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Bilevel Capacitated Fixed Charge Transportation Problem

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Abstract

Bilevel programming is a two-stage optimization problem where the constraint region of the first level problem is implicitly determined by another optimization problem. This paper is divided into two sections. In section I of this paper, we consider the bilevel programming problem in which both the leader and the followers' problem are capacitated fixed charge transportation problems with bounds on total availabilities at sources and total destination requirements. In section II, we restrict the transportation flow to a known specified level. Both the problems are converted into a standard fixed charge transportation problems. The algorithms based on the concept that the optimal solution of Bilevel programming problem lies at an extreme point are presented to solve the problems which are illustrated with the help of an example.

Keywords: Bilevel programming problem, Non-convex optimization, Fixed charge transportation problem, capacitated transportation problem, Restricted flow.

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Introduction

Bilevel programming problem has been developed and studied by Bialas and Karwan [5, 6] in the year 1982, 1984; Candler and Townsley [9] in 1982; Bard [1, 2,3] in the year 1982, 84, 92. The bilevel programming structure has been used to model problems concerning traffic signal optimization [19], structural design [22] and genetic algorithms [11]. The transportation problem is a subclass of linear programming problem. There are different types of transportation problems and the simplest of them was presented by Hitchcock in 1941 [13], along with a constructive solution and later independently by Koopman in 1947 [17]. Brigden [8] in 1974 considered the transportation problem with mixed constraints.

The fixed charge transportation problem (FCTP) is a non-convex transportation problem. It was originally formulated by Dantzig and Hirsch [12] in 1954. K.G. Murthy [18] solved the (FCTP) by ranking the extreme points. Sandrock [21] gave a simple algorithm for solving a (FCTP), Basu et.al [4] gave an algorithm for solving a (FCTP). Khanna et al. [15, 16] developed techniques for solving the transportation problem when the flow is either restricted or enhanced.

Capacitated transportation problem with bounds on total availabilities at sources and total destinations, find their applications in a variety of real world problems like telecommunication networks, production-distribution system, rail and urban-road system where finite capacity of resources such as vehicles, docks, parking places etc. have to be taken into account and equipment capacity, location shipping and receiving constraints are typically experienced. Charnes and Klingman [10], Verma and Puri [23] have discussed minimization of dead-Mileage assessed in terms of running buses from various depots to starting points. If total flow in transportation problem with bounds on sources and destinations are considered, them the resulting problem makes the transportation model more realistic. Kassay [14] gave an operator method for solving capacitated transportation problem. Later on, Bit et al. [7], Zheng et al. [25] and Rachev and Olkin [20] worked on capacitated transportation problems.

SECTION I

A Bilevel Capacitated Fixed Charge Transportation Problem (BCFCTP) is defined as

(BCFCTP): $\underset{X_1}{\text{Min}}\, Z_1 = c_1^T X_1 + c_2^T X_2 + F_1$

where X_2 solves, for a given X_1

$\underset{X_2}{\text{Min}}\, Z_2 = d_1^T X_1 + d_2^T X_2 + F_2$

subject to

$$\left.\begin{aligned} a_i \le \sum_{j\in J} x_{ij} \le A_i, \quad & \forall\, i \in I \\ b_j \le \sum_{i\in I} x_{ij} \le B_j, \quad & \forall\, j \in J \end{aligned}\right\} \tag{1}$$

$$\ell_{ij} \le x_{ij} \le u_{ij} \ \text{ \& integers } \ \forall\, i \in I_1,\ j \in J_1 \tag{2}$$

$$\ell'_{ij} \le x_{ij} \le u'_{ij} \ \text{ \& integers } \ \forall\, i \in I_2,\ j \in J_2 \tag{3}$$

where $c_1 = [c_{ij}]$, $i \in I_1 = \{1, 2, .., m_1\}$; $j \in J_1 = \{1, 2, ..., n_1\}$

$c_2 = [c_{ij}]$, $i \in I_2 = \{m_1+1, .., m\}$; $j \in J_2 = \{n_1+1, ..., n\}$

$d_1 = [d_{ij}]$, $i \in I_1$, $j \in J_1$ and $d_2 = [d_{ij}]$, $i \in I_2$, $j \in J_2$

$I = I_1 \cup I_2 = \{1, 2, .., m\}$

$J = J_1 \cup J_2 = \{1, 2, ..., n\}$

$X_1 = [x_{ij}]$, $i \in I_1$, $j \in J_1$ and $X_2 = [x_{ij}]$; $i \in I_2$, $j \in J_2$ are the variables controlled by the upper level and lower level problems respectively.

Here, ℓ_{ij} is assumed to be non-negative for all $i \in I$, $j \in J$. Let ℓ_{ij} , u_{ij} and ℓ'_{ij} , u'_{ij} be the minimum and maximum number of units to be transported from the ith origin to the jth destination for the upper level and for the lower level problems respectively.

Here, a_i $(i \in I)$ and A_i $(i \in I)$ are the bounds on the goods available at the ith origin and b_j $(j \in J)$ and $B_j (j \in J)$ are the bounds on the demands at the jth destinations. c_{ij} and d_{ij} $(i \in I, j \in J)$ are per unit costs of transportation of goods from the ith origin to the jth destination of the upper level and the lower level problems respectively.

$F_1 = \sum_{i \in I} F_i$ is the total fixed cost for the upper level problem and F_i is the fixed cost associated with origin i for the upper level problem and $F_2 = \sum_{i \in I} F'_i$ is the total fixed cost for the lower level problem and F'_i is the fixed cost associated with origin i for the lower level problem.

For formulation of F_i $(i \in I)$, assume that F_i $(i \in I)$ has p number of steps so that $F_i = \sum_{\ell=1}^{p} \delta_{i\ell} F_{i\ell}$, $i \in I$ where

$$\delta_{i\ell} = 1 \text{ if } \sum_{j=1}^{n} x_{ij} > A_{i\ell},\ i \in I,\ \ell \in L = \{1, 2, ..., p\}$$

$$= 0 \text{ otherwise.}$$

Here, $0 < A_{i_1} < A_{i_2} < ... < A_{i_p}$.

$A_{i_1}, A_{i_2}, \dots, A_{i_p}$ $(i \in I)$ are constants and $F_{i\ell}\,(i \in I,\ \ell \in L)$ are fixed costs.

Similarly, for formulation of $F_i'(i \in I)$, assume that $F_i'(i \in I)$ has q number of steps, so that $F_i' = \sum_{k=1}^{q} \delta_{ik}' F_{ik}'$, $i \in I$,

where $\delta_{ik}' = 1$ if $\sum_{j=1}^{n} x_{ij} > B_{ik}$, $i \in I$, $k \in K = \{1, 2, \dots, q\}$

$= 0$ otherwise.

Here, $0 < B_{i_1} < B_{i_2} < \dots < B_{i_q}$

$B_{i_1}, B_{i_2}, \dots, B_{i_q}$ $(i \in I)$ are constants and $F_{ik}'\,(i \in I,\ k \in K)$ are fixed costs.

Algorithmic Development for (BCFCTP)

To solve the problems (BCFCTP), we separate it into two problems, upper level capacitated fixed charge transportation problem (UCP) and lower level capacitated fixed charge transportation problem (LCP), defined as

(UCP) : $\min\limits_{X_1} Z_1 = c_1^T X_1 + c_2^T X_2 + F_1$

subject to (1) and (2).

(LCP) : $\min\limits_{X_2} Z_2 = d_1^T X_1 + d_2^T X_2 + F_2$, for a given X_1

subject to (1) and (3).

Problem (UCP)

Consider the upper level capacitated fixed charge transportation problem as

(UCP) : $$\text{Min } Z_1 = \sum_{i \in I_1} \sum_{j \in J_1} c_{ij} x_{ij} + \sum_{i \in I_2} \sum_{j \in J_2} c_{ij} x_{ij} + \sum_{i \in I} F_i$$

subject to (1) and (2)

or $$\text{Min } Z_1 = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} F_i$$

subject to (1) and (2).

In order to solve problem (UCP), consider the related transportation problem (RUCP) as follows:

(RUCP) : $$\text{Min } Z_1' = \sum_{i \in I'} \sum_{j \in J'} c_{ij}' t_{ij} + \sum_{i \in I'} F_i'$$

subject to

$$\sum_{j \in J'} t_{ij} = A_i' \quad \forall i \in I' \tag{4}$$

$$\sum_{i \in I'} t_{ij} = B_j' \quad \forall j \in J' \tag{5}$$

where $\ell_{ij} \le t_{ij} \le u_{ij} \quad \forall i \in I, j \in J$

$$0 \le t_{m+1,j} \le B_j - b_j, \quad \forall j \in J$$

$$0 \le t_{i,n+1} \le A_i - a_i, \quad \forall i \in I$$

$t_{m+1,n+1} \ge 0$ and integers,

$$A_i' = A_i \quad \forall i \in I; \qquad A_{m+1}' = \sum_{j \in J} B_j,$$

$$B_j' = B_i \quad \forall j \in J; \qquad B_{n+1}' = \sum_{i \in I} A_i,$$

$$c_{ij}' = c_{ij} \;\; \forall i \in I, j \in J; \quad c_{m+1,j}' = c_{i,n+1}' = c_{m+1,n+1}' = 0 \;\; \forall i \in I, j \in J$$

$I' = \{1, 2, ..., m, m+1\}$, $J' = \{1, 2, ..., n, n+1\}$.

$F_i' = F_i, \ \forall\, i \in I$

$F_{m+1} = 0.$

It can be shown that problems (UCP) and (RUCP) are equivalent [24].

Theorem 1. The value of the objective function of problem (UCP) at a feasible solution is equal to the value of the objective function (RUCP) at its corresponding feasible solution and conversely.

Proof: Let $\{t_{ij}\} : i \in I', j \in J'$ and $\{x_{ij}\};\ i \in I, j \in J$ be corresponding feasible solutions of problem (UCP) and (RUCP) respectively. Then

Z_1' = objective function value of (RUCP) at $\{t_{ij}\} : i \in I', j \in J'$

$$= \sum_{i \in I'} \sum_{j \in J'} c_{ij}' t_{ij} + \sum_{i \in I'} F_i$$

$$= \sum_{i \in I} \sum_{j \in J} c_{ij}' t_{ij} + \sum_{i \in I} c_{i,n+1}' t_{i,n+1} + \sum_{j \in J} c_{m+1}' t_{m+1,j} + c_{m+1,n+1}' t_{m+1,n+1} + \sum_{i \in I} F_i + F_{m+1}$$

$$= \sum_{i \in I} \sum_{j \in J} c_{ij} t_{ij} + \sum_{i \in I} F_i \qquad (\because c_{i,n+1}' = c_{m+1}' = c_{m+1,n+1}' = 0,\ c_{ij}' = c_{ij}\ \forall i \in I,\ j \in J$$

$$\text{and } F_{m+1} = 0,\ F_i' = F_i\ \ \forall i \in I)$$

$$= \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} F_i \qquad (\because t_{ij} = x_{ij},\ i \in I,\ j \in J)$$

= objective function value of problem (UCP) at $\{x_{ij}\}$; $i \in I, j \in J$.

Similarly, the converse can be proved.

To find the Optimal Solution of the Problem (RUCP)

In order to solve the problem (RUCP), an additional source and an additional destination have been added. The method moves from an $(m+1)\times(n+1)$ capacitated fixed charge transportation problem to an $(m\times n)$ capacitated fixed charge transportation problem.

Let $\{t'_{ij}\}$, $i \in I'$, $j \in J'$ be a basic solution of the problem (RUCP), with respect to the variable cost using upper bounding simplex method. Let B be the basis matrix and N_1 and N_2 denote the set of non-basic cells (i, j) which are at their lower bounds and upper bounds respectively. Find the corresponding fixed cost. Let it be denoted by F_1^k (current), where $F_1^k(\text{current}) = \sum_{i \in I} F_i$.

Also, $(A_{ij}^k)_1 = (c'_{ij})_k \times (E_{ij})_k$, where $(c'_{ij})_k = c'_{ij} - u_i^k - v_j^k, \forall (i, j) \notin B, u_i^k, v_j^k$ ($i \in I'$, $j \in J'$) are the dual variables and $(A_{ij}^k)_1$, is the change in the cost of the upper level that occurs on introducing a non-basic cell (i, j) with value $(E_{ij})_k$ into the basis by making reallocations.

Find $(F_{ij}^k)_1$ (Difference) = $(F_{ij}^k)_1$(NB) − F_1^k (current), where $(F_{ij}^k)_1$(NB) is the total fixed cost obtained on introducing the cell (i, j) into the basis.

Find $(\Delta_{ij}^k)_1 = (F_{ij}^k)_1$ (Difference) + $(A_{ij}^k)_1$, $\forall$ (i, j) $\notin$ B.

Optimality Criterion for the Problem (RUCP)

The basic feasible solution $\{t'_{ij}\}$, $i \in I'$, $j \in J'$ for the problem (RUCP) with basis matrix B will be an optimal basic feasible solution if $(\Delta_{ij}^k)_1 \geq 0 \ \forall (i, j) \in N_1$ and $(\Delta_{ij}^k)_1 \leq 0 \ \forall (i, j) \in N_2$.

If $(\Delta_{pq}^k)_1 < 0$ for some $(p, q) \in N_1$, then one can move from one basic feasible solution to another basic feasible solution on entering the cell (p, q) $\in N_1$ into the basis which undergoes change by an amount θ_{pq} given by

$$\theta_{pq} = \min\{u_{pq} - \ell_{pq};\ t'_{pq} - \ell_{pq} \text{ for all basic cells with } (-\theta) \text{ entry in the } \theta\text{-loop};\ u_{pq} - t'_{pq} \text{ for all basic cells with } (+\theta) \text{ entry in the } \theta\text{-loop}\}$$

Similarly, when non-basic variable $t'_{pq} \in N_2$ undergoes change by an amount θ_{pq}, the solution can be improved.

Optimal Solution for the Problem (UCP)

Since (RUCP) and (UCP) are equivalent problem, therefore, optimal solution for (RUCP) yields the optimal solution for (UCP). Let the optimal solution of the upper level capacitated fixed charge transportation problem be denoted by $X^* = (X_1^*, X_2^*)$, with the value of the objective function as Z_1^*. Putting the value of $X_1 = X_1^*$ in the lower level capacitated fixed charge transportation problem (LCP), its related problem (RLCP) is formulated and solved by the method explained above. Let $\hat{X}_2$ be its optimal solution with the value of the objective function as $\hat{Z}_2$.

If $X_2^* = \hat{X}_2$, then X^* is the optimal solution of the given problem (BCFCTP).

If $X_2^* \neq \hat{X}_2$, then find an alternate optimal solution to the problem (UCP). If there exists an alternate solution $X^{**} = (X_1^{**}, X_2^{**})$, repeat the above process for $X_1 = X_1^{**}$. Let $\hat{\hat{X}}_2$ be optimal solution of (LCP).

If $X_2^{**} = \hat{\hat{X}}_2$, then X_2^{**} is the optimal solution of given (BCFCTP). If not, then test for other alternate solutions till we get the optimal solution of (BCFCTP). This process must end in a finite number of steps because the solution of (BCFCTP) lies on an extreme point which are finite in number.

Algorithm for Solving (BCFCTP)

Step 1: Consider the Bilevel Capacitated Fixed Charge Transportation Problem (BCFCTP).

Step 2: Separate the problem (BCFCTP) into the problems (UCP) and (LCP).

Step 3: Set K = 0, where K is the number of iterations in the algorithm.

Step 4: Set K = K + 1, K = 0, 1, 2, ...

Step 5: To solve (UCP), formulate its related capacitated fixed charge transportation problem (RUCP), by introducing additional rows and columns respectively. Find a basic feasible solution of this problem with respect to the variable costs only. Find the corresponding basic feasible solution of (UCP).

Step 6: Find the corresponding fixed cost. Let it be denoted by F_1^K (current), where $F_1^K \text{ (current)} = \sum_{i \in I} F_i$.

Also, find $(A_{ij}^K)_1 = (c'_{ij})_k \times (E_{ij})_k$ and

$(F_{ij}^K)_1$ (Difference) = $(F_{ij}^K)_1$ (NB) - F_1^K (Current).

Step 7: Find $(\Delta_{ij}^K)_1 = (F_{ij}^K)_1$ (Difference) + $(A_{ij}^K)_1$, $\forall$ (i, j) $\notin$ B.

If $(\Delta_{ij}^K)_1 \geq 0 \ \forall (i, j) \in N_1$ and $(\Delta_{ij}^K)_1 \leq 0 \ \forall (i, j) \in N_2$, then the optimal solution of (RUCP) is obtained. Go to step 9.

If $(\Delta_{pq}^K) < 0$ for some (p, q)$\in N_1$ or $(\Delta_{pq}^K) > 0$ for some (p, q) $\in N_2$, then go to step 8.

Step 8: If $(\Delta_{pq}^K) < 0$, for some (p, q) $\in N_1$, enter the cell (p, q) into the basis which undergoes change by an amount given by

$\theta_{pq} = \text{Min}\{u_{pq} - \ell_{pq};\ t'_{pq} - \ell_{pq}$ for all basic cells with (–θ) entry in the θ-loop; $u_{pq} - t'_{pq}$ for all basic cells with (+θ) entry in the θ -loop}

Go to step 6.

Similarly, if $(\Delta_{pq}^K) > 0$, for some (p, q) $\in N_2$, improve the solution by an amount θ_{pq} and go to step 6.

Step 9: Let the optimal solution of the problem (UCP) be denoted by $X_1^* = (X_1^*, X_2^*)$.

Step 10: For a given $X_1 = X_1^*$, solve the problem (LCP). Formulate its related capacitated fixed charge transportation problem (RLCP) and solve it by the method explained above. Let $\hat{X}_2$ be its optimal solution**.**

Step 11: If $X_2^* = \hat{X}_2$, then X^* is an optimal solution of the given problem (BCFCTP).

If $X_2^* \neq \hat{X}_2$ find all possible alternate solutions of (UCP) and go to step 9.

The procedure is repeated till an optimal solution of (BCFCTP) is obtained.

Example 1: Consider the following capacitated bilevel fixed charge transportation problem as

$$\underset{X_1}{\text{Minimize}}\ Z_1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}x_{ij} + \sum_{i=1}^{3} F_i$$

where X_2 solves

$$\underset{X_2}{\text{Minimize}}\ Z_2 = \sum_{i=1}^{3} \sum_{j=1}^{4} d_{ij}x_{ij} + \sum_{i=1}^{3} F_i'$$

subject to

$$5 \le \sum_{j=1}^{4} x_{1j} \le 26, \qquad 9 \le \sum_{j=1}^{4} x_{2j} \le 20$$

$$18 \le \sum_{j=1}^{4} x_{3j} \le 24, \qquad 4 \le \sum_{i=1}^{3} x_{i1} \le 10, \tag{6}$$

$$10 \le \sum_{i=1}^{3} x_{i2} \le 22, \qquad 10 \le \sum_{i=1}^{4} x_{i3} \le 25, \qquad 20 \le \sum_{i=1}^{3} x_{i4} \le 27,$$

where

$$3 \le x_{11} \le 15,\ 1 \le x_{12} \le 10,\ 0 \le x_{13} \le 15,\ 2 \le x_{14} \le 8,$$

$$1 \le x_{21} \le 10,\ 4 \le x_{22} \le 12,\ 1 \le x_{23} \le 14,\ 0 \le x_{24} \le 7, \qquad (7)$$

$$1 \le x_{31} \le 5,\ 0 \le x_{32} \le 12,\ 1 \le x_{33} \le 11,\ 0 \le x_{34} \le 14$$

are the bounds on the upper level problem, and

$$1 \le x_{11} \le 10,\ 4 \le x_{12} \le 12,\ 1 \le x_{13} \le 14,\ 0 \le x_{14} \le 7,$$

$$1 \le x_{21} \le 5,\ 0 \le x_{22} \le 12,\ 1 \le x_{23} \le 11,\ 0 \le x_{24} \le 14, \qquad (8)$$

$$0 \le x_{31} \le 6,\ 0 \le x_{32} \le 16,\ 0 \le x_{33} \le 15,\ 0 \le x_{34} \le 7$$

are the bounds on the lower level problem.

Here, $X_1 = (x_{11}, x_{12}, x_{13}, x_{14})$ are the variables controlled by the leader. $X_2 = (x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34})$ are the variables controlled by the follower.

The above problem is separated into two problems.

The upper level problem (UCP) is

$$\underset{X_1}{\text{Minimize}}\ Z_1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij} + \sum_{i=1}^{3} F_i$$

subject to (6) and (7).

The lower level problem (LCP) is

$$\underset{X_2}{\text{Minimize}}\ Z_2 = \sum_{i=1}^{3} \sum_{j=1}^{4} d_{ij} x_{ij} + \sum_{i=1}^{3} F_i'$$

subject to (6) and (8).

Table (1) and (2) give the values of the variable cost c_{ij} $(i = 1, 2, 3;\ j = 1, 2, 3, 4)$ and d_{ij} $(i = 1, 2, 3;\ j = 1, 2, 3, 4)$ for the upper level and lower level problems respectively.

2	2	1	2
3	2	1	2
4	2	3	1

Table 1

1	2	1	2
4	5	3	3
3	2	2	1

Table 2

The fixed costs for the upper level problem are

$F_{11} = 10$, $F_{12} = 10$, $F_{13} = 5$, $F_{14} = 5$, $F_{21} = 5$, $F_{22} = 5$, $F_{23} = 10$,

$F_{24} = 5$, $F_{31} = 10$, $F_{32} = 5$, $F_{33} = 5$, $F_{34} = 10$

The total cost which is to be minimized is given by

$$\left(\sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} + \sum_{i=1}^{3} F_i\right), \text{ where } \sum_{\ell=1}^{3} \delta_{i\ell}F_{i\ell},\ i = 1,2,3$$

where $\delta_{i1} = 1$ if $\sum_{j=1}^{4} x_{ij} > 20 \qquad i = 1,2,3$

$= 0$ otherwise

$\delta_{i2} = 1$ if $\sum_{j=1}^{4} x_{ij} > 8 \qquad i = 1,2,3$

$= 0$ otherwise

$\delta_{i3} = 1$ if $\sum_{j=1}^{4} x_{ij} > 20 \qquad i = 1,2,3$

$= 0$ otherwise

$\delta_{i4} = 1$ if $\sum_{j=1}^{4} x_{ij} > 7 \qquad i = 1,2,3$

$= 0$ otherwise

The fixed costs for the lower level problem are

$$F'_{11}=10 \qquad F'_{12}=5 \qquad F'_{13}=5 \qquad F'_{14}=10$$

$$F'_{21}=5 \qquad F'_{22}=5 \qquad F'_{23}=10 \qquad F'_{24}=5$$

$$F'_{31}=10 \qquad F'_{32}=10 \qquad F'_{33}=5 \qquad F'_{34}=5$$

The total cost which is to be minimized is given by

$$\left(\sum_{i=1}^{3}\sum_{j=1}^{4} d_{ij}x_{ij}+\sum_{i=1}^{3} F'_i\right), \text{ where } F'_i=\sum_{k=1}^{3} \delta'_{ik}F'_{ik},\ i=1,2,3$$

where $\delta'_{i1}=1$ if $\sum_{j=1}^{4} x_{ij}>5 \qquad i=1,2,3$

= 0 otherwise

$\delta'_{i2}=1$ if $\sum_{j=1}^{4} x_{ij}>12 \qquad i=1,2,3$

= 0 otherwise

$\delta'_{i3}=1$ if $\sum_{j=1}^{4} x_{ij}>5 \qquad i=1,2,3$

= 0 otherwise

$\delta'_{i4}=1$ if $\sum_{j=1}^{4} x_{ij}>20 \qquad i=1,2,3$

= 0 otherwise

Related standard transportation problem for the upper level problem is given as

(RUCP) $\qquad \sum_{i=1}^{4}\sum_{j=1}^{5} c'_{ij}t_{ij}$

subject to

$$\sum_{j=1}^{5} t_{ij}=A'_i=26, 20, 24, 84; \qquad i=1,2,3,4$$

$$\sum_{i=1}^{4} t_{ij}=B'_j=10, 22, 25, 27, 70; \qquad j=1,2,3,4,5$$

$3 \le t_{11} \le 15,\ \ 1 \le t_{12} \le 10,\ \ 0 \le t_{13} \le 15,\ 2 \le t_{14} \le 8,$

$1 \le t_{21} \le 10,\ \ 4 \le t_{22} \le 12,\ \ 1 \le t_{23} \le 14,\ \ 0 \le t_{24} \le 7,$

$1 \le t_{31} \le 5,\ \ 0 \le t_{32} \le 12,\ \ 1 \le t_{33} \le 11,\ \ 0 \le t_{34} \le 14,$

$0 \le t_{14} \le \le 21,\ \ 0 \le t_{24} \le 11,\ \ 0 \le t_{34} \le 6$ and $t_{44} \ge 0.$

c'_{ij} (i = 1, 2, 3, 4; j = 1, 2, 3, 4, 5) are given as depicted in the following tabular form

2	2	1	2	0
3	2	1	2	0
4	2	3	1	0
0	0	0	0	0

Table 3

The initial basic feasible solution for the upper level problem using upper bounding simplex method is given in Table 4.

						u_i
	2 $\underline{3}$ 2	2 4	1 $\overline{15}$ 1	2 3	0 1	0
	3 $\underline{1}$ 3	2 $\underline{4}$ 0	1 $\underline{1}$ 1	2 3	0 $\overline{11}$ 0	0
	4 $\underline{1}$ 4	2 2	3 $\underline{1}$ 3	1 $\overline{14}$ –1	0 $\overline{6}$ 0	0
	0 5	0 $\overline{12}$ –2	0 8	0 $\overline{7}$ –2	0 52	0
v_j	0	2	0	2	0	

Table 4

Calculate $(c_{ij})_1 = c_{ij} - u'_i - v'_j \quad \forall (i, j) \notin B$

and $\quad (\Delta_{ij})_1 = (F'_{ij})_1 (\text{Difference}) + (A'_{ij})_1, \quad \forall (i, j) \notin B$

which is given in Table 5.

(i,j)	(1,1)	(1,3)	(2,1)	(2,2)	(2,3)	(2,5)	(3,1)	(3,3)	(3,4)	(3,5)	(4,2)	(4,4)
$(A_{ij})_1$	2	7	3	0	1	0	4	3	−3	0	−2	−2
$(\Delta_{ij})_1$	2	22	3	0	1	5	4	3	−3	0	−2	−2

Table 5

Since $(\Delta_{ij})_1 \geq 0$ for upper bounded variable corresponding to the cell (1,3), therefore, x_{13} enters the basis.

Proceeding as above, the optimal solution for the upper level problem so obtained is $X_1^* = (3, 4, 8, 3)$ and $X_2^* = (1, 4, 1, 3, 1, 2, 1, 14)$.

Putting the values of $X_1 = X_1^*$ in the lower level problem and solving by the same procedure as above, the optimal solution for the lower level problem is $\hat{X}_2 = (1, 4, 1, 3, 1, 2, 1, 14)$.

Since $X_2^* = \hat{X}_2$, therefore, the optimal solution for the bilevel problem is (3, 4, 8, 3, 1, 4, 1, 3, 1, 2, 1, 14), with $Z_1 = 71$, $F_1 = 40$ and $Z_2 = 84$, $F_2 = 30$.

SECTION II

Problem (BCFCTP) with Restricted Flow

If in problem (BCFCTP), total flow is specified, that is, $\sum_{i \in I} \sum_{j \in J} x_{ij} = P$, then problem can be stated as

$$(\text{BCFCTPF}): \underset{X_1}{\text{Min}}\, Z_1 = c_1^T X_1 + c_2^T X_2 + F_1$$

where X_2 solves, for a given X_1

$$\underset{X_2}{\text{Min}}\, Z_2 = d_1^T X_1 + d_2^T X_2 + F_2$$

subject to

$$\left.\begin{aligned} a_i \le \sum_{j\in J} x_{ij} \le A_i, \quad & \forall i \in I \\ b_j \le \sum_{i\in I} x_{ij} \le B_j, \quad & \forall j \in J \end{aligned}\right\} \tag{9}$$

$$\ell_{ij} \le x_{ij} \le u_{ij} \quad \& \text{ integers} \quad \forall i \in I_1,\ j \in J_1 \tag{10}$$

$$\ell'_{ij} \le x_{ij} \le u'_{ij} \quad \& \text{ integers} \quad \forall i \in I_2,\ j \in J_2 \tag{11}$$

$$\sum_{i\in I} \sum_{j\in J} x_{ij} = P, \left(P < \text{Min}\left\{ \sum_{i\in I} A_i, \sum_{j\in J} B_j \right\} \right) \tag{12}$$

$I = I_1 \cup I_2 = \{1, 2, .., m\}$ denote the origins,

$J = J_1 \cup J_2 = \{1, 2, ..., n\}$ denote the destinations.

The symbols defined in the above problem are same as defined in Section I.

Algorithm Development for (BCFCTPF)

To solve the problem (BCFCTPF), we separate it into two problems, upper level capacitated fixed charge transportation problem with restricted flow (UCPF) and lower level capacitated fixed charge transportation problem with restricted flow (LCPF), defined as

(UCPF): $\underset{X_1}{\text{Min}}\, Z_1 = c_1^T X_1 + c_2^T X_2 + F_1$

subject to (9), (10) and (12).

$\underset{X_2}{\text{Min}}\, Z_2 = d_1^T X_1 + d_2^T X_2 + F_2$, for a given X_1

subject to (9), (11) and (12).

The flow constraint in the problem (BCFCTPF) implies that a total of $\left(\sum_{i \in I} A_i - P\right)$ of source reserves has to be kept at the various sources and a total of $\left(\sum_{j \in J} B_j - P\right)$ of destination slacks is to be retained at the various destinations. Therefore, an extra destination to receive the source reserves and an extra source to fill up the destination slacks are introduced. Hence, the Related Bilevel capacitated Fixed Charge Transportation Problem with Restricted Flow for upper level and lower level problems are defined.

To solve the problem (UCPF) the related capacitated fixed charge transportation problem with restricted flow is formulated with an additional supply point and an additional destination point, defined as

(RUCPF): $\text{Min}\, Z_1 = c_1'^T Y_1 + c_2'^T Y_2 + F_1'$

subject to

$$\sum_{j \in J'} y_{ij} = A_i' \quad \forall i \in I' = I \cup \{m+1\}$$

$$\sum_{i \in I'} y_{ij} = B_j' \quad \forall j \in J' = J \cup \{n+1\}$$

$$\ell_{ij} \le y_{ij} \le u_{ij} \quad \forall i \in I, j \in J$$

$$0 \le y_{i,n+1} \le A_i - a_i; \ \ i \in I$$

$0 \le y_{m+1,j} \le B_j - b_j;\ j \in J;\ \ y_{m+1,n+1} \ge 0.$

$c'_{ij} = c_{ij},\ i \in I, j \in J;\ c'_{i,n+1} = c'_{m+1,j} = 0, \qquad i \in I, j \in J,$

$c'_{m+1,n+1} = M.$

$A'_i = A_i,\ i \in I, \qquad B'_j = B_j,\ \ j \in J$

$A'_{m+1} = \sum_{j \in J} B_j - P, \quad B'_{n+1} = \sum_{i \in I} A_i - P$

$F'_{m+1} = 0.$

Definition 1: A feasible solution $\{y_{ij}\}_{I' \times J'}$ to solve the problem (RUCPF) is called a convex feasible solution (cfs) if $y_{m+1,n+1} = 0$.

Theorem 2 [24]: There is a one-one correspondence between the feasible solution to (UCPF) and the corner feasible solution to (RUCPF).

Theorem 3: Let $\{x_{ij}\}_{I \times J}$ be a feasible solution of (UCPF) and let its corresponding cfs be $\{y_{ij}\}_{I' \times J'}$. Then the values of the objective function of (UCPF) and (RUCPF) are equal.

Proof: The value of the objective function (RUCPF) at the feasible solution

$\{y_{ij}\}$ is $= \sum_{i \in I'} \sum_{j \in J'} c'_{ij} y_{ij} + \sum_{i \in I'} F'_i$

$$= \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ij} + \sum_{i \in I} c'_{i,n+1}, y_{i,n+1} + \sum_{j \in J} c'_{m+1,j} y_{m+1,j}$$

$$+ c'_{m+1,n+1} y_{m+1,n+1} + \sum_{i \in I} F'_i + F'_{m+1}$$

$$= \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} F_i \qquad (\because c'_{m+1,j} = c'_{i,n+1} = 0, c'_{m+1,n+1} = 0,\ y_{m+1,n+1} = 0)$$

= the value of the objective function of (UCPF) at the corresponding feasible solution $\{x_{ij}\}$.

Theorem 4: The optimal solutions of (UCPF) and (RUCPF) are equivalent.

Optimal Solution for the Problem (UCPF)

Solve the problem (RUCPF) with respect to the variable cost using upper bounding simplex method. The fixed cost for the problem (RUCPF) is calculated by the method explained in section I.

On solving the problem (RUCPF), let $Y^* = \{y_{ij}^*\},\ i \in I', j \in J'$ be the basic feasible solution. It will be optimal if $(\Delta_{ij}^K)_1 \geq 0\ \forall (i,j) \in N_1$ and $(\Delta_{ij}^K)_1 \leq 0\ \forall (i,j) \in N_2$. The optimal solution $\{x_{ij}^*\}$ to the problem (UCPF) is then derived using the following transformation.

$$y_{ij} = x_{ij}, \qquad i \in I,\ j \in J$$

$$y_{i,n+1} = A_i - \sum_{j} x_{ij}, \quad i \in I$$

$$y_{m+1,j} = B_j - \sum_{i \in I} x_{ij}, \quad j \in J$$

$$y_{m+1,n+1} = 0 .$$

Let the optimal solution of the problem (BCFCTPF) be denoted by $X^* = (X_1^*, X_2^*)$, with the value of the objective function as Z_1^*. Putting the value of $X_1 = X_1^*$ in the lower level capacitated fixed charge transportation problem (LCPF), its related problem (RLCPF) is formulated by the method explained above. Let $\hat{X}_2$ be its optimal solution with the value of the objective function as $\hat{Z}_2$.

If $X_2^* = \hat{X}_2$, then X^* is the optimal solution of the given problem (BCFCTPF).

If $X_2^* \neq \hat{X}_2$, then find an alternate solution of the problem (BCFCTPF). If not, then test for other alternate solutions till we get the optimal solution of (BCFCTPF). This process must end in a finite number of steps because the solution of (BCFCTPF) lies on an extreme point which are finite in number.

Example 2: Consider the bilevel capacitated fixed charge transportation problem, defined in Example 1. The bounds on the upper level and the lower level problems are same as defined in example 1. The fixed costs for the upper level problem are

$F_{11} = 5$, $F_{12} = 10$, $F_{13} = 5$, $F_{14} = 10$,

$F_{21} = 5$, $F_{22} = 5$, $F_{23} = 5$, $F_{24} = 10$

$F_{31} = 5$, $F_{32} = 5$, $F_{33} = 10$, $F_{34} = 10$

The total cost which is to be minimized is given by $\left(\sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} + \sum_{i=1}^{3} F_i\right)$,

where $\sum_{\ell=1}^{3} \delta_{i\ell}F_{i\ell}, \quad i = 1, 2, 3.$

where $\delta_{i1} = 1$ if $\sum_{j=1}^{4} x_{ij} > 5 \quad i = 1, 2, 3$
$= 0$ otherwise

$\delta_{i2} = 1$ if $\sum_{j=1}^{4} x_{ij} > 12 \quad i = 1, 2, 3$
$= 0$ otherwise

$\delta_{i3} = 1$ if $\sum_{j=1}^{4} x_{ij} > 15 \quad i = 1, 2, 3$
$= 0$ otherwise

$\delta_{i4} = 1$ if $\sum_{j=1}^{4} x_{ij} > 22 \quad i = 1, 2, 3$
$= 0$ otherwise

The fixed costs for the lower level problem are

$F'_{11} = 5$	$F'_{12} = 5$	$F'_{13} = 10$	$F'_{14} = 10$
$F'_{21} = 10$	$F'_{22} = 10$	$F'_{23} = 5$	$F'_{24} = 5$
$F'_{31} = 5$	$F'_{32} = 5$	$F'_{33} = 5$	$F'_{34} = 5$

The total cost which is to be minimized is given by

$$\left(\sum_{i=1}^{3}\sum_{j=1}^{4} d_{ij}x_{ij} + \sum_{i=1}^{3} F_i'\right), \text{ where } F_i' = \sum_{k=1}^{3} \delta_{ik}' F_{ik}', \quad i = 1, 2, 3$$

where $\delta_{i1}' = 1 \text{ if } \sum_{j=1}^{4} x_{ij} > 6 \qquad i = 1, 2, 3$

= 0 Otherwise

$\delta_{i2}' = 1 \text{ if } \sum_{j=1}^{4} x_{ij} > 7 \qquad i = 1, 2, 3$

= 0 Otherwise

$\delta_{i3}' = 1 \text{ if } \sum_{j=1}^{4} x_{ij} > 10 \qquad i = 1, 2, 3$

= 0 Otherwise

$\delta_{i4}' = 1 \text{ if } \sum_{j=1}^{4} x_{ij} > 15 \qquad i = 1, 2, 3$

= 0 Otherwise

Consider the above problem with additional flow constraint $\sum_{i=1}^{3}\sum_{j=1}^{4} x_{ij} = 45$.

Formulate the related standard transportation problem for the upper level problem with additional flow constraint. The optimal solution for the upper level problem so obtained by the upper bounding simplex method is $X_1^* = (3, 4, 8, 3)$ and X_2^* = (1, 4, 1, 3, 1, 2, 14).

Putting the values of X_2^* in the lower level problem and solving by the method explained above, we get $\hat{X}_2$ = (1, 4, 1, 3, 1, 2, 14).

Since $X_2^* = \hat{X}_2$, therefore the optimal solution for the bilevel problem is (3, 4, 8, 3, 1, 4, 1, 3, 1, 2, 1, 14), with Z_1 = 71, F_1 = 45 and Z_2 = 84, F_2 = 40.

CONCLUSIONS

The algorithm moves from one extreme point to another extreme point in (BCFCTP) as well as in (BCFCTPF). Since the extreme points are finite in number, therefore, the procedure must end in a finite number of steps and the optimal solutions of both the problems (BCFCTP) and (BCFCTPF) lies on an extreme point.

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