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Retailer's replenishment policies under trade credit depending on the ordered quantity

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Abstract

In this paper we have developed an economic order quantity (EOQ) model with partially permissible delay in payments for defectives items, in which a supplier frequently offers its retailers a permissible delay linked to order quantity and the demand rate of the item, is assumed to be a function of both the selling price and credit period offered by retailer to his customer. The purpose of this paper is to maximize the retailer's profit by determining the optimal cycle time and the optimal selling price. Finally, numerical example is presented to illustrate the theoretical results followed by the sensitivity of parameters on the optimal solution.

Keywords: EOQ; delay in payment; two level trade credit policy; partially permissible delay in payment; defective items

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1. Introduction

The traditional economic order quantity model is based on the assumption that the retailer paid for the items immediately after the items are received. However, in practice, the supplier may provide the retailer many incentives such as a cash discount to motivate faster payment and stimulate sales, or a permissible delay in payment to attract new customer and increase the sales. [Goyal, 1985] is the first to establish an economic order quantity model under the condition of permissible delay in payments. [Aggarwal and Jaggi, 1995] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. [Jamal et al., 1997] further generalized the model with shortages. [Teng, 2002] amended Goyal's model by considering the difference between unit price and unit cost. All above models assumed that the supplier would offer the retailer a delay period but the retailer would not offer the trade credit period to his customer. But in most business transactions, this assumption is debatable. [Huang, 2003] extends [Goyal,1985] to provide a fixed trade credit period M between the supplier and the retailer and a maximal trade credit period N ($M > N$) between the retailer and the customer. Basically, the inventory model of [Goyal, 1985] is a supply chain of two stages (the supplier and the retailer). [Huang, 2003] generalizes [Goyal, 1985] to the supply chain of three stages the supplier, the retailer and customers. [Jaggi et al., 2007] developed inventory model in demand is assumed to be a function of selling price and length of credit period offered by the retailer. In the present competitive world business to attract more sales supplier frequently offer a permissible delay in payment if the retailer orders more than or equal to predetermined quantity. [Chung and Liao, 2004] studied a similar lot sizing problem under supplier's trade credits depending on retailer's order quantity. [Chang, 2004] extended [Chung and Liao, 2004] by taking into account inflation and finite time horizon. They assumed that the supplier only offers the retailer fully permissible delay in payments if the retailer orders a sufficient quantity. Otherwise, permissible delay in payments would not be permitted. We know that this policy of the supplier to stimulate the demands from the retailer is very practical. But this is just an extreme case. That is, the retailer would obtain 100% permissible delay in payments if the retailer ordered a large enough quantity. Otherwise, 0% permissible delay in payments would happen. In reality, the supplier can relax this extreme case to offer the retailer partially permissible delay in payments rather than 0% permissible delay in payments when the order quantity is smaller than a predetermined quantity. That is, the retailer must make a partial payment to the supplier when the order is received to enjoy some portion of the trade credit. Then, the retailer must pay off the remaining balances at the end of the permissible delay period. For example, the supplier provides 100% delay payment permitted if the retailer ordered a sufficient quantity, otherwise only α% ($0 \leq \alpha \leq 100$) delay payment permitted. [Ouyang et al., 2009] developed An economic order quantity model for deteriorating items with partially permissible delay in payments linked to order quantity. Recently, [Huang, 2007] established an EOQ model in which the supplier offers a partially delay in payments when the order quantity is smaller than the predetermined quantity W.

Previous studies related to trade credits were focused on determining optimal ordering policy for a retailer in which the quality-related issues are not taken into account. Namely, it is implicitly assumed that all items produced are non-defective. However, in a real production environment, it is often observed that some defective items are produced due to an imperfect production process or other factors. [Salameh and Jaber, 2000] developed an EOQ model for circumstances where a fraction of the ordered lot is of imperfect quality and has a uniform distribution. Their model assumed that shortages are not permitted to occur. [Goyal and Cardenas-Barron, 2002] reworked on the paper by [Salameh and Jaber, 2000] and presented a practical approach to find out the optimal lot size. [Chung and Huang, 2012] develop a inventory model of the retailer to allow items with imperfect quality under permissible delay in payments. [Ouyang et al. 2012] extended [Huang, 2007] EOQ model with partially permissible delay in payments to consider defective items.

In this paper we have developed economic order quantity (EOQ) model with partially permissible delay in payments for defectives items, in which a supplier frequently offers its retailers a permissible delay linked to order quantity and the demand rate of the item, is assumed to be a function of both the selling price and credit period offered by retailer to his customer.

2. Model Description

The retailer orders Q units of non-defective items at the beginning of the cycle. Due to an imperfect production process, the supplier produces some defective items. Based on past statistics, the supplier knows its defective rate is γ in an order lot, where $0 \le \gamma < 1$. Hence, the supplier immediately delivers $q = Q/(1-\gamma) \ge Q$ items to the retailer in the same shipment. For simplicity, we assume the lead time is zero. Upon arrival, the retailer inspects all q items with the inspection rate of x items per year. After inspection, the defective items are discovered and returned to the supplier at the time of the next replenishment. If the retailer's order quantity of non-defective items is greater than or equal to Q_d, then the supplier offers fully delay payment to the retailer ,i.e. pay cQ after M time periods from the time the order is filled.. Otherwise, as the order is filled, the retailer must make a partial payment, $(1-\beta)cQ$, to the supplier ,where $0 \le \beta \le 1$. Then the retailer must pay off the remaining balances,βcQ, at the end of the trade credit period. The retailer also offers the full trade credit period N to his customer to settle the account.

The demand rate consists of (i) regular cash-demand, which is inversely proportional to selling price and (ii) credit-demand, which is inversely proportional to selling price and directly proportional to the credit period offered by the retailer. Hence, demand rate at any time t can be represented as

$$D(t,p) = \begin{cases} k_1 p^{-e} + k_2 p^{-e}(N-t)^{\alpha} & 0 \le t \le N \\ k_1 p^{-e} & N \le t \le T \end{cases}$$

Where $k_1 > 0$, $k_2 > 0$,$\alpha > 0$ and $e > 1$

3. Notations and Assumptions

The following notations and assumptions are adopted to develop the mathematical model

3.1 Notations

A ordering cost per order

h_1 holding cost per non-defective item per year excluding interest charges

h_2 holding cost per defective item per year excluding interest charges where $h_2 \le h_1$

x inspection rate (or inspected items) per year

s inspection cost per item

c purchase cost per item

p selling price per item, where p > c

M delay period in payment offered by the supplier

N delay period in payment offered by the retailer, where $N \le T$ and also $N \le M$

I_e interest earned per dollar per year

I_p interest charged per dollar per year

Q retailer's order quantity of non-defective items per order

T replenishment cycle time in years

T_d time interval that Q_d units are depleted to zero due to demand

TP retailer's optimal total profit per year

3.2 Assumption

- During the inspection period, the on-hand non-defective inventory is larger than or equal to the demand.
- For simplicity, it is assumed that $N \leq M$. It is also assumed that the customers would settle their accounts only on the last day of the credit period N .
- The shortages are not permitted.

4. Mathematical formulation

A batch of Q units is purchased from the supplier at the beginning of the cycle. As the time passes, the inventory level depletes due to demand. Thus, the changes of inventory level can be represented by the following differential equation

$$\frac{dI_1}{dt} = -k_1 p^{-e} - k_2 p^{-e}(N-t)^{\alpha} \qquad 0 \leq t \leq N \qquad (1)$$

$$\frac{dI_2}{dt} = -k_1 p^{-e} \qquad N \leq t \leq T \qquad (2)$$

With the conditions $I_1(0) = Q$, $I_1(N) = I_2(N)$ and $I_2(T) = 0$. The solutions of Eq.(1) and Eq.(2) are

$$I_1(t) = k_1 p^{-e}(T-t) + k_2 p^{-e} \frac{(N-t)^{\alpha+1}}{(\alpha+1)} \qquad 0 \leq t \leq N \qquad (3)$$

$$I_2(t) = k_1 p^{-e}(T-t) \qquad N \leq t \leq T \qquad (4)$$

The order quantity for each cycle is

$$Q = \int_0^T D(t)\, dt = k_1 p^{-e} T + k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)} \qquad (5)$$

From Eq.(5),we can obtain the time interval that Q_d units are depleted to zero due to demand as

$$T_d = \frac{1}{k_1 p^{-e}}\left[Q_d - k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}\right] \qquad (6)$$

The retailer's annual profit consist of the following elements

Annual sales revenue $= \frac{pQ}{T} = \frac{p}{T}\left(k_1 p^{-e} T + k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}\right)$

Annual purchasing cost $= \frac{cQ}{T} = \frac{c}{T}\left(k_1 p^{-e} T + k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}\right)$

Annual ordering cost $= \frac{A}{T}$

Annual inspection cost $=\frac{sq}{T}=\frac{s}{T(1-\gamma)}\left(k_1p^{-e}T+k_2p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}\right)$

Annual stock holding cost for the non-defective items and some waiting for inspection items

$$=\frac{h_1}{T}\left[\int_0^N I_1(t)\,dt+\int_N^T I_2(t)\,dt+\frac{\gamma q^2}{2x}\right]$$

$$=\frac{h_1}{T}\left[k_1p^{-e}\frac{T^2}{2}+k_2p^{-e}\frac{N^{\alpha+2}}{(\alpha+1)(\alpha+2)}+\frac{\gamma}{2(1-\gamma)^2x}\left(k_1p^{-e}T+k_2p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}\right)^2\right]$$

Annual stock holding cost for the defective items

$$=\frac{h_2}{T}\left[\gamma qT-\frac{\gamma q^2}{2x}\right]$$

$$=\frac{h_2}{T}\frac{\gamma}{(1-\gamma)}\left(k_1p^{-e}T+k_2p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}\right)\left[T-\frac{1}{2(1-\gamma)x}\left(k_1p^{-e}T+k_2p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}\right)\right]$$

Computation for interests charged and earned (i.e. (g) and (h)) are based on the values of Q and Q_d , there are two cases: (1) $Q < Q_d$ and (2) $Q \geq Q_d$.

Case 1 $\boldsymbol{Q < Q_d}$ (i.e., $\boldsymbol{T < T_d}$)

In this case, the order quantity is less than Q_d, we know that the retailer must pay $c(1-\beta)Q$ at initial time and pay off remaining balance $c\beta Q$ at time M. According to the values of M, N, T and T_d, we have the following possible sub-cases: (i) $N \leq M \leq T < T_d$, (ii) $N \leq T \leq M \leq T_d$, and (iii) $N \leq T < T_d \leq M$

Sub case 1.1 $N \leq M \leq T < T_d$

The retailer accumulates revenue from cash sales during the period $(0, M)$ and also from credit sales during (N, M) into an account that earns an interest rate of I_e. So in the period $(0, M)$ the total revenue generated due to cash sales is $\int_0^M k_1p^{-e}t\ dt$ and from credit sales during the time period (N, M) is $\int_N^M k_2p^{-e}\ \frac{N^{\alpha+1}}{(\alpha+1)}\ dt$

Annual interest earned is $=\frac{I_ep}{T}\left[\int_0^M k_1p^{-e}t\ dt+\int_N^M k_2p^{-e}\ \frac{N^{\alpha+1}}{(\alpha+1)}\ dt\right]$

$$=\frac{I_ep}{T}\left[k_1p^{-e}\frac{M^2}{2}+k_2p^{-e}\ \frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right]$$

On the other hand, since because, $M \le T$, the retailer still has some inventory on hand at t=M. The retailer pay off all unit sold and keep his/ser profit and starts paying for interest charges on the item in stock with rate I_p.

Annual interest payable is= $\frac{I_p c}{T}\left[(1-\beta)QM + \int_M^T I_2(t)\, dt\right]$

$= \frac{I_p c}{T}\left[(1-\beta)\left(k_1 p^{-e} T + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}\right)M + k_1 p^{-e}\frac{(T-M)^2}{2}\right]$

Sub case 1.2 $\quad N \le T \le M \ \le \ T_d$

Here the credit period M is more than or equal to the cycle , so the retailer earns interest on cash sales during the period$(0, M)$ and also on credit sales during (N, M). Therefore

(f) Annual interest earned is $= \frac{I_e p}{T}\left[\int_0^T k_1 p^{-e} t \ dt + \int_N^T k_2 p^{-e}\ \frac{N^{\alpha+1}}{(\alpha+1)}\ dt + \int_T^M Q\ dt\right]$

$= \frac{I_e p}{T}\left[k_1 p^{-e}\frac{(2M-T)T}{2} + k_2 p^{-e}\ \frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right]$

Since $T \le M$, the retailer sells all items and receives all returns from customer before pays purchasing amount to the supplier. Hence

Annual interest payable is= $\frac{I_p c}{T}(1-\beta)\left(k_1 p^{-e} T + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}\right)M$

Sub case 1.3 $\quad N \le T < T_d \le \ M$

The interests charged and earned in sub-case 1.3 are the same as those in sub-case 1.2.

Case 2 $Q \ge Q_d$ $(i.e.\ T \ge T_d)$

In this case, the order quantity is greater than or equal to Q_d, we know that the delay in payments is permitted for all purchase cost. That is, the retailer does not pay any purchase cost at initial time, he/she is permitted to pays off all purchase cost cQ to the supplier at time M. Based on the values of M, T and T_d, we have the following possible sub-cases: (i)$N \le T_d \le T \le M$,(ii) $T_d \le N \le T \le M$ (iii) $T_d \le N \le M \le T$ (iv)$N \ \le T_d \le M \le T$ (v) $N \le M \le T_d \le T$

Sub case 2.1 $N \le T_d \le T \le M$

Since $N \le T_d \le T \le M$, the retailer faces no interest charged. On the other hand, the retailer earns interest on cash sales during the period$(0, M)$ and also on credit sales during(N, M). Therefore

(f) Annual interest earned is $= \frac{I_e p}{T}\left[\int_0^T k_1 p^{-e} t \ dt + \int_N^T k_2 p^{-e} \ \frac{N^{\alpha+1}}{(\alpha+1)} \ dt + \int_T^M Q \ dt\right]$

$= \frac{I_e p}{T}\left[k_1 p^{-e} \frac{(2M-T)T}{2} + k_2 p^{-e} \ \frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right]$

Annual interest payable is = 0

Sub case 2.2 $T_d \le N \le T \le M$

The interests charged and earned in sub-case 2.2 are the same as those in sub-case 2.1.

Sub case 2.3 $T_d \le N \le M \le T$

The retailer accumulates revenue from cash sales during the period $(0, M)$ and also from credit sales during (N, M) into an account that earns an interest rate of I_e. Therefore

Annual interest earned is $= \frac{I_e p}{T}\left[\int_0^M k_1 p^{-e} t \ dt + \int_N^M k_2 p^{-e} \ \frac{N^{\alpha+1}}{(\alpha+1)} \ dt\right]$

$= \frac{I_e p}{T}\left[k_1 p^{-e} \frac{M^2}{2} + k_2 p^{-e} \ \frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right]$

On the other hand , since after the due date M the retailer still has some inventory on hand , As a result interest charged for the item kept in stock.

Annual interest payable is$= \frac{I_p c}{T}\int_M^T I_2(t) \ dt = \frac{I_p c}{T}\left[k_1 p^{-e} \frac{(T-M)^2}{2}\right]$

Sub case 2.4 $N \le T_d \le M \le T$

The interests charged and earned in sub-case 2.4 are the same as those in sub-case 2.3.

Sub case 2.5 $N \le M \le T_d \le T$

The interests charged and earned in sub-case 2.5 are the same as those in sub-case 2.3.

As a result the retailer's annual profit

$TP(T,p)$ = Annual sales revenue −Annual purchasing cost − Annual ordering cost −Annual inspection cost− Annual stock holding cost for the non-defective items and some waiting for inspection items − Annual stock holding cost for the defective items + Annual interest earned – Annual interest payable

Case 1 $Q < Q_d$ (i.e., $T < T_d$)

$$TP_1(T,p) = \begin{cases} TP_{11}(T,p) & N \leq M \leq T < T_d \\ TP_{12}(T,p) & N \leq T \leq M \leq T_d \\ TP_{13}(T,p) & N \leq T < T_d \leq M \end{cases}$$

Where

$$TP_{11}(T,p) = S_1 - I_p c k_1 p^{-e} M(1-\beta) + I_p c k_1 p^{-e} M - \frac{1}{T}\left[S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c\left((1-\beta)M k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)} + k_1 p^{-e}\frac{M^2}{2}\right)\right] - T\left[S_3 + \frac{I_p c}{2}k_1 p^{-e}\right] \quad (7)$$

$$TP_{12}(T,p) = TP_{13}(T,p) = S_1 + I_e p k_1 p^{-e} M - I_p c(1-\beta)k_1 p^{-e} M - \frac{1}{T}\left[S_2 - I_e p k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N) + I_p c(1-\beta)k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}M\right] - T\left[S_3 + \frac{I_e p}{2}k_1 p^{-e}\right] \quad (8)$$

Where

$$S_1 = \left[\left(p - c - \frac{s}{(1-\gamma)}\right)k_1 p^{-e} - h_1\frac{\gamma k_1 k_2 p^{-2e} N^{\alpha+1}}{x(1-\gamma)^2(\alpha+1)} + h_2\frac{k_2 p^{-e} N^{\alpha+1}\gamma}{(\alpha+1)(1-\gamma)}\left(\frac{k_1 p^{-e}}{x(1-\gamma)} - 1\right)\right] \quad (9)$$

$$S_2 = \left[\left(c - p + \frac{s}{(1-\gamma)}\right)\frac{k_2 p^{-e} N^{\alpha+1}}{(\alpha+1)} + A + h_1\frac{k_2 p^{-e} N^{\alpha+1}}{(\alpha+1)}\left(\frac{\gamma k_2 p^{-e} N^{\alpha+1}}{2x(1-\gamma)^2(\alpha+1)} + \frac{N}{(\alpha+2)}\right) - h_2\frac{\gamma k_2^2 p^{-2e} N^{2(\alpha+1)}}{2x(1-\gamma)^2(\alpha+1)^2}\right] \quad (10)$$

$$S_3 = \left[h_1\left(\frac{k_1^2 p^{-2e}\gamma}{2x(1-\gamma)^2} + \frac{k_1 p^{-e}}{2}\right) - h_2 k_1 p^{-e}\frac{\gamma}{(1-\gamma)}\left(\frac{k_1 p^{-e}}{2x(1-\gamma)} - 1\right)\right] \quad (11)$$

At $T = M$,we find that $TP_{11}(M,p) = TP_{12}(M,p)$ for $M \leq T_d$,hence $TP_1(T,p)$ is a continuous function and well defined.

Case 2 $Q \geq Q_d$ $(i.e.\ T \geq T_d)$

$$TP_2(T) = \begin{cases} TP_{21}(T) & N \leq T_d \leq T \leq M \\ TP_{22}(T) & T_d \leq N \leq T \leq M \\ TP_{23}(T) & T_d \leq N \leq M \leq T \\ TP_{24}(T) & N \leq T_d \leq M \leq T \\ TP_{25}(T) & N \leq M \leq T_d \leq T \end{cases}$$

Where

$$TP_{21}(T,p) = TP_{22}(T,p) = S_1 + I_e p k_1 p^{-e} M - \frac{1}{T}\left[S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right] - T\left[S_3 + \frac{I_e p}{2} k_1 p^{-e}\right] \tag{12}$$

$$TP_{23}(T,p) = TP_{24}(T,p) = TP_{25}(T,p) = S_1 + I_p c k_1 p^{-e} M - \frac{1}{T}\left[S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c k_1 p^{-e}\frac{M^2}{2}\right] - T\left[S_3 + \frac{I_p c}{2} k_1 p^{-e}\right] \tag{13}$$

At $T = M$, we find that $TP_{21}(M,p) = TP_{23}(M,p)$for $M \geq T_d$, hence $TP_2(T,p)$ is a continuous function and well defined.

5. Determination of optimal replenishment time for a fixed price

In this section, we find the optimal replenishment time for the case of $Q < Q_d$, and then the other case of $Q \geq Q_d$.

Case 1 $\boldsymbol{Q < Q_d}$ **(i.e.,** $\boldsymbol{T < T_d}$ **)**

Sub case 1.1 $N \leq M \leq T < T_d$

For any given value of p, optimal value of T (say T_{11}),which maximizes $TP_{11}(T,p)$ is obtained from the equation $\frac{\partial TP_{11}(T,p)}{\partial T} = 0$. Thus,

$$T_{11}(p) = \sqrt{\frac{S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c\left((1-\beta)M k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)} + k_1 p^{-e}\frac{M^2}{2}\right)}{S_3 + \frac{I_p c}{2} k_1 p^{-e}}} \tag{14}$$

$TP_{11}(T,p)$ is strictly concave on $T > 0$ if $\frac{\partial^2 TP_{11}(T,p)}{\partial T^2} < 0$,which gives

$$S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c\left((1-\beta)Mk_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)} + k_1 p^{-e}\frac{M^2}{2}\right) > 0$$

To ensure that $M \leq T_{11} < T_d$, we substitute (14) into inequality $M \leq T_{11} < T_d$, and obtain if and only if $\Delta_1 \leq S_2 < \Delta_2$, then $M \leq T_{11} < T_d$ where

$$\Delta_1 = M^2 S_3 + I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) - I_p c(1-\beta)Mk_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)} \qquad (15)$$

$$\Delta_2 = T_d^2\left(S_3 + \frac{I_p c}{2}k_1 p^{-e}\right) + I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) - I_p c\Big((1-$$

$$\beta)Mk_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)} + k_1 p^{-e}\frac{M^2}{2}\Big) \qquad (16)$$

Based on the above results , the following lemma can be obtained

Lemma 1 For $N \leq M \leq T_d$

a) if $\Delta_1 \leq S_2 < \Delta_2$,then $T = T_{11}$ is the optimal value which maximizes $TP_{11}(T,p)$
b) if $S_2 < \Delta_1$, then $T = M$maximizes $TP_{11}(T,p)$.
c) if $S_2 \geq \Delta_2$, then the value of T maximizes $TP_{11}(T,p)$ does not exist.

Proof: If $S_2 < \Delta_1$, then for any $T_2 > T_1 \geq M$

$$TP_{11}(T_2,p) - TP_{11}(T_1,p) = \left(S_3 + \frac{I_p c}{2}k_1 p^{-e}\right)(T_1 - T_2) + \Bigg[S_2 - I_e p\Bigg(k_1 p^{-e}\frac{M^2}{2} +$$

$$k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\Bigg) + I_p c\left((1-\beta)Mk_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)} + k_1 p^{-e}\frac{M^2}{2}\right)\Bigg]\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$< (M^2 - T_1 T_2)\left(\frac{T_2 - T_1}{T_1 T_2}\right)\left(S_3 + \frac{I_p c}{2}k_1 p^{-e}\right) < 0$$

Hence, $TP_{11}(T,p)$ is a strictly decreasing function on the half-closed interval $[M,\infty)$.Consequently, $TP_{11}(T,p)$ has a maximum value at the boundary point $T = M$. Likewise, if $S_2 \geq \Delta_2$, then for any given $T_1 < T_2 < T_d$, we obtain the following results:

$$TP_{11}(T_2,p) - TP_{11}(T_1,p) \geq \left(S_3 + \frac{I_p c}{2}k_1 p^{-e}\right)(T_d^2 - T_1 T_2)\left(\frac{T_2 - T_1}{T_1 T_2}\right) > 0$$

Hence, $TP_{11}(T,p)$ is a strictly increasing function on the open interval $(0, T_d)$. As a result, the value of T which maximizes $TP_{11}(T,p)$ does not exist.

Sub case 1.2 $\quad N \le T \le M \le T_d$

Likewise, we obtain optimal value of T (sayT_{12})which maximizes $TP_{12}(T,p)$ as

$$T_{12}(p) = \sqrt{\frac{S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) + I_p c(1-\beta) k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)} M}{S_3 + \frac{I_e p}{2} k_1 p^{-e}}} \tag{17}$$

$TP_{12}(T,p)$ is strictly concave on $T > 0$ if

$$S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) + I_p c(1-\beta) k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)} M > 0$$

To ensure that $N \le T_{12} \le M$, we substitute (17) into inequality $N \le T_{12} \le M$, and obtain

if and only if $\Delta_3 \le S_2 \le \Delta_1$, then $N \le T_{12} \le M$ where

$$\Delta_3 = \left(S_3 + \frac{I_e p}{2} k_1 p^{-e}\right) N^2 + I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) - I_p c(1-\beta) k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)} M \tag{18}$$

and Δ_1 is defined as in (15)

Based on the above results , the following lemma can be obtained

Lemma 2 For $N \le M \le T_d$

a) if $\Delta_3 \le S_2 \le \Delta_1$,then $T = T_{12}$ is the optimal value which maximizes $TP_{12}(T,p)$.
b) if $S_2 < \Delta_3$, then $T = N$ maximizes $TP_{12}(T,p)$.
c) if $S_2 > \Delta_1$, then $T = M$ maximizes $TP_{12}(T,p)$.

Proof. If $S_2 > \Delta_1$, then for any $T_1 < T_2 \le M$

$$TP_{12}(T_2,p) - TP_{12}(T_1,p) = \left(S_3 + \frac{I_p c}{2} k_1 p^{-e}\right)(T_1 - T_2) + \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\left[S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) + I_p c(1-\beta) k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)} M\right] > (M^2 - T_1 T_2)\left(\frac{T_2 - T_1}{T_1 T_2}\right)\left(S_3 + \frac{I_p c}{2} k_1 p^{-e}\right) > 0$$

Hence, $TP_{12}(T,p)$ is a strictly increasing function on the half-closed interval $(0,M]$. Consequently, $TP_{12}(T,p)$ has a maximum value at the boundary point $T = M$.

If $S_2 < \Delta_3$,then for any $T_2 > T_1 \ge N$

$$TP_{12}(T_2,p) - TP_{12}(T_1,p) < (N^2 - T_1 T_2)\left(\frac{T_2 - T_1}{T_1 T_2}\right)\left(S_3 + \frac{I_p c}{2} k_1 p^{-e}\right) < 0$$

Hence, $TP_{12}(T,p)$ is a strictly decreasing function on the half-closed interval $[N,\infty)$.Consequently, $TP_{12}(T,p)$ has a maximum value at the boundary point $T = N$.

Sub case 1.3 $N \le T < T_d \le M$

Since$TP_{12}(T,p) = TP_{13}(T,p)$, we obtain the optimal value of T (sayT_{13}) which maximizes $TP_{13}(T,p)$, as following

$$T_{13}(p) = T_{12}(p) = \sqrt{\frac{S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) + I_p c(1-\beta) k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)} M}{S_3 + \frac{I_e p}{2} k_1 p^{-e}}} \tag{19}$$

$TP_{13}(T,p)$ is strictly concave on $T > 0$ if

$$S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) + I_p c(1-\beta) k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)} M > 0$$

To ensure that $N \le T_{13} < T_d$, we substitute (19) into inequality $N \le T_{13} < T_d$, and obtain if and only if $\Delta_3 \le S_2 < \Delta_4$, then $N \le T_{13} < T_d$ where

$$\Delta_4 = \left(S_3 + \frac{I_e p}{2} k_1 p^{-e}\right) T_d^2 + I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) - I_p c(1-\beta) k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)} M \tag{20}$$

and Δ_3 is defined as in (18)

Based on the above results , the following lemma can be obtained

Lemma 3 For $N \le T_d \le M$

a) if $\Delta_3 \le S_2 < \Delta_4$,then $T = T_{13}$ is the optimal value which maximizes $TP_{13}(T,p)$
b) if $S_2 < \Delta_3$, then $T = N$ maximizes $TP_{13}(T,p)$.
c) if $S_2 \ge \Delta_4$, then the value of T maximizes $TP_{13}(T,p)$ does not exist.

Proof. If $S_2 < \Delta_3$,then using lemma 2 ,we find that , $TP_{13}(T,p)$ has a maximum value at the boundary point $T = N$.

If $S_2 \ge \Delta_4$, then for any $T_1 < T_2 < T_d$

$$TP_{13}(T_2,p) - TP_{13}(T_1,p) \ge \left(T_d^2 - T_1 T_2\right)\left(\frac{T_2 - T_1}{T_1 T_2}\right)\left(S_3 + \frac{I_p c}{2} k_1 p^{-e}\right) > 0$$

Hence, $TP_{13}(T,p)$ is a strictly increasing function on the open interval $(0, T_d)$. As a result, the value of T which maximizes $TP_{13}(T,p)$ does not exist.

Case 2 $Q \geq Q_d$ $(i.e.\ T \geq T_d)$

Sub case 2.1 $N \leq T_d \leq T \leq M$

The optimal value of T (say T_{21})which maximizes $TP_{21}(T,p)$ is

$$T_{21}(p) = \sqrt{\frac{S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N)}{S_3 + \frac{I_e p}{2} k_1 p^{-e}}} \tag{21}$$

$TP_{21}(T,p)$ is strictly concave on $T > 0$ if $S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) > 0$

To ensure that $T_d \leq T_{21} \leq M$, we substitute (21) into inequality $T_d \leq T_{21} \leq M$, and obtain if and only if $\Delta_6 \leq S_2 \leq \Delta_5$, then $T_d \leq T_{21} \leq M$ where

$$\Delta_5 = M^2 \left(S_3 + \frac{I_e p}{2} k_1 p^{-e}\right) + I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) \tag{22}$$

$$\Delta_6 = T_d^2 \left(S_3 + \frac{I_e p}{2} k_1 p^{-e}\right) + I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) \tag{23}$$

Based on the above results , the following lemma can be obtained

Lemma 4 For $N \leq T_d \leq M$

a) if $\Delta_6 \leq S_2 \leq \Delta_5$,then $T = T_{21}$ is the optimal value which maximizes $TP_{21}(T,p)$
b) if $S_2 < \Delta_6$, then $T = T_d$ maximizes $TP_{21}(T,p)$.
c) if $S_2 > \Delta_5$, then $T = M$ maximizes $TP_{21}(T,p)$.

proof: the proof is similar to lemma 1.

Sub case 2.2 $T_d \leq N \leq T \leq M$

Since$TP_{21}(T,p) = TP_{22}(T,p)$, we obtain the optimal value of T (sayT_{22}) which maximizes $TP_{22}(T,p)$, as following

$$T_{22}(p) = T_{21}(p) = \sqrt{\frac{S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N)}{S_3 + \frac{I_e p}{2} k_1 p^{-e}}} \tag{24}$$

$TP_{22}(T,p)$ is strictly concave on $T > 0$ if $S_2 - I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)}(M-N) > 0$

To ensure that $N \leq T_{22} \leq M$, we substitute (24) into inequality $N \leq T_{22} \leq M$, and obtain if and only if $\Delta_7 \leq S_2 \leq \Delta_5$, then $N \leq T_{22} \leq M$ where

$\Delta_7 = N^2 \left(S_3 + \frac{I_e p}{2} k_1 p^{-e}\right) + I_e p k_2 p^{-e} \frac{N^{\alpha+1}}{(\alpha+1)} (M - N)$ (25)

and Δ_5 is defined as in (22)

Based on the above results , the following lemma can be obtained

Lemma 5 For $T_d \leq N \leq M$

a) if $\Delta_7 \leq S_2 \leq \Delta_5$,then $T = T_{22}$ is the optimal value which maximizes $TP_{22}(T,p)$
b) if $S_2 < \Delta_7$, then $T = N$ maximizes $TP_{22}(T,p)$.
c) if $S_2 > \Delta_5$, then $T = M$ maximizes $TP_{22}(T,p)$.

proof: the proof is similar to lemma2.

Sub case 2.3 $T_d \leq N \leq M \leq T$

The optimal value of T (say T_{23})which maximizes $TP_{23}(T,p)$ is

$$T_{23}(p) = \sqrt{\frac{S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c k_1 p^{-e}\frac{M^2}{2}}{S_3 + \frac{I_p c}{2} k_1 p^{-e}}} \quad (26)$$

$TP_{23}(T,p)$ is strictly concave on $T > 0$ if

$$S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c k_1 p^{-e}\frac{M^2}{2} > 0$$

To ensure that $M \leq T_{23}$, we substitute (26) into inequality $M \leq T_{23}$, and obtain if and only if $S_2 \geq \Delta_5$, then $M \leq T_{23}$ where Δ_5 is defined as in (22)

Based on the above results , the following lemma can be obtained

Lemma 6 For $T_d \leq N \leq M$

a) if $S_2 \geq \Delta_5$,then $T = T_{23}$ is the optimal value which maximizes $TP_{23}(T.p)$.
b) if $S_2 < \Delta_5$, then $T = M$ maximizes $TP_{23}(T.p)$.

proof : If $S_2 < \Delta_5$ and for any $T_1 > T_2 \geq M$

$$TP_{23}(T_2,p) - TP_{23}(T_1,p) < (M^2 - T_1 T_2)\left(\frac{T_2 - T_1}{T_1 T_2}\right)\left(S_3 + \frac{I_p c}{2} k_1 p^{-e}\right) < 0$$

Hence, $TP_{12}(T,p)$ is a strictly decreasing function on the half-closed interval $[M,\infty)$. , $TP_{23}(T,p)$ has a maximum value at the boundary point $T = M$.

Sub case 2.4 $N \leq T_d \leq M \leq T$

Since $TP_{24}(T.p) = TP_{23}(T.p)$, we obtain the optimal value of T (say T_{24}) which maximizes $TP_{24}(T,p)$, as following

$$T_{24}(p) = T_{23}(p) = \sqrt{\frac{S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c k_1 p^{-e}\frac{M^2}{2}}{S_3 + \frac{I_p c}{2} k_1 p^{-e}}} \tag{27}$$

$TP_{24}(T,p)$ is strictly concave on $T > 0$ if

$$S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c k_1 p^{-e}\frac{M^2}{2} > 0$$

To ensure that $M \leq T_{24}$, we substitute (27) into inequality $M \leq T_{24}$, and obtain if and only if $S_2 \geq \Delta_5$, then $M \leq T_{24}$ where Δ_5 is defined as in (22)

Based on the above results , the following lemma can be obtained

Lemma 7 For $N \leq T_d \leq M$

a) if $S_2 \geq \Delta_5$,then $T = T_{24}$ is the optimal value which maximizes $TP_{24}(T,p)$.
b) if $S_2 < \Delta_5$, then $T = M$ maximizes $TP_{24}(T,p)$.

proof: the proof is similar to lemma 6.

Sub case 2.5 $N \leq M \leq T_d \leq T$

Since $TP_{25}(T,p) = TP_{24}(T,p) = TP_{23}(T,p)$, we obtain the optimal value of T (say T_{25}) which maximizes $TP_{24}(T,p)$, as following

$$T_{25}(p) = T_{24}(p) = T_{23}(p) = \sqrt{\frac{S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c k_1 p^{-e}\frac{M^2}{2}}{S_3 + \frac{I_p c}{2} k_1 p^{-e}}} \tag{28}$$

$TP_{25}(T,p)$ is strictly concave on $T > 0$ if

$$S_2 - I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) + I_p c k_1 p^{-e}\frac{M^2}{2} > 0$$

To ensure that $T_d \leq T_{25}$, we substitute (28) into inequality $T_d \leq T_{25}$, and obtain if and only if $S_2 \geq \Delta_8$, then $T_d \leq T_{25}$ where

$$\Delta_8 = \left(S_3 + \frac{I_p c}{2}k_1 p^{-e}\right)T_d^2 + I_e p\left(k_1 p^{-e}\frac{M^2}{2} + k_2 p^{-e}\frac{N^{\alpha+1}}{(\alpha+1)}(M-N)\right) - I_p c k_1 p^{-e}\frac{M^2}{2} \tag{29}$$

Based on the above results , the following lemma can be obtained

Lemma 8 For $N \leq M \leq T_d$

a) if $S_2 \geq \Delta_8$,then $T = T_{25}$ is the optimal value which maximizes $TP_{25}(T,p)$.
b) if $S_2 < \Delta_8$, then $T = T_d$ maximizes $TP_{25}(T,p)$.

proof: the proof is similar to lemma 6.

Combine Lemmas1-8, and let T^* denote the optimal replenishment time, we can obtain the following main results.

Theorem 1 For $N \leq M \leq T_d$ and fixed p

(1) if $S_2 < \Delta_3$, then $TP(T^*,p) = \max\{TP_{11}(M,p), TP_{12}(N,p), TP_{25}(T_d,p)\}$. Hence, T^* is N *or* M *or* T_d associate with larger profit.
(2) if $\Delta_3 \leq S_2 < \Delta_1$, then $TP(T^*,p) = \max\{TP_{12}(T_{12},p), TP_{25}(T_d,p)\}$. Hence, T^* is T_{12} *or* T_d associate with larger profit.
(3) if $S_2 = \Delta_1$, then $TP(T^*,p) = \max\{TP_{11}(T_{11},p), TP_{12}(T_{12},p), TP_{25}(T_d,p)\}$. Hence, T^* is T_{11}or T_{12} or T_d associate with larger profit.
(4) if $\Delta_1 < S_2 < \Delta_2$,then $TP(T^*,p) = \max\{TP_{11}(T_{11},p), TP_{25}(T_d,p)\}$ .Hence, T^* *is* T_{11} *or* T_d associate with larger profit.
(5) if $\Delta_2 \leq S_2 < \Delta_8$, then $TP(T^*,p) = \max\{TP_{12}(M,p), TP_{25}(T_d,p)\}$ .Hence, T^* is M *or* T_d associate with larger profit.
(6) If $S_2 \geq \Delta_8$,then $TP(T^*,p) = \max\{TP_{12}(M,p), TP_{25}(T_{25},p)\}$. Hence, T^* *is* M *or* T_{25} associate with larger profit.

Proof It is immediately follows from the fact that $TP_{11}(M,p) = TP_{12}(M,p)$ for $N \leq M \leq T_d$ and the lemma 1,2 and 8.

Theorem 2 For $N \leq T_d \leq M$ and fixed p

(1) if $S_2 < \Delta_3$, then $TP(T^*) = max\{TP_{13}(N,p), TP_{21}(T_d,p), TP_{24}(M,p)\}$. Hence, T^* is N *or* M *or* T_d associate with larger profit.
(2) if $\Delta_3 \leq S_2 < \Delta_4$,then $TP(T^*,p) = max\{TP_{13}(T_{13},p), TP_{21}(T_d,p), TP_{24}(M,p)\}$. Hence, T^* is T_{13} *or* T_d *or* M associate with larger profit.
(3) if $\Delta_4 \leq S_2 < \Delta_6$,then $TP(T^*,p) = max\{TP_{21}(T_d,p), TP_{24}(M,p)\}$. Hence, T^* is T_d *or* M associate with larger profit.
(4) if $\Delta_6 \leq S_2 < \Delta_5$,then $TP(T^*,p) = TP_{21}(T_{21},p)$. Hence, T^* is T_{21} associate with larger profit.
(5) if $S_2 = \Delta_5$,then $TP(T^*,p) = max\{TP_{21}(T_{21},p), TP_{24}(T_{24},p)\}$. Hence, T^* is T_{21} *or* T_{24} associate with larger profit.
(6) if $S_2 > \Delta_5$, then $TP(T^*,p) = TP_{24}(T_{24},p)$. Hence, T^* is T_{24} associate with larger profit.

Proof It is immediately follows from the fact that $TP_{21}(M,p) = TP_{24}(M,p)$ for

$N \leq T_d \leq M$ and the lemma 3,4 and 7.

Theorem 3 For $T_d \leq N \leq M$ and fixed p

(1) if $S_2 < \Delta_7$,then $TP(T^*,p) = max\{TP_{22}(N,p), TP_{23}(M,p)\}$. Hence, T^* is N *or* M associate with larger profit .
(2) if $\Delta_7 \leq S_2 < \Delta_5$,then $TP(T^*,p) = TP_{22}(T_{22},p)$.Hence, T^* is T_{22} associate with larger profit .
(3) if $S_2 = \Delta_5$,then $TP(T^*,p) = max\{TP_{22}(T_{22},p), TP_{23}(T_{23},p)\}$ .Hence, T^* is T_{22} *or* T_{23} associate with larger profit
(4) if $S_2 > \Delta_5$,then $TP(T^*,p) = TP_{23}(T_{23},p)$ .Hence, T^* is T_{23} associate with larger profit .

Proof It is immediately follows from the fact that $TP_{22}(M,p) = TP_{23}(M,p)$ for $T_d \leq N \leq M$ And the lemma 5 and 6.

6. Solution Methodology

Each $TP_{ij}(T,p)$ is continuous function of p over the set $(0,\infty)$. It is clear that TP_{ij} is not maximum if $p = 0$ or ∞, because $TP_{ij} \rightarrow 0$ as $p \rightarrow \infty$ and demand function $D(t,p) \rightarrow \infty$ as $p \rightarrow 0$ which is impossible.

Using Eq.[6] , we obtain that the inequalities $N \leq M \leq T_d,$, $N \leq T_d \leq M$ and $T_d \leq N \leq M$ imply $p_2 \leq p$, $p_1 \leq p \leq p_2$ and $p \leq p_1$ respectively, where

$$p_1 = [\frac{k_1}{Q_d}\{N + \frac{k_2 N^{\alpha+1}}{k_1(\alpha+1)}\}]^{1/e} \text{ and } p_2 = [\frac{k_1}{Q_d}\{M + \frac{k_2 N^{\alpha+1}}{k_1(\alpha+1)}\}]^{1/e}.$$

To find optimal total profit , we adopt the following algorithm

6.1 **Algorithm 1**

1. Convert the problem of maximizing $TP_{ij}(T,p)$ into single variable problem $TP_{ij}(T(p),p)$ by substituting $T_{ij} = T_{ij}(p)$
2. Determine p_1 and p_2
3. Do a full search over p.
   a) If $p \leq p_1$ using Theorem 3, determine the total profit .
   b) If $p_1 \leq p \leq p_2$ using Theorem 2, determine the total profit .
   c) If $p \geq p_2$ using Theorem 1, determine the total profit .
4. Note value of p and corresponding value of total profit . Select the p which gives maximum value of total profit TP.

7.Numerical Examples

In order to illustrate the above solution procedure, we consider the following examples.

Example 1. Given A=\$10000 per order,k_1=9800000000, k_2=400000000, c=\$900 per unit, h1=\$21per unit, h2=\$15 per unit ,s=\$50 per unit , γ=0.01, β=0.6, I_e=.15, I_p=.45, Q_d=20 units ,α =3, x=45; M=150/365 yr = 0.411 yr ,N=100/365 yr =0.274 yr , e=2.5.

Using propose algorithm, we obtain p_1 =\$ 1783.19 , p_2=\$2097.11. Fig 1 shows that TP is strictly concave function of p. As a result, we are sure that the maximum total profit obtained from the proposed algorithm is indeed the global optimum solution. Using algorithm 1 , we obtained the following optimal results: Optimal cycle length (T^*) =T_{25} =0.75041 years, optimal selling price (p^*)= \$ 2455.871per unit and the optimal total profit (TP^*) =\$ 27900.3178 per year.

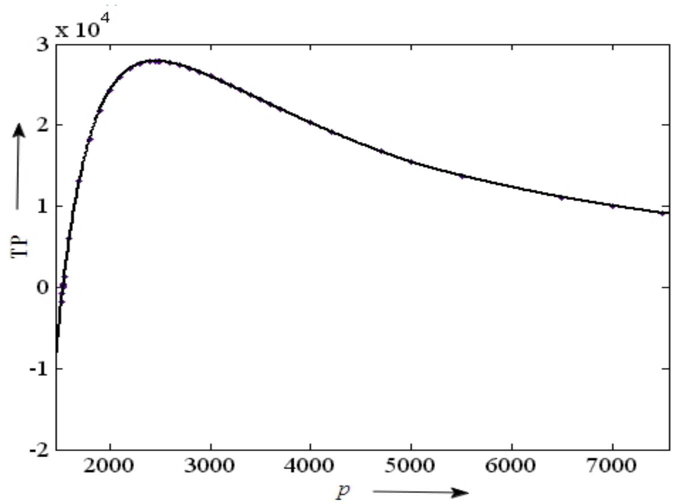

Fig 1 Retailer's optimal total profit for various value of $\boldsymbol{p}$

To study the effects of change in the value of parameters Q_d , γ, M, N,α we use the data of Example .

Table 1 Optimal solutions for different values of Q_d

Q_d	T_d	p*	T*	Q*	q*	TP*
10	0.30493	2455.871	T_{24}=0.75041	24.60621	24.85476	27900.3178
20	0.60993	2455.871	T_{25}=0.75041	24.60621	24.85476	27900.3178
30	0.90533	2445.550	T_d=0.90533	30.00000	30.30303	27375.9683
40	1.36001	2565.019	T_{11}=0.81727	24.03791	24.28072	25832.2750
50	1.36001	2565.019	T_{11}=0.81727	24.03791	24.28072	25832.2750

Table 1 indicates that when $Q_d$is greater than or equal to 30 units, the retailer will take the partial delay in payment(i.e., the optimal order quantity $Q^*< Q_d$) instead of the fully delay in payment.

Table 2 Optimal solutions for different values of γ

γ	T_d	P*	T*	Q*	q*	TP*
0.10	0.61551	2464.85	T_{25}=0.75461	24.51930	27.24367	27692.1935
0.15	0.61918	2470.70	T_{25}=0.75734	24.46234	28.77922	27557.8608
0.20	0.62334	2477.33	T_{25}=0.76041	24.39760	30.49700	27407.0618

Table 2 indicates that as γ increases , T_d^*,T^* , Q^* and p^* increase but TP^* decreases.

Table 3 Optimal solutions for different values of M and N

M	N	T_d	p*	T*	Q*	q*	TP*
80/365	50/365	0.67677	2560.093	T_{25}=0.81280	24.01982	24.26245	25547.1024
	60/365	0.67676	2560.077	T_{25}=0.81277	24.01979	24.26242	25547.3299
	70/365	0.67673	2560.052	T_{25}=0.81275	24.01974	24.26236	25547.7014
100/365	50/365	0.65552	2527.639	T_{25}=0.79279	24.18790	24.43222	26201.6515
	60/365	0.65551	2527.623	T_{25}=0.79277	24.18787	24.43219	26201.8905
	70/365	0.65549	2527.598	T_{25}=0.79274	24.18781	24.43213	26202.2809
120/365	50/365	0.63603	2497.298	T_{25}=0.77453	24.35522	24.60124	26870.6886
	60/365	0.63602	2497.283	T_{25}=0.77452	24.35518	24.60120	26870.9391
	70/365	0.63599	2497.258	T_{25}=0.77449	24.35512	24.60114	26871.3484
140/365	50/365	0.61831	2469.227	T_{25}=0.75813	24.52266	24.77037	27552.0669
	60/365	0.61829	2469.212	T_{25}=0.75811	24.52263	24.77033	27552.3288
	70/365	0.61827	2469.186	T_{25}=0.75808	24.52257	24.77027	27552.7567

Table 3 shows that as N increases while all other parameters remain unchanged, there is marginal decrease in T_d^*,T^* , Q^* and p^* but there marginal is increase in TP^* implies that offering credit period to his customer would be more profitable for retailer. While as M increases, T_d^*,T^* , Q^* and p^* decrease but there is increase in TP^*.

Table 4 Optimal solutions for different values of α

α	T_d	p*	T*	Q*	q*	TP*
1	0.60491	2450.157	T_{25}=0.74417	24.59279	24.84120	28001.1836
2	0.60917	2455.012	T_{25}=0.74947	24.60420	24.85272	27915.4546
3	0.60993	2455.871	T_{25}=0.75041	24.60621	24.85476	27900.3178
4	0.61008	2456.045	T_{25}=0.75060	24.60661	24.85516	27897.2646

Table 4 presents that as α increases , T_d^*,T^* , Q^* and p^* increase but TP^* decreases.

8. Conclusion

In this paper we develop an inventory model under two level trade credit policy with defective items by considering the following situations simultaneously: (1) the supplier may offer a partial permissible delay in payments even if the order quantity is less than Q_d.(2) retailer offer the credit period to his customer in order to stimulate their demand.(3) demand is a function of both the selling price and credit period. A solution procedure is proposed which gives the decision rule for obtaining retailer's cycle length and selling price to maximize the retailer's profit .Finally, numerical example is presented to illustrate the theoretical results followed by the sensitivity of parameters on the optimal solution.

The proposed model can be extended in several ways. For instance, we may extend the model for deteriorating items. Also, we could generalize the model to allow for shortages, quantity discounts, time discount and inflation rates, and others.

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