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Entropy and Similarity Measures For Interval-Valued Intuitionistic Fuzzy Sets Based on Intuitionism and Fuzziness

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Abstract. Many authors have developed lots of entropies and similarity measures for interval-valued intuitionistic fuzzy sets(IvIFSs). In this paper, we first review some famous entropies for IvIFSs. Then, a new entropy for IvIFSs based on intuitionism and fuzziness is proposed and proved to satisfy the axiomatic requirements given by Liu and Zhang. Furthermore, a new class of similarity measure for IvIFSs is developed based on the above proposed entropy. Besides, a numerical example is demonstrated to verify the efficiency of the developed entropy.

Keywords. Interval-valued intuitionistic fuzzy sets; Entropy; Similarity measure

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1 Introduction

To treat the imperfect and uncertain information in real world, Zadeh[1] introduced the concept of fuzzy set(FS), which can depict the fuzziness of nature by the membership degree and the nonmembership degree. Then, the classical FS theory is extended by many researchers from different perspective, and many extensions are developed. Among them, initionistic fuzzy set(IFS) proposed by Atanassov[2], is one of the most important extensions, characterized by a membership degree, a non-membership degree and a hesitancy degree, which make the IFS more flexible than FS to deal with uncertainty. In addition to IFS, interval valued fuzzy set(IvFS) introduced by Zadeh[3] and vague set(VS) proposed by Gau and Buchrer[4] are also famous generalizations of classical FS. Later, Deschrijver and Kerre[5] pointed out that IvFS is equivalent to IFS, and Bustince and Burillo[6] showed that VS is also equivalent to IFS.

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Then, the IFS is further extended to interval-valued intuitionistic fuzzy set(IvIFS) by Atanassov and Gargov[7].

Entropy and similarity measures are two hot topics in fuzzy set theory, which have been applied in various fields, such as group decision making, medical diagnosis, etc. The notion of entropy was firstly introduced by Zadeh[9], which is used to estimate the fuzziness of FSs. Then, the entropy theory is investigated by many researchers. Many entropy formulas have been proposed. Recently, some important principles with which the entropy should comply are proposed, such as in [10], Vlachos and Sergiadis pointed out that entropy for IFSs should measure both intuitionism and fuzziness of an IFS, and in [11], Mao, Yao, and Wang showed that entropy should increase along with the fuzziness under the same intuitionism, and equivalently, it should decrease along with the weakened intuitionism under the same fuzziness. In fact, the above principles are also applicable to the entropies for IvIFSs.

Similarity measure for fuzzy sets can indicate the similarity degree between fuzzy sets and can be viewed as the counterpart of distance measure which depicts the discrepancy between fuzzy sets. Li and Cheng[12] firstly gave the axiomatic definition of the similarity measure for IFSs and defined a similarity measure for IFSs, and discussed its application to pattern recognitions. Wei, Wang and Zhang[13] have proposed a new similarity measure based on the entropy measure for IvIFSs. Many literatures indicate that entropy for IvIFSs has strong relationship with similarity measure between IvIFSs. Some authors proved that entropy and similarity measure for IvIFSs can be transformed by each other, such as Wei, Wang and Zhang[13], Zhang et al[14].

In this paper, we propose a new entropy for IvIFSs based on both intuitionism and fuzziness, which not only satisfy the axiomatic requirements given by Liu and Zhang[15], but also fulfill the above mentioned two principles. Then, based on the new proposed entropy for IvIFSs and the relationship between entropy and similarity measure, we develop a new similarity measure for IvIFSs, which has simple structure.

We organize the paper as follows. In Section 2, a brief introduction to the basic notions of IvIFSs and some widely used entropies are presented. In Section 3, a new entropy for IvIFSs is presented and proved to meet the entropy axiom of IvIFSs, and a new similarity measure for IvIFSs is also proposed by the transformation method between entropy and similarity measure. In Section 4, a example is given to compare the proposed entropy with some existing entropies. Conclusions are given in Section 5.

2 Preliminaries

In this section, the basic concepts of IvIFSs which will be used in the following analysis are introduced.

Definition 1.[6] An interval-valued intuitionistic fuzzy set(IvIFS) A in the finite universe X is expressed by the form

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\},$$

where $\mu_A(x) = [\mu_A^-(x), \mu_A^+(x)] \in [0,1]$ is called membership interval of element x to IvIFS A, while $v_A(x) = [v_A^-(x), v_A^+(x)] \in [0,1]$ is the non-membership interval of that element to the set A, and the condition $0 \le \mu_A^+(x) + v_A^+(x) \le 1$ must hold for any $x \in X$.

For convenience of notations, we denote by IvIFS(X) the set of all the IvIFS in X.

We call the interval

$$[1 - \mu_A^+(x) - v_A^+(x), 1 - \mu_A^-(x) - v_A^-(x)],$$

abbreviated by $[\pi_A^-(x), \pi_A^+(x)]$ and denoted by $\pi_A(x)$, the interval-valued intuitionistic index of x in A, which is a hesitancy degree of x to A.

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a discrete universe and $A = \{\langle x_i, \mu_A(x_i), v_A(x_i)\rangle | x_i \in X\}$, $B = \{\langle x_i, \mu_B(x_i), v_B(x_i)\rangle | x_i \in X\} \in$ IvIFS(X), then some operations can be defined as follows:

$$A^C = \{\langle x_i, [v_A^-(x_i), v_A^+(x_i)], [\mu_A^-(x_i), \mu_A^+(x_i)]\rangle | x_i \in X\};$$

$$A \subseteq B, \text{ iff } [\mu_A^-(x_i), \mu_A^+(x_i)] \le [\mu_B^-(x_i), \mu_B^+(x_i)], \text{ and } [v_A^-(x_i), v_A^+(x_i)] \ge [v_B^-(x_i) v_B^+(x_i)] \quad x_i \in X.$$

Definition 2. [15] A real function E : IvIFS$(X) \to [0,1]$ is named an entropy on IvIFS, if E satisfies all the following properties:

(P1) $E(A) = 0$ (minimum) iff A is a crisp set;

(P2) $E(A) = 1$ (maximum) iff $\mu_A(x_i) = v_A(x_i)$, $\forall x_i \in X$;

(P3) $E(A) \le E(B)$ if A is less fuzzy than B, which is defined as

$$\mu_A(x_i) \le \mu_B(x_i), v_A(x_i) \ge v_B(x_i), \text{ for } \mu_B(x_i) \le v_B(x_i);$$

$$\mu_A(x_i) \ge \mu_B(x_i), v_A(x_i) \le v_B(x_i), \text{ for } \mu_B(x_i) \ge v_B(x_i),$$

for any $x_i \in X$;

(P4) $E(A) = E(A^C)$.

Definition 3.[14] A real-valued function S : IvIFS$(X) \times$ IvIFS$(X) \to [0,1]$ is called a similarity measure on IvIFS(X), if it satisfies the following axiomatic requirements:

(S1) $0 \le S(A,B) \le 1$; (S2) $S(A,B) = 1$ iff $A = B$;

(S3) $S(A,B) = S(B,A)$; (S4) If $A \subseteq B \subseteq C$, then $S(A,C) \le S(A,B) \wedge S(B,C)$.

Many entropy formulas for IvIFSs have been proposed, which can be divided into three classes.

Class 1. The entropies based on the intuitionism $\pi_A^-(x_i) + \pi_A^+(x_i)$, such as:

$$E_{B1}(A) = \frac{1}{2n} \sum_{i=1}^{n} (\pi_A^-(x_i) + \pi_A^+(x_i)) \tag{1}$$

which can be viewed as the generalized form of the entropy for IFSs proposed by Burillo and Bustince in [8].

Class 2. The entropies based on the fuzziness $|\mu_A^-(x_i) - v_A^-(x_i)| + |\mu_A^+(x_i) - v_A^+(x_i)|$, such as:

$$E_{\mathrm{ZJ}}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A^-(x_i) \wedge v_A^-(x_i) + \mu_A^+(x_i) \wedge v_A^+(x_i)}{\mu_A^-(x_i) \vee v_A^-(x_i) + \mu_A^+(x_i) \vee v_A^+(x_i)}. \tag{2}$$

Class 3. The entropies based on both the intuitionism and the fuzziness, such as.

$$E_{\mathrm{WW}}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A^-(x_i) \wedge v_A^-(x_i) + \mu_A^+(x_i) \wedge v_A^+(x_i) + \pi_A^-(x_i) + \pi_A^+(x_i)}{\mu_A^-(x_i) \vee v_A^-(x_i) + \mu_A^+(x_i) \vee v_A^+(x_i) + \pi_A^-(x_i) + \pi_A^+(x_i)}. \tag{3}$$

3 Entropy and Similarity Measure for IvIFSs

In the following, we shall introduce a new entropy formula and an induced similarity measure for IvIFSs. The new entropy is defined as follows:

$$E_{\mathrm{NEW}}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{2 + \pi_A^-(x_i) + \pi_A^+(x_i) - |\mu_A^-(x_i) - v_A^-(x_i)| - |\mu_A^+(x_i) - v_A^+(x_i)|}{2 + \pi_A^-(x_i) + \pi_A^+(x_i)}. \tag{4}$$

Obviously, from the definition of the new entropy, we have the following conclusions:

Theorem 1. The mapping $E_{\mathrm{NEW}}(A)$, defined by (4), is a real-valued continuous function being increasing with respect to the intuitionism $\pi_A^-(x_i) + \pi_A^+(x_i)$ under the same fuzziness $\delta_A(x_i)$, and decreasingly with the fuzziness $\delta_A(x_i)$ under the same intuitionism $\pi_A^-(x_i) + \pi_A^+(x_i)$, where $\delta_A(x_i) = |\mu_A^-(x_i) - v_A^-(x_i)| + |\mu_A^+(x_i) - v_A^+(x_i)|$.

Proof. It is obvious.

Theorem 2. The mapping $E_{\mathrm{NEW}}(A)$, defined by (4), is an entropy measure for IvIFSs, i.e., it satisfies all the requirements in Definition 2.

Proof. We only need to prove that $E_{\mathrm{NEW}}(A)$ satisfies the conditions (P1)-(P4) in Definition 2.

(P1) If A is a crisp set, then for any $x_i \in X$, we know

$$\mu_A(x_i) = [1,1], v_A(x_i) = [0,0] \quad \text{or} \quad \mu_A(x_i) = [0,0], v_A(x_i) = [1,1],$$

then $\pi_A(x_i) = [0,0]$, thus from (4), we obtain that $E_{\mathrm{NEW}}(A) = 0$.

On the other hand, now suppose that $E_{\mathrm{NEW}}(A) = 0$. Since every term in the summation of (4) is non-negative, we deduce that every term should be zero, i.e.,

$$|\mu_A^-(x_i) - v_A^-(x_i)| + |\mu_A^+(x_i) - v_A^+(x_i)| = 2 + \pi_A^-(x_i) + \pi_A^+(x_i).$$

From $\mu_A(x_i) \in [0,1], v_A(x_i) \in [0,1]$ and $\mu_A^+(x_i) + v_A^+(x_i) \le 1$, it is easy to deduce that

$$|\mu_A^-(x_i) - v_A^-(x_i)| = 1 \quad \text{and} \quad |\mu_A^+(x_i) - v_A^+(x_i)| = 1.$$

Thus

$$\mu_A(x_i) = [1,1], v_A(x_i) = [0,0] \quad \text{or} \quad \mu_A(x_i) = [0,0], v_A(x_i) = [1,1],$$

for any $x_i \in X$. This indicates that A is a crisp set.

(P2) Let $\mu_A(x_i) = v_A(x_i)$ for $x_i \in X$, i.e., $\mu_A^-(x_i) = v_A^-(x_i)$ and $\mu_A^+(x_i) = v_A^+(x_i)$. From Equation(4), we obtain $E_{\text{NEW}}(A) = 1$.

Now suppose that $E_{\text{NEW}}(A) = 1$, and then also from (4), we have

$$\frac{2 + \pi_A^-(x_i) + \pi_A^+(x_i) - |\mu_A^-(x_i) - v_A^-(x_i)| - |\mu_A^+(x_i) - v_A^+(x_i)|}{2 + \pi_A^-(x_i) + \pi_A^+(x_i)} = 1.$$

Thus

$$\mu_A^-(x_i) = v_A^-(x_i) \text{ and } \mu_A^+(x_i) = v_A^+(x_i),$$

for each $x_i \in X$, which implies that $\mu_A(x_i) = v_A(x_i)$ for each $x_i \in X$.

(P3) Suppose that $\mu_B(x_i) \leq v_B(x_i)$, i.e., $\mu_B^-(x_i) \leq v_B^-(x_i)$, $\mu_B^+(x_i) \leq v_B^+(x_i)$ and $\mu_A(x_i) \leq \mu_B(x_i), v_A(x_i) \geq v_B(x_i)$, i.e.,

$$\mu_A^-(x_i) \leq \mu_B^-(x_i), \mu_A^+(x_i) \leq \mu_B^+(x_i), v_A^-(x_i) \geq v_B^-(x_i), v_A^+(x_i) \geq v_B^+(x_i) \tag{5}$$

for each $x_i \in X$. Firstly, we prove that

$$\frac{v_A^-(x_i) - \mu_A^-(x_i) + v_A^+(x_i) - \mu_A^+(x_i)}{2 + \pi_A^-(x_i) + \pi_A^+(x_i)} \geq \frac{v_B^-(x_i) - \mu_B^-(x_i) + v_B^+(x_i) - \mu_B^+(x_i)}{2 + \pi_B^-(x_i) + \pi_B^+(x_i)}.$$

Assume that the above inequality does not right. Then we have

$$\frac{v_A^-(x_i) - \mu_A^-(x_i) + v_A^+(x_i) - \mu_A^+(x_i)}{2 + \pi_A^-(x_i) + \pi_A^+(x_i)} < \frac{v_B^-(x_i) - \mu_B^-(x_i) + v_B^+(x_i) - \mu_B^+(x_i)}{2 + \pi_B^-(x_i) + \pi_B^+(x_i)},$$

That is

$$\begin{aligned} & (v_A^-(x_i) - \mu_A^-(x_i) + v_A^+(x_i) - \mu_A^+(x_i))(2 + \pi_B^-(x_i) + \pi_B^+(x_i)) \\ < \quad & (v_B^-(x_i) - \mu_B^-(x_i) + v_B^+(x_i) - \mu_B^+(x_i))(2 + \pi_A^-(x_i) + \pi_A^+(x_i)). \end{aligned}$$

From the above inequality, we can deduce that

$$\begin{aligned} & (\mu_A^-(x_i) + \mu_A^+(x_i))(2v_B^-(x_i) + 2v_B^+(x_i) - 4) + (v_A^-(x_i) + v_A^+(x_i)) \times \\ & (4 - 2\mu_B^-(x_i) - 2\mu_B^+(x_i)) + 4(\mu_B^-(x_i) + \mu_B^+(x_i)) - 4(v_B^-(x_i) + v_B^+(x_i)) \\ < \quad & 0. \end{aligned} \tag{6}$$

From inequality (5), we have $\mu_A^-(x_i) + \mu_A^+(x_i) \leq \mu_B^-(x_i) + \mu_B^+(x_i)$ and $v_A^-(x_i) + v_A^+(x_i) \geq v_B^-(x_i) + v_B^+(x_i)$. Thus

$$(\mu_A^-(x_i) + \mu_A^+(x_i))(2v_B^-(x_i) + 2v_B^+(x_i) - 4) \geq (\mu_B^-(x_i) + \mu_B^+(x_i))(2v_B^-(x_i) + 2v_B^+(x_i) - 4)$$

and

$$(v_A^-(x_i)+v_A^+(x_i))(4-2\mu_B^-(x_i)-2\mu_B^+(x_i)) \geq (v_B^-(x_i)+v_B^+(x_i))(4-2\mu_B^-(x_i)-2\mu_B^+(x_i)).$$

From inequality (5) again, we have $v_A^-(x_i)+v_A^+(x_i) \geq v_B^-(x_i)+v_B^+(x_i)$ and $\mu_B^-(x_i)+\mu_B^+(x_i) \geq \mu_A^-(x_i)+\mu_A^+(x_i)$. Thus

$$(v_A^-(x_i)+v_A^+(x_i))(\mu_B^-(x_i)+\mu_B^+(x_i))-(v_B^-(x_i)+v_B^+(x_i))(\mu_A^-(x_i)+\mu_A^+(x_i)) \geq 0.$$

Substitute the above three inequalities into the left side of (6), we obtain that

$$\begin{aligned}
& (\mu_A^-(x_i)+\mu_A^+(x_i))(2v_B^-(x_i)+2v_B^+(x_i)-4)+(v_A^-(x_i)+v_A^+(x_i)) \times \\
& (4-2\mu_B^-(x_i)-2\mu_B^+(x_i))+4(\mu_B^-(x_i)+\mu_B^+(x_i))-4(v_B^-(x_i)+v_B^+(x_i)) \\
\geq\ & (\mu_B^-(x_i)+\mu_B^+(x_i))(2v_B^-(x_i)+2v_B^+(x_i)-4)+(v_B^-(x_i)+v_B^+(x_i))(4-2\mu_B^-(x_i)-2\mu_B^+(x_i)) \\
& +4(\mu_B^-(x_i)+\mu_B^+(x_i))-4(v_B^-(x_i)+v_B^+(x_i)) \\
=\ & 0,
\end{aligned}$$

which contradicts with the inequality (6). From this contradiction we get

$$\frac{v_A^-(x_i)-\mu_A^-(x_i)+v_A^+(x_i)-\mu_A^+(x_i)}{2+\pi_A^-(x_i)+\pi_A^+(x_i)} \geq \frac{v_B^-(x_i)-\mu_B^-(x_i)+v_B^+(x_i)-\mu_B^+(x_i)}{2+\pi_B^-(x_i)+\pi_B^+(x_i)}.$$

for all $x_i \in X$. Thus,

$$\frac{-|v_A^-(x_i)-\mu_A^-(x_i)|-|v_A^+(x_i)-\mu_A^+(x_i)|}{2+\pi_A^-(x_i)+\pi_A^+(x_i)} \leq \frac{-|v_B^-(x_i)-\mu_B^-(x_i)|-|v_B^+(x_i)-\mu_B^+(x_i)|}{2+\pi_B^-(x_i)+\pi_B^+(x_i)}.$$

Then, from (4), we have $E_{\mathrm{NEW}}(A) \leq E_{\mathrm{NEW}}(B)$.

Similarly, when $\mu_B(x_i) \leq v_B(x_i)$, and $\mu_A(x_i) \leq \mu_B(x_i), v_A(x_i) \geq v_B(x_i)$ for each $x_i \in X$, we can also prove that $E_{\mathrm{NEW}}(A) \leq E_{\mathrm{NEW}}(B)$.

(P4) It is clear that $A^C = \{\langle x_i, [v_A^-(x_i), v_A^+(x_i)], [\mu_A^-(x_i), \mu_A^+(x_i)]\rangle | x_i \in X\}$, i.e., $\mu_{A^C}(x_i) = v_A(x_i) = [v_A^-(x_i), v_A^+(x_i)]$, and $v_{A^C}(x_i) = \mu_A(x_i) = [\mu_A^-(x_i), \mu_A^+(x_i)]$. By (4), we have $E_{\mathrm{NEW}}(A^C) = E_{\mathrm{NEW}}(A)$. This completes the proof.

Now, we will give a similarity measure for IvIFSs based on the entropy defined by Eq.(4). For two given IvIFSs

$$A = \{\langle x, [\mu_A^-(x), \mu_A^+(x)], [v_A^-(x), v_A^+(x)]\rangle | x \in X\}, B = \{\langle x, [\mu_B^-(x), \mu_B^+(x)], [v_B^-(x), v_B^+(x)]\rangle | x \in X\},$$

let

$$AB1 = |\mu_A^- - \mu_B^-| \vee |v_A^- - v_B^-|, AB2 = |\mu_A^+ - \mu_B^+| \vee |v_A^+ - v_B^+|$$

where $\vee$ stands for max operator, and μ_A^- denotes $\mu_A^-(x)$, (x) is omitted, the same as other notions such as $\mu_A^+, v_A^-, v_A^+, \mu_B^-, \mu_B^+, v_B^-, v_B^+$. Zhang et al.[14] proposed a method about how to transform the entropy to the similarity measure. Firstly, they define a new IvIFS $M(A,B)$ from A,B as follows:

$$M(A,B) = \{\langle x, [\mu^-_{M(A,B)}(x), \mu^+_{M(A,B)}(x)], [v^-_{M(A,B)}(x), v^+_{M(A,B)}(x)]\rangle | x \in X\},$$

where

$$\mu^-_{M(A,B)}(x) = \mu^+_{M(A,B)}(x) = \frac{1}{2}[1 + \min\{AB1, AB2\}];$$
$$v^-_{M(A,B)}(x) = v^+_{M(A,B)}(x) = \frac{1}{2}[1 - \max\{AB1, AB2\}]$$

Then $M(A,B)$ is an IvIFS on X.

The following theorem is Theorem 4 in [14].

Theorem 3. Let E be an entropy of IvIFSs, for $A, B \in \text{IvIFS}(X)$, then $E(M(A,B))$ is a similarity measure of IvIFSs A and B.

Assume $X = \{x_1, x_2, \ldots, x_n\}$ is a finite universe, and the entropy $E_{\text{NEW}}(A)$ is defined by (4), then we can get the following similarity measure by Theorem 3.

Known by the definition of $M(A,B)$, we obtain $\mu^-_{M(A,B)}(x_i) \geq v^-_{M(A,B)}(x_i), \mu^+_{M(A,B)}(x_i) \geq v^+_{M(A,B)}(x_i)$. Thus

$$\begin{aligned} & S(A,B) = E(M(A,B)) \\ = & \frac{1}{n}\sum_{i=1}^{n} \frac{2 + \pi^-_{M(A,B)}(x_i) + \pi^+_{M(A,B)}(x_i) - \mu^-_{M(A,B)}(x_i) + v^-_{M(A,B)}(x_i) - \mu^+_{M(A,B)}(x_i) + v^+_{M(A,B)}(x_i)}{2 + \pi^-_{M(A,B)}(x_i) + \pi^+_{M(A,B)}(x_i)} \\ = & \frac{1}{n}\sum_{i=1}^{n} \frac{2 - 2\min\{AB1, AB2\}}{2 - \min\{AB1, AB2\} + \max\{AB1, AB2\}}. \end{aligned}$$

From Theorem 3, the function S defined by the above equality is a similarity measure for IvIFS(X).

4 Comparative Example

We consider three IvIFSs A_1, A_2, A_3 and compare the proposed new entropy measure Equation (4) with the existing ones (1)-(3). Assume that $X = \{x\}$ and the four IvIFSs $A_i (i = 1, 2, 3)$ is defined as follows:

$$A_1 = \{\langle x, [0.7, 0.7], [0.2, 0.2]\rangle\}, A_2 = \{\langle x, [0.6, 0.6], [0.3, 0.3]\rangle\}, A_3 = \{\langle x, [0.4, 0.4], [0.2, 0.2]\rangle\}.$$

Intuitively, the entropy of them should be:

$$E(A_1) < E(A_2) < E(A_3). \tag{7}$$

The comparison results are listed in Table 1. Table 1 indicates the facts that only E_{WW} and E_{NEW} meet the requirement of inequality (7), and other entropies aforementioned are not consistent with our intuition. The reason maybe that entropies E_{WW} and E_{NEW} are based on the fuzziness and intuitionism.

Table 1: Comparison the degree of fuzziness with different entropy measures

	$E_{\mathrm{B1}}(A)$	$E_{\mathrm{ZJ}}(A)$	$E_{\mathrm{WW}}(A)$	$E_{\mathrm{NEW}}(A)$
A_1	0.1	0.2857	0.3750	0.5455
A_2	0.1	0.5	0.5714	0.7273
A_3		0.5	0.75	0.8571

5 Concluding Remarks

In this paper, we proposed a new kind of entropy measure for IvIFSs based on fuzziness and intuitionism, and a new similarity measure for IvIFSs is also induced. Some comparisons are made to show the efficiency of the proposed measures.

References

[1] Zadeh L.A., Fuzzy Sets. Information and Control, 1965, 8, pp. 338-353.

[2] Atanassov K., Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 1986, 20, pp. 87-96.

[3] Zadeh L. A., The concept of a linguistic variable and its application to approximate reasoning-I, Information Sciences, 1975, 8, pp. 199-249.

[4] Gau W. L., Buehrer D. J., Vague sets, IEEE Transactions on systems, Man and Cybernetics, 1993, 23(2), pp. 610-614.

[5] G. Deschrijver, E.E. Kerre, On the relationship between some extensions of fuzzy set theory, Fuzzy Sets and Systems, 2003, 133, pp. 227-235.

[6] Burillo P., Bustince H., Vague sets are intuitionistic sets, Fuzzy Sets and Systems, 1996, 79, pp. 403-405.

[7] Atanassov K., Gargov G., Interval-valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 1989, 31(3), pp. 343-349.

[8] Burillo P., Bustince H., Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets, Fuzzy Sets and Systems, 1996, 118(3), pp.305-316.

[9] Zadeh L. A. , Fuzzy sets and systems, Proceedings of the Symposium on Systems Theory, Polytechnic Institute of Brooklyn, New York, 29-37 (1965).

[10] Vlachos I. K., Sergiadis G. D., Intuitionistic fuzzy information-applications to pattern recognition, Pattern Recognition Letters, 2007, 28, pp. 197-206.

[11] Mao J.J., Yao D.B., Wang C.C., A novel cross-entropy and entropy measures of IFSs and their applications. Knowledge-Based Systems, 2013, 48, pp. 37-45.

[12] Li D. F., Cheng C. T., New similarity measure of intuitionistic fuzzy sets and application to pattern recongnitions, Pattern Recognition Letters, 2002, 23, pp. 221-225.

[13] Wei C.P, Wang P., Zhang Y.Z., Entropy, similarity measure of interval-valued intuitionistic fuzzy sets and their applications. Information Sciences, 2011, 181, pp. 4273-4286.

[14] Zhang Q.S., Jiang S.Y., Jia B.G. and Luo S.H., Some information measures for interval-valued intuitionistic fuzzy sets. Information Sciences, 2010, 180, pp. 5130-5145.

[15] Liu X.D., Zhang S.H., Entropy and subsethood for general interval-valued intuitionistic fuzzy sets, in: L. Wang, Y. Jin (Eds.), FSKD 2005, vol. LNAI 3613, Springer-Verlag, Berlin Heidelberg, 2005, pp. 42-52.