The Use of the Markov Chain Monte Carlo Method in Deriving the Elevator Round Trip Time under Incoming Traffic Conditions and a Single Entrance

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Abstract

The round trip time is the basis for designing elevator systems. There are a number of different methods for calculating the round trip time, both analytical and numerical.

As the building and the conditions of the traffic become more complicated, analytical methods become intractable. Numerical methods offer an attractive alternative for calculating the round trip time.

Monte Carlo simulation has been used to find the value of the round trip time. The use of the Markov Chain Monte Carlo Method is a viable alternative. This paper derives the formulae necessary to build the transition probability matrix for the elevator during a round trip under incoming traffic conditions and a single entrance. It then provides a numerical example illustrating the practical use of the method to evaluate the value of the round trip time.

Nomenclature

τ is the elevator round trip time in s
d_f is the typical height of one floor in metres
H is the highest reversal floor (where floors are numbered 0, 1, 2….N)
J_{ij} *is the event whereby the elevator travels between the two floors i and j without stopping in between in a round trip*
P is the number of passengers in the car when it leaves the ground floor
$P(J_{ij})$ *is the probability of the elevator travelling between the two floors i and j without stopping in between in a round trip*
$P(S_i)$ *is the probability of the elevator stopping at floor i in a round trip*
S is the number of stops in a round trip
S_i *is the event whereby the elevator stops at floor i in a round trip*
t_{ao} is the door advance opening time in seconds (where the door starts opening before the car comes to a complete standstill)
t_{dc} is the door closing time in seconds
t_{do} is the door opening time in seconds
t_f is the time taken to complete a one floor journey in seconds
t_{pi} is the passenger boarding time in seconds
t_{po} is the passenger alighting time in seconds
t_{sd} is the motor start delay in seconds
v is the rated speed in metres per second

1.0 INTRODUCTION

This paper introduces a methodology for evaluating the elevator round trip time by using the Markov Chain Monte Carlo method (MCMC). The round trip time is the time taken by the elevator to complete a full round trip during which it picks up passengers from the main entrance, delivers them to their destinations in the building and then returns back to the main entrance.

Evaluating the round trip time is critical to the design of elevator traffic systems, whereby it can be used to find the required number of elevators in a building, based on quantitative and qualitative user requirements [1]. Evaluating the round trip be achieved using analytical-equation-based methods [2, 3] or by numerical methods [4]. Analytical methods are restricted in their scope of application as they cannot deal with cases where the top speed is not attained in one floor journey and where floor heights are not equal, although there is some work in that area [5].

The advantage of numerical methods is that they can be applied under any of the special cases such as top speed not attained in one floor journey, unequal floor heights, unequal floor populations and multiple entrances. The only numerical method currently used in evaluating the round trip time is the Monte Carlo method [4].

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This paper introduces the Markov Chain Monte Carlo (MCMC) method as an additional numerical tool in evaluating the elevator round trip time. The method introduced in this paper is restricted to the case of a single entrance building.

Section 2 introduces the methodology for deriving the transition probability matrix. Section 3 develops the required equations, and introduces the kinematics matrix. A step by step guide to using the MCMC method in evaluating the round trip is presented in section 4. A numerical example on the evaluation of the round trip time using the MCMC method is given in section 5. Some conclusions on the application of the method are drawn in section 6.

2.0 THE TRANSITION PROBABILITY MATRIX

In order to use the Markov Chain Monte Carlo method for evaluating the round trip time it is first necessary to derive the transition probability matrix [6]. Each element in the transition probability matrix provides the probability that the elevator will move to a floor *j* from floor *i* given that the elevator is currently on floor *i*. This is shown below:

$$P_{ij} = P\left(J_{ij} \mid S_i\right) \tag{1}$$

A general format for the matrix is shown in Table 1 below. It can be seen that the diagonal is zero, as the elevator cannot stay on the same floor. As the traffic is incoming, the traffic cannot move to a floor below it, except to return to the ground floor. The upper triangle of the matrix represents the probabilities of the elevator moving to an upper floor. The first column represents the probability of returning back to the ground floor.

Table 1: Initial transition probability matrix.

	G	**1**	**2**	**...**	**N-2**	**N-1**	**N**
G	0	$P(J_{G1}\|S_0)$	$P(J_{G2}\|S_0)$	...	$P(J_{GN-2}\|S_0)$	$P(J_{GN-1}\|S_0)$	$P(J_{GN}\|S_0)$
1	$P(J_{1G}\|S_1)$	0	$P(J_{12}\|S_1)$	...	$P(J_{1N-2}\|S_1)$	$P(J_{1N-1}\|S_1)$	$P(J_{1N}\|S_1)$
2	$P(J_{2G}\|S_2)$	0	0	...	$P(J_{2N-2}\|S_2)$	$P(J_{2N-1}\|S_2)$	$P(J_{2N}\|S_2)$
...	...	...	...	0	...	...	...
N-2	$P(J_{N-2G}\|S_{N-2})$	0	0	...	0	$P(J_{N-2N-1}\|S_{N-2})$	$P(J_{N-2N}\|S_{N-2})$
N-1	$P(J_{N-1G}\|S_{N-1})$	0	0	...	0	0	$P(J_{N-1N}\|S_{N-1})$
N	$P(J_{NG}\|S_N)$	0	0	...	0	0	0

Taking the general expression for each cell and expanding it, gives:

$$P_{ij} = P\left(J_{ij} \mid S_i\right) = \frac{P\left(J_{ij} \cap S_i\right)}{P\left(S_i\right)} \tag{2}$$

Thus the transition probability from floor *i* to floor *j* is equal to the probability of a journey from *i* to *j* without stopping at the floors in between them and a stop at floor *i*, divided by the probability of stopping at floor *i* in a round trip. But it is worth noting that the event "*journey from floor i to j in a round trip*" is a subset of the event "*stopping at floor i in a round trip*", as by definition in order for a journey from *i* to *j* to take place, the elevator must stop at floor *i* to start with. This is shown in equation (3) shown below:

$$\left(J_{ij} \cap S_i\right) \subset S_i \tag{3}$$

But if an event *A* is a subset of another event *B*, then their intersection is event *A*. So the probability in (2) can be simplified as shown in equation (4) below:

$$P\left(J_{ij} \cap S_i\right) = P\left(J_{ij}\right) \tag{4}$$

Substituting (4) in (2) gives the important result:

$$P_{ij} = \frac{P(J_{ij})}{P(S_i)} \tag{5}$$

Thus the values in each row must be divided by the probability of stopping at a floor. The ground floor is the only entrance. Thus by definition the elevator has to stop at that floor in order to pick the *P* passengers up. Thus the probability of the elevator stopping at this floor in one round trip is 1.

$$P(S_0) = 1 \tag{6}$$

Table 2: Final transition probability matrix.

	G	1	2	...	N-2	N-1	N
G	0	$P(J_{G1})$	$P(J_{G2})$	...	$P(J_{GN-2})$	$P(J_{GN-1})$	$P(J_{GN})$
1	$P(H=1)/P(S_1)$	0	$P(J_{12})/P(S_1)$	...	$P(J_{1N-2})/P(S_1)$	$P(J_{1N-1})/P(S_1)$	$P(J_{1N})/P(S_1)$
2	$P(H=2)/P(S_2)$	0	0	...	$P(J_{2N-2})/P(S_2)$	$P(J_{2N-1})/P(S_2)$	$P(J_{2N})/P(S_2)$
...	...	...	...	0	...	...	...
N-2	$P(H=N-2)/P(S_{N-2})$	0	0	...	0	$P(J_{N-2N-1})/P(S_{N-2})$	$P(J_{N-2N})/P(S_{N-2})$
N-1	$P(H=N-1)/P(S_{N-1})$	0	0	...	0	0	$P(J_{N-1N})/P(S_{N-1})$
N	1	0	0	...	0	0	0

The first column represents the probability of a certain floor being the highest reversal floor in a round trip given that the elevator stopped at that floor. This is shown in equation (7) below:

$$P(H = i \mid S_i) = \frac{P(H = i \cap S_i)}{P(S_i)} \tag{7}$$

This is equal to the probability of the i^{th} floor being the highest reversal floor and a stop at the i^{th} floor, divided by the probability of stopping at the i^{th} floor in a round trip. But it is worth noting that the event "the i^{th} floor being the highest reversal floor" is a subset of the event "stopping at the i^{th} floor in a round trip", as by definition in order for the i^{th} floor to be the highest reversal floor, the elevator must stop at the i^{th} floor to start with. This is shown below:

$$(H = i) \subset S_i \tag{8}$$

But if an event *A* is a subset of another event *B*, then their intersection is event *A*. So the probability in (7) can be simplified as shown in equation (9) below:

$$P(H = i \cap S_i) = P(H = i) \tag{9}$$

Substituting (9) in (7) gives the important result:

$$P(H = i \mid S_i) = \frac{P(H = i)}{P(S_i)} \tag{10}$$

Thus the elements in the first column can be derived by dividing the probability of a floor being the highest reversal floor divided by the probability of stopping at that floor. The final modified transition probability matrix is shown in Table 2.

Thus in order to derive the transition probability matrix, it is necessary to have formulae for the probability of the following events:

- the probability of a journey between two floors without stopping at the intermediate floors between them.
- the probability of a floor being the highest reversal floor.
- the probability of stopping at a floor.

These three formulae are derived in the next section.

3.0 EQUATIONS

It has been shown that the probability of a journey taking place between two floors *i* and *j* (without stopping at any of the floor between them) in a round trip is given by the following expression shown in (11) below [5]:

$$P(J_{ij}) = \left(1 - \sum_{k=i+1}^{j-1}\left(\frac{U_k}{U}\right)\right)^P - \left(1 - \sum_{k=i}^{j-1}\left(\frac{U_k}{U}\right)\right)^P - \left(1 - \sum_{k=i+1}^{j}\left(\frac{U_k}{U}\right)\right)^P + \left(1 - \sum_{k=i}^{j}\left(\frac{U_k}{U}\right)\right)^P \tag{11}$$

For the special case, where the elevator starts from the ground floor (i.e., the single entrance), equation (11) is reduced to the special case equation shown in (12) below [5]:

$$P(J_{0j}) = \left(1 - \sum_{k=1}^{j-1}\left(\frac{U_k}{U}\right)\right)^P - \left(1 - \sum_{k=1}^{j}\left(\frac{U_k}{U}\right)\right)^P \tag{12}$$

The probability of the elevator stopping at a floor *i* during a round trip depends on the population on that floor and the number of passengers boarding the car, as shown in equation (13) below [5]:

$$P(S_i) = 1 - \left(1 - \frac{U_i}{U}\right)^P \tag{13}$$

The ground floor is the only entrance. Thus by definition the elevator has to stop at that floor in order to pick the *P* passengers up. Thus the probability of the elevator stopping at this floor in one round trip is 1.

$$P(S_0) = 1 \tag{14}$$

As can be seen in the transition probability matrix (for a single entrance arrangement), the first column represents the probability that a certain floor be the highest reversal floor, given that a stop has taken place on that floor during a round trip. The formula for the probability of a floor being the highest reversal floor is shown below in equation (15) [5]:

$$P(H = i) = \left(\sum_{k=1}^{i}\left(\frac{U_k}{U}\right)\right)^P - \left(\sum_{k=1}^{i-1}\left(\frac{U_k}{U}\right)\right)^P \tag{15}$$

It is also necessary to calculate the travelling time between floors. The travelling time between any two floors *i* and *j* separated by a distance *d*, with a rated speed of *v*, rated acceleration *a,* and rated jerk *j* can be calculated as shown in equations (16), (17) and (18) for the three different conditions (rated speed attained; rated speed not attained but rated acceleration attained; rated speed not attained and rated acceleration not attained, respectively) [7]:

Rated speed attained	If $d \geq \left(\frac{a^2 v + v^2 j}{aj}\right)$ then $t = \frac{d}{v} + \frac{v}{a} + \frac{a}{j}$	(16)
Rated speed not attained but rated acceleration attained	If $\frac{2a^3}{j^2} \leq d < \left(\frac{a^2 v + v^2 j}{aj}\right)$ then $t = \frac{a}{j} + \sqrt{\frac{4d}{a} + \left(\frac{a}{j}\right)^2}$	(17)
Neither rated speed nor rated acceleration attained	If $d < \frac{2a^3}{j^2}$ then $t = \left(\frac{32d}{j}\right)^{\frac{1}{3}}$	(18)

This produces a two dimensional matrix that shows the time required to travel from the floor *i* to the floor *j*. A representation of such a matrix is shown below. This matrix is necessary when calculating the round trip time.

Table 3: Kinematics matrix.

	G	**1**	**2**	**...**	**N-2**	**N-1**	**N**
G	*0*	t_{01}	t_{02}	...	t_{0N-2}	t_{0N-1}	t_{0N}
1	t_{10}	*0*	t_{12}	...	t_{1N-2}	t_{1N-1}	t_{1N}
2	t_{20}	t_{21}	*0*	...	t_{2N-2}	t_{2N-1}	t_{2N}
...	...	...	...	*0*	...	...	...
N-2	t_{N-20}	t_{N-21}	t_{N-22}	...	*0*	t_{N-2N-1}	t_{N-2N}
N-1	t_{N-10}	t_{N-11}	t_{N-12}	...	t_{N-1N-2}	*0*	t_{N-1N}
N	t_{N0}	t_{N1}	t_{N2}	...	t_{NN-2}	t_{NN-1}	*0*

As can be seen in the table, the diagonal is zero, as no time is required to move from a floor to the same floor. It can also be seen that the upper triangle of the matrix above the diagonal is a mirror image of the lower triangle below the diagonal.

4.0 METHOD OF CALCULATING THE ROUND TRIP TIME

In this section an overview is given of the steps taken in order to evaluate the round trip time using the Markov Chain Monte Carlo method:

1. Develop the kinematics matrix.
2. Develop the transition probability matrix.
3. Extract the pdf (probability density function) for each floor of the building form the corresponding row in the transition probability matrix.
4. Convert each pdf obtained in step 3 into a cdf (cumulative distribution function).
5. Assuming that the starting position for the elevator is the main entrance (floor G or 0) draw a random sample from the cdf produced from the first row of the transition probability matrix. This will produce the next destination of the elevator.
6. Using the row that corresponds to the next destination, use the cdf of that row to generate the next destination.
7. Repeat step 6 until the elevator returns back to the ground floor. This forms a complete round trip.
8. Using the kinematics matrix developed in 1 above, calculate the travelling time component of the round trip.
9. Based on the number of stops in the journey found in 7 above, calculate the door time component of the round trip.
10. Based on the number of passengers on which the transition probability matrix was developed, calculate the passenger transition component of the round trip.
11. The round trip is then sum of the three terms in 8, 9 and 10.

12. Repeat steps 5 to 11 a certain number of trials (e.g., 10 000), and then take the average of all trials, thus obtaining the value of the round trip time.

5.0 NUMERICAL EXAMPLE

In this section a numerical example is introduced to illustrate the practical application of the MCMC method. A building has 5 floors (N) above the main entrance (ground floor). The elevator car fills up with 6 passengers, thus P=6, at the ground floor. The building has equal floor height, d_f=4.5 m. The door opening time (t_{do}) is 2 s and the door closing time (t_{dc}) is 3 s. The passenger transfer time into the elevator (t_{pi}) is 1.2 s and out of the elevator (t_{po}) is 1.2. The rated speed (v) is 1.6 ms^{-1}; the rated acceleration (a) is 1 ms^{-2}; the rated jerk (j) is 1 ms^{-3}.

The transition probability matrix is developed in Table 4 below.

Table 4: Transition probability matrix for the building (5 decimal places).

	G	1	2	3	4	5
G	0	0.73786	0.21549	0.04256	0.00403	0.00006
1	0.00004	0	0.70800	0.23437	0.05221	0.00538
2	0.00542	0	0	0.70800	0.23437	0.05221
3	0.05763	0	0	0	0.70800	0.23437
4	0.29200	0	0	0	0	0.70800
5	1	0	0	0	0	0

…as expected the sum of the elements in any row is 1, the diagonal is zero and the lower triangle is zero (except for the first column).

The pdf and cdf for each row is generated and then used to carry out random sampling in order generate a full round trip. Generating random numbers and then using them to generate the movement of the elevator in the building as follows:

Random()=0.244: 0 to 1
Random()=0.746: 1 to 3
Random()=0.796: 3 to 5
And then it must go from 5 to 0

So the full journey becomes: 0 to 1; 1 to 3; 3 to 5; 5 to 0.

In order to calculate the first term of the round trip, the travelling time, it is necessary to find the kinematic matrix. This represents the time required for the elevator to travel between any two floors starting at rated speed, rated acceleration and rated jerk. These times can be calculated using the three equations (16), (17) and (18). They are shown in Table 5 below for this building and the elevator kinematic parameters.

Table 5: Kinematic matrix for the building.

	G	1	2	3	4	5
G	0	5.4125	8.2250	11.0375	13.8500	16.6625
1	5.4125	0	5.4125	8.2250	11.0375	13.8500
2	8.2250	5.4125	0	5.4125	8.2250	11.0375
3	11.0375	8.2250	5.4125	0	5.4125	8.2250
4	13.8500	11.0375	8.2250	5.4125	0	5.4125
5	16.6625	13.8500	11.0375	8.2250	5.4125	0

The round trip time comprises three components. The first of these components is the travelling time.

$$\begin{aligned}\tau_T &= \sum t_{ij} = t_{0-1} + t_{1-3} + t_{3-5} + t_{5-0} \\ &= 5.4125 + 8.225 + 8.225 + 16.6625 = 38.525\,\mathrm{s}\end{aligned} \tag{19}$$

The second term is the door time. This can be calculated by multiplying the number of stops during the journey by the door opening and closing time. The number of stops in this case is 4 (stops at 0, 1, 3 and 5):

$$\tau_D = S \cdot \left(t_{dc} + t_{do}\right) = 4 \cdot \left(3 + 2\right) = 20\,\mathrm{s} \tag{20}$$

The third and final term in the round trip time equation represents the passenger boarding and alighting time and is denoted by τ_P. It is easy to calculate by multiplying the number of passengers by the sum of the boarding and alighting time per passenger, as shown below:

$$\tau_P = P \cdot \left(t_{pi} + t_{po}\right) = 6 \cdot \left(1.2 + 1.2\right) = 14.4\,\mathrm{s} \tag{21}$$

Adding all three terms of the round trip time, gives the first trial of the round trip time as shown below:

$$\tau = \tau_T + \tau_D + \tau_P = 72.925\,\mathrm{s} \tag{22}$$

Repeating this procedure 10 000 times gives a final value of the round trip time of 76.3231 s. The exact value using the analytical equation is 76.4031 s.

The true power of the method becomes obvious where no analytical equation exists for calculating the round trip time where any or all of the following conditions exist such as where the top speed is not attained in one floor journey or where the floor heights are not equal.

7.0 CONCLUSIONS

A new method for numerically calculating the round trip time for an elevator system using the Markov Chain Monte Carlo method has been presented. A clear set of steps are outlined for evaluating the value of the round trip time. The equations for developing the transition probability matrix of the elevator movements during a round trip have been derived. A numerical practical example has been given illustrating the use of the method to evaluate the round trip time for one trial.

The method is very powerful in cases where the analytical equations do not exist for the special building conditions, such as the top speed not attained in one floor trip and for unequal floor heights.

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