{l=1}^{L} \frac{\widetilde{h_{sl}} C_l T_1^2}{2} \tag{11}$$

4.2 Inventory model of raw material in producer's warehouse:

The inventory level of raw material at the producer's warehouse at time t, q_{PW} determine by the linear differential equation

$$\frac{dq_{PW}}{dt} = -U, \qquad (i+1)T_1 \le t \le (i+2)T_1, \quad i = 0, 1, 2, ..., n-2. \tag{12}$$

As shown in the figure-1 the inventory conditions for the model are:

$$q_{PW}((i+1)T_1) = Q_{PW} \text{ and } q_{PW}((i+2)T_1) = 0 \text{ for } i = 0, 1, 2, ..., n-2$$

Therefore using the condition $q_{PW}((i+2)T_1) = 0$ the inventory at any time t is given by:

$$q_{PW}(t) = U\{(i+2)T_1 - t\} \tag{13}$$

Using the condition $q_{PW}((i+1)T_1) = Q_{PW}$ we get

$$Q_{PW} = UT_1 \tag{14}$$

Holding cost of raw material is

$$\begin{aligned} &= \widetilde{h}_{PW} \int_{(i+1)T_1}^{(i+2)T_1} q_{PW}(t)dt \\ &= \frac{\widetilde{h}_{PW} U T_1^2}{2} \end{aligned} \tag{15}$$

So total holding cost of raw material is

$$\begin{aligned} \widetilde{H}_{PW} &= \sum_{i=0}^{n-2} \frac{\widetilde{h}_{PW} U T_1^2}{2} \\ &= (n-2)\frac{\widetilde{h}_{PW} U T_1^2}{2} \end{aligned} \tag{16}$$

Purchasing cost of raw materials=$p_{PW}Q_{PW}$

Total purchasing cost of raw materials()

$$\begin{aligned} \widetilde{P}_{PW} &= \sum_{i=0}^{n-2} p_{PW} Q_{PW} \\ &= (n-2) p_{PW} Q_{PW} \\ &= (n-2) p_{PW} U T_1 \end{aligned} \tag{17}$$

The total raw material cost for the producer's warehouse is the sum of the set up cost, purchasing cost of raw material and holding cost as follows:

$$\widetilde{TC}_{PW} = \widetilde{A}_{PW} + (n-2)p_{PW}UT_1 + (n-2)\frac{\widetilde{h}_{PW}UT_1^2}{2} \tag{18}$$

4.3 Inventory model of producer's finished goods:

The finished goods inventory level for the producer with unknown production rate U is described by the following differential equation:

$$\frac{dq_P}{dt} = (1-\lambda^{-iT_1})U, \qquad (i+1)T_1 \le t \le (i+2)T_1, \quad i=0,1,2,...,n-2 \tag{19}$$

The boundary conditions are $q_P\{(i+1)T_1\}=0$ and $q_P\{(i+2)T_1\}=Q_P$
Therefore using the condition $q_P\{(i+1)T_1\}=0$ the inventory at any time t is given by:

$$q_P = (1-\lambda^{-iT_1})U\{t-(i+1)T_1\} \tag{20}$$

Using the condition $q_P\{(i+2)T_1\}=Q_P$, we get,

$$Q_P = (1-\lambda^{-iT_1})UT_1 \tag{21}$$

In this case holding cost is

$$\begin{aligned} &= \widetilde{h}_P \int_{(i+1)T_1}^{(i+2)T_1} q_P(t)dt \\ &= \frac{\widetilde{h}_P(1-\lambda^{-iT_1})UT_1^2}{2} \end{aligned} \tag{22}$$

So total holding cost of raw material is

$$\begin{aligned} \widetilde{H}_{PW} &= \sum_{i=0}^{n-2} \frac{\widetilde{h}_P(1-\lambda^{-iT_1})UT_1^2}{2} \\ &= \widetilde{h}_P \frac{UT_1^2}{2}\left[(n-2) - \frac{1-(\lambda^{-T_1})^{n-1}}{1-\lambda^{-T_1}}\right] \end{aligned} \tag{23}$$

Production cost=$p_P Q_P$
Total production cost

$$\begin{aligned} \widetilde{P}_P &= \sum_{i=0}^{n-2} p_P Q_P \\ &= (n-2)(1-\lambda^{-iT_1})p_P Q_P \\ &= p_P UT_1\left[(n-2) - \frac{1-(\lambda^{-T_1})^{n-1}}{1-\lambda^{-T_1}}\right] \end{aligned} \tag{24}$$

The total cost for the producer due to finished goods can be expressed as the sum of the setup cost, production cost and holding cost as follows:

$$\widetilde{TC}_P = \widetilde{A}_P + p_P UT_1\left[(n-2) - \frac{1-(\lambda^{-T_1})^{n-1}}{1-\lambda^{-T_1}}\right] + \widetilde{h}_P \frac{UT_1^2}{2}\left[(n-2) - \frac{1-(\lambda^{-T_1})^{n-1}}{1-\lambda^{-T_1}}\right] \tag{25}$$

4.4 Inventory model for the retailers for finished goods :

If $q_{Rk}(t)$ be the inventory of the finished goods at any time t for the retailer with imprecise demand $\widetilde{D}_k$ and if t_{b_i} be the time of shortage for the k-th retailer in the i-th cycle, the governing differential equations are:

$$\frac{dq_{Rk}}{dt} = -\widetilde{D}_k, \qquad (i+2)T_1 \leq t \leq (i+3)T_1, \quad i = 0, 1, 2, ..., n-2. \tag{26}$$

As shown in the figure-1 the inventory conditions for the model are:

$$\begin{aligned} & q_{Rk}((i+2)T_1) = Q_{Ri} \\ & q_{Rk}((i+2)T_1 + T_R) = 0 \\ \text{and} \quad & q_{Rk}((i+3)T_1) = S_{Rk} \text{ for } i = 0, 1, 2, ..., n-2 \end{aligned} \tag{27}$$

Therefore using the condition $q_{Rk}\{(i+1)T_1\} = 0$ the inventory at any time t is given by:

$$q_{Rk} = \widetilde{D}_k\{(i+2)T_1 - t\} + Q_{RK}, \qquad (i+2)T_1 \leq t \leq (i+2)T_1 + T_R, \quad i = 0, 1, ..., n-2 \tag{28}$$

Using the condition $q_{Rk}\{(i+2)T_1 + T_R\} = 0$, we get,

$$Q_{Rk} = \widetilde{D}_k T_R \tag{29}$$

Using the condition $q_{Rk}((i+3)T_1) = S_R$, we get,

$$q_{Rk} = \widetilde{D}_k\{(i+3)T_1 - t\} + S_R, \qquad (i+2)T_1 + T_R \leq t \leq (i+3)T_1, \quad i = 0, 1, ..., n-2 \tag{30}$$

Now using $q_{Rk}\{(i+2)T_1 + T_R\} = 0$, we get,

$$S_{Rk} = \widetilde{D}_k(T_1 - T_R) \tag{31}$$

In this case holding cost is

$$\begin{aligned} &= \widetilde{h}_{Rk} \int_{(i+2)T_1}^{\{(i+2)T_1+T_R\}} q_{Rk}(t)dt \\ &= \widetilde{h}_{Rk} \frac{Q_{Rk}T_R}{2} \end{aligned} \tag{32}$$

So total holding cost of finished goods is

$$\begin{aligned}\widetilde{H}_R &= \sum_{i=0}^{n-2}\sum_{k=1}^{K}\widetilde{h}_{Rk}\frac{Q_{Rk}T_R}{2} \\ &= (n-2)\sum_{k=1}^{K}\widetilde{h}_{Rk}\frac{Q_{Rk}T_R}{2} \qquad (33)\end{aligned}$$

corresponding shortage cost

$$\begin{aligned}&= C_{3k}\int_{\{(i+2)T_1+T_R\}}^{(i+3)T_1}[\tilde{D}_k\{(i+3)T_1-t\}+S_{Rk}]dt \\ &= \frac{C_{3k}S_{Rk}(T_1-T_R)}{2} \qquad (34)\end{aligned}$$

Total shortage cost($\widetilde{TS}_R$)

$$\begin{aligned}&= \sum_{k=1}^{K}\sum_{i=0}^{n-2}C_{3k}(S_{Rk}T_R+\frac{\widetilde{D}_kT_R^2}{2}) \\ &= \sum_{k=1}^{K}(n-2)\frac{C_{3k}S_{Rk}(T_1-T_R)}{2} \qquad (35)\end{aligned}$$

The purchasing cost=$\widetilde{p}_{Rk}Q_{Rk}$
Total purchasing cost($\widetilde{TP}_R$)

$$\begin{aligned}&= \sum_{k=1}^{K}\sum_{i=0}^{n-2}\widetilde{p}_{Rk}Q_{Rk} \\ &= \sum_{k=1}^{K}(n-2)\widetilde{p}_{Rk}\widetilde{D}_kT_R \qquad (36)\end{aligned}$$

The total cost for the retailer due to finished goods can be expressed as the sum of the setup cost, production cost, holding cost and shortage cost as follows:

$$\widetilde{TC}_R = \sum_{k=1}^{K}\widetilde{A}_{Rk}+\sum_{k=1}^{K}(n-2)\widetilde{p}_{Rk}\widetilde{D}_kT_R+(n-2)\widetilde{h}_{Rk}\frac{Q_{Rk}T_R}{2}+\sum_{k=1}^{K}(n-2)\frac{C_{3k}S_{Rk}(T_1-T_R)}{2} \qquad (37)$$

Integrated Model:

Assuming the whole system is owned and managed by a single concern / management the problem reduces to a single objective minimization problem as:

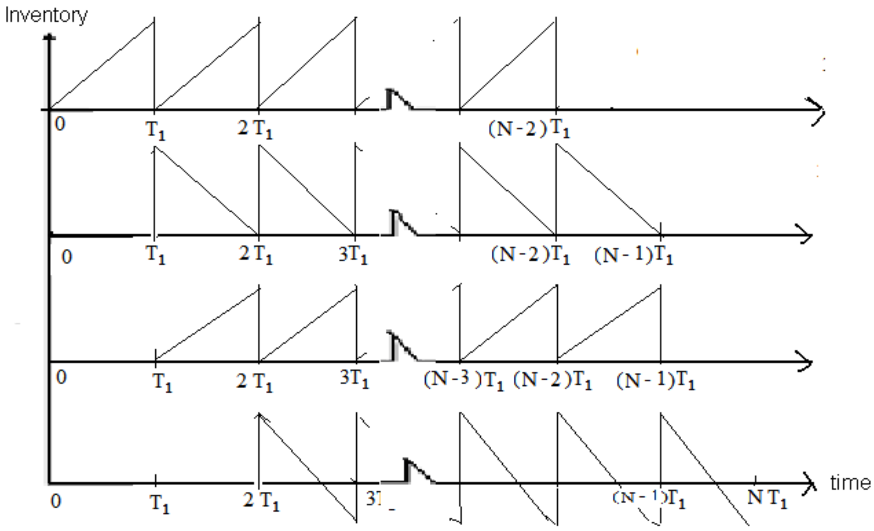

Figure 1: Production-Inventory Model of a SCM

$$\min \widetilde{TC}_I \approx \min \left\{ \widetilde{TC}_s + \widetilde{TC}_{PW} + \widetilde{TC}_P + \widetilde{TC}_R \right\} \tag{38}$$

Subject to $Q_P + Q_{PW} \leq \widetilde{W}$............ ....$(C-1)$

$t_0' + t_1' Q_S \leq \widetilde{T}_{11}$....... ....$(C-2)$

$t_0'' + t_1'' Q_R \leq \widetilde{T}_{21}$....... ....$(C-3)$

5 Procedure for Defuzzification

Since $\widetilde{TC}_I$ is fuzzy in nature minimize $\widetilde{TC}_I$ is not well defined. So instead of minimize $\widetilde{TC}_I$ one can minimize F such that necessity of the event $\widetilde{TC}_I < F$ exceeds some predefined level $\alpha(0 < \alpha < 1)$ according to companies requirement. Similarly as fuzzy constraints are also not well defined, necessity of the constraints (C-1,C-2,C-3) must exceed some predefined level $\alpha_i(0 < \alpha_i < 1)(i = 1, 2, 3)$ as proposed by Maiti and Maiti [13]. Then the problem reduces to

$$\left.\begin{array}{l} \text{Minimize} \quad F \\ \text{Subject to } \mathrm{Nes}(\widetilde{TC}_I < F) > \alpha \\ \qquad \mathrm{Nes}(Q_P + Q_{PW} \leq \widetilde{W}) > \alpha_1 \\ \qquad \mathrm{Nes}(t_0' + t_1' Q_S \leq \widetilde{T}_{11}) > \alpha_2 \\ \qquad \mathrm{Nes}(t_0'' + t_1'' Q_R \leq \widetilde{T}_{21}) > \alpha_3 \end{array}\right\} \tag{39}$$

Now, let us consider $\tilde{h}_s = (h_{s1}, h_{s2}, h_{s3})$, $\tilde{A}_s = (A_{s1}, A_{s2}, A_{s3})$, $\tilde{h}_{PW} = (h_{PW1}, h_{PW2}, h_{PW3})$, $\tilde{A}_{PW} = (A_{PW1}, A_{PW2}, A_{PW3})$, $\tilde{h}_P = (h_{P1}, h_{P2}, h_{P3})$, $\tilde{A}_P = (A_{P1}, A_{P2}, A_{P3})$, $\tilde{D} = (D_1, D_2, D_3)$, $\tilde{C}_3 = (C_{31}, C_{32}, C_{33})$, $\tilde{h}_R = (h_{R1}, h_{R2}, h_{R3})$, $\tilde{A}_R = (A_{R1}, A_{R2}, A_{R3})$, $\tilde{p}_R = (p_{R1}, p_{R2}, p_{R3})$, $\tilde{W} = (W_1, W_2, W_3)$, $\tilde{T}_{11} = (T_{111}, T_{112}, T_{113})$, $\tilde{T}_{21} = (T_{211}, T_{212}, T_{213})$, as TFNs then $\widetilde{TC}_I$ becomes a TFN $(TC_{I1}, TC_{I2}, TC_{I3})$. Then using Lemma-1 and Lemma-2 the above problem reduces to:

$$\left.\begin{array}{ll} \text{Minimize} & F \\ \text{Subject to} & \alpha TC_{I3} + (1-\alpha)TC_{I2} \leq F \\ & \frac{W_2-(Q_P+Q_{PW})}{W_2-W_1} \geq \alpha_1 \\ & \frac{T_{112}-(t_0^{'}+t_1^{'}Q_s)}{T_{112}-T_{111}} \geq \alpha_2 \\ & \frac{T_{212}-(t_0^{''}+t_1^{''}Q_s)}{T_{212}-T_{211}} \geq \alpha_3 \end{array}\right\} \tag{40}$$

which is equivalent to

$$\left.\begin{array}{ll} \text{Minimize} & F = \alpha TC_{I3} + (1-\alpha)TC_{I2} \\ \text{Subject to} & \frac{W_2-(Q_P+Q_{PW})}{W_2-W_1} \geq \alpha_1 \\ & \frac{T_{112}-(t_0^{'}+t_1^{'}Q_s)}{T_{112}-T_{111}} \geq \alpha_2 \\ & \frac{T_{212}-(t_0^{''}+t_1^{''}Q_s)}{T_{212}-T_{211}} \geq \alpha_3 \end{array}\right\} \tag{41}$$

For some assumed parametric values we get the optimal value of the problem by using LINGO software.

6 Numerical Experiment:

In this section, the author consider two different examples to illustrate the proposed supply-chain model.

Example-1: Consider a SCM consisting of single supplier (L=1), producer and single retailer (K=1) with fuzzy demand (110,120,130). The relevant cost data are shown in Table-6.1.

Table-6.1: Fuzzy Input data:

SCM	Ordering cost	Purchasing/Prod cost	Holding cost	defective cost	Shortage rate	Transportation cost
Supplier	(45, 50, 55)	(18, 20, 22)	(3.5, 4, 4.5)	–	–	(5, 0.5)
Prod. warehouse	(75, 80, 85)	(35, 40, 45)	(4.5, 5, 5.5)	–	–	–
Prod. centre	(95, 100, 105)	(4.5, 5, 5.5)	(5.5, 6, 6.5)	0.02	–	
Retailer	(85, 90, 95)	(45, 50, 55)	(4.5, 5, 5.5)	–	(1.6, 2.0, 2.3)	(5, 0.5)

In addition to this data, the following parameters are also assumed :
$\tilde{W}$=(2.4,2.5,2.6) , $\tilde{T_{11}}$=(11,12,13.5) , $\tilde{T_{21}}$=(11,12,13.5) and predefined necessity levels $\alpha = 0.9$, $\alpha_1 = 0.8$, $\alpha_2 = 0.8$, $\alpha_3 = 0.8$
We used the software package Mathematica (by Wolfram Research,Inc.)to obtain the optimal and heuristic solutions. The optimization option we used within the package is the Generelised Reduced Gradient (GRG) method which provides quite effectively results, such as: the optimal number of shipments is N^*=10, optimal time length of each cycle T_1^*=0.1028 , T_R^*=0.1000 and the corresponding optimal rates are C^*=121.21, U^*=121.21 with optimal quantity QS^*=12.46, QR^*=12.30. The total cost under this solution is TC_I^*=15990.52 for the optimal total cost of supplier TC_s^*=2064.15, for producer TC_P^*= (4091.92+4583.57) and for retailer TC_r^*=5250.87.

Example-2: Consider a SCM consisting of two suppliers (L=2), producer and two retailers (K=2) for the respective imprecise market demands (100, 108,118) and (124, 130, 140). The relevant cost parameters are shown in Table-6.2.

Table-6.2: Fuzzy Input data:

SCM	Ordering cost	Purchasing/Prod cost	Holding cost	defective rate	Demand rate	Transportation cost
Suppliers'	(45, 50, 55)	(18, 20, 22)	(3.5, 4, 4.5)	–	–	(5, 0.5)
	(45, 50, 55)	(18, 20, 22)	(3.5, 4, 4.5)	–	–	(5, 0.5)
Prod. warehouse	(75, 80, 85)	(35, 40, 45)	(4.5, 5, 5.5)	–	–	–
Prod. centre	(95, 100, 105)	(4.5, 5, 5.5)	(5.5, 6, 6.5)	0.02	–	
Retailers'	(85, 90, 95)	(45, 50, 55)	(4.5, 5, 5.5)	–	(110, 120, 130)	(5, 0.5)
	(85, 90, 95)	(45, 50, 55)	(4.5, 5, 5.5)	–	(110, 120, 130)	(5, 0.5)

The additional data are remain same as example-1.
The solution procedure provides the optimal number of shipments is N^*=8, optimal time length of each cycle T_1^*=0.1110 , T_R^*=0.1022 and the corresponding optimal rates are C_l^*=(116.48, 126.32), U^*=121.42 with optimal quantity QS_l^*=(11.90,12.05), QR_k^*=(12.05,12.72). The total cost under this solution is TC_I^*=23071.18 for the optimal total cost of supplier TC_s^*=(2012.35, 2065.46)), for producer TC_P^*= (4112.26 + 4502.73) and for retailer TC_r^*=(5078.20, 5299.67).

7 Discussion:

This paper addresses the optimal order quantity placed by the retailers, production rate of the producer and the collection or production rate of the suppliers to minimize the supply-chain cost. Here, it is seen that all these control variables directly or indirectly depend on the demand by the customers placed to the retailers.

From mathematical representation and numerical results, it is also observed that (i) producers product amount in a cycle = sum of order quantities of all the retailers for a single cycle = total demand of the customers during the cycle time to all retailers, (ii) The stored amount of raw-materials by the producer= sum of the collection or production amount of all the suppliers during the cycle period, (iii) the production rate of the finished goods is proportional to the decay rate of the raw-materials.

8 Conclusion:

Here, fuzzy chance constraints on the transportation costs for both producer, retailers and also a space constraint for producer (to contain finished goods and different raw materials) is considered. The different supply-chain costs are imprecise in nature. The EOQ business process by the different retailers also may follow shortage of the finished goods. Considering all these real-life scenarios the supply-chain model is optimized for a finite number of sub-intervals in the finite time horizon.

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References

[1] Cachon. G.P and Zipkin. P.H (1999), *Competitive and cooperative inventory policies in a two-stage supply chain*, Management Science, **45**,936-953.

[2] Chen, C.T. and Huang, S.F. (2006), *Order fulfilment ability analysis in the supply chain system with fuzzy operation times*, International Journal of Production Economics,**101**, 185-193.

[3] Chen, C.T., Lin, C.T. and Huang, S.F. (2006), *A fuzzy approach for supplier evaluation and selection in supply chain management*, International Journal of Production Economics,**102**, 289-301.

[4] Cohen, M.A. and Lee,H.L., (1988), *Strategic analysis of integrated production-distributionvsystems-models and methods,*Operations Research,**36**, 216-228.

[5] Das, B., Maity, K., Mondal, S. and Maiti, M., (2005), *A Supply-chain production control problem via genetic algorithm.* Proc, International Conference on Operation Research and Developement-V. 279-286.

[6] Das, B., Maity, K., and Maiti, M., (2007), *A Two Warehouse Supply-Chain Model under Possibility / Necessity / Credibility measure.* Mathematical and Computer Modelling. 46, 398-409.

[7] Hill. R. M.,Seifbarghy. M., Smith. D.K.,(2007), *A two-echolon inventory model with lost sales.* European Journal of Operational research.181, 753-766.

[8] Jaber. M. Y and Bonney. M., (1998), *The effects of learning and forgetting on the optimal lot size quantity of intermittent production runs.* Production Planning and Control (9).20-27.

[9] Keachie. E. C and Fontana. R. J (1966), *Production lot sizes under a learning effect.* Management Science 13.B102-108

[10] Kumar, M., Vrat, P.,and Shan, R., (2004), *A fuzzy goal programming approach for vendor selection problem in a supply chain*, Computers and Industrial Engineering , 46 , 69-85.

[11] Liang, T.F.,(2007), *Applying fuzzy goal programming to production/transportation planning decisions in supply chain.*, International Journal of Systems Science, 38, 293-304.

[12] Liu, B. and Iwamura, K., *Chance constraint Programming with fuzzy parameters*, Fuzzy Sets and Systems, 94(1998), 227-237.

[13] Maiti, M. K. and Maiti, M.,(2006), *Fuzzy Inventory Model with Two Warehouses under Possibility Constraints*, Fuzzy sets and Systems, 157(1), 52-73.

[14] Nair,A., Closs, D. J.,(2006), *An examination of the impact of coordinating supply chain policies and price markdowns on short life cycle product retail performance*,International Journal of Production Economics, **102**, 379-392.

[15] Peidro, D., Mula, J., Polar, R., Verdegay, J-L., (2009), *Fuzzy Optimization for supply chain planning under supply, demand and process uncertainties* , Fuzzy sets and Systems, **Volume 160 Issue 18,** 2640-2657.

[16] Petrovic, D., *Simulation of supply chain behaviour and performance in an uncertain environment*, International Journal of Production Economics, **71**(2001), 429-438.

[17] Rau, H., Wu, M.-Y. and Wee, H.-M.,(2004), *Deteriorating item inventory model with shortage due to supplier in an integrated supply chain*, International Journal of Systems Science, **35**, 293-303.

[18] Sarmah S.P., Acharya. D., Goyal. S.K.,(2008), *Coordination of a single-manufacturer/multi-buyer supply chain with credit option* . International Journal of Production Economics ,111, 676-685

[19] Santoso,T., Ahmed,S., Goetschalckx, M. and Shapiro, A., (2005), *A stochastic programming approach for supply chain network design under uncertainty.* European Journal of Operational Research, **167**, 96-115.

[20] Viswanathan,S. and Piplani,R., (2001), *Coordinating supply chain inventories through common replenishment epochs.* European Journal of Operational Research, **129**, 277-286.

[21] Wang, J., and Shu, Y.F.,(2005), *Fuzzy decision modelling for supply chain management*, Fuzzy Sets and Systems, **150**, 107-127.

[22] Xie, Y., Petrovic, D. and Bumham, K.,(2006), *A heuristic procedure for the two-level control of serial supply chains under fuzzy customer demand*, International Journal of Production Economics, **102**, 37-50.

[23] Zadeh, L. A.,(1965), *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems, **1**, 3-28.

[24] Zimmermann, H.-J.,(1976), *Description and optimization of fuzzy systems.* International Journal of General Systems, **2** , 209-215.

[25] Zimmermann, H.-J.,(1978), *Fuzzy programming and linear programming with several objective functions.* Fuzzy Sets and Systems, **1**, 45-56.