AMO - Advanced Modeling and Optimization, Volume 15, Number 3, 2013 *

N-Vehicle Cost Varying Transportation Problem

Arpita Panda[1]† and Chandan Bikash Das[2]

[1]Department of Mathematics

Sonakhali Girls's High School, Sonakhali, Daspur, Paschim Midnapore, West Bengal, India

E-mail: arpita201277@yahoo.co.in

[2]Department of Mathematics,Tamralipta Mahavidyalaya, Tamluk, Purba Midnapore-721636,

West Bengal, India, E-mail: cdas_bikash@yahoo.co.in

Abstract

In this paper we represent a transportation problem whose transportation cost is varying due to capacity of N-vehicles as well as transport quantities. The N-vehicle multi-objective cost varying transportation problem is a Bi-level Mathematical programming model. To solve this model, use north west corner rule for determining initial basic feasible solution and then set up unit cost (which varies in each iteration) for each cost matrix corresponding to each objective by proper choice of vehicles with our proposed algorithm. Apply optimality test for determining optimal solution for each objective separately. Numerical examples are presented to illustrate the model.

Keywords: Transportation Problem, Basic Cell, Non-basic Cell, North West Corner Rule.

***AMO-Advanced Modeling and Optimization. ISSN: 1841 - 4311**

†Corresponding author

1 Introduction

Transportation problem is a special class of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destination of demand in such a manner that the total transportation cost is minimized. in order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values but in real life applications unit transportation cost may be vary due to capacity of vehicles which are transport the commodities from sources to destinations according their demands.
The basic transportation problem was originally developed by Hitchock [3]in 1941. Efficient methods of solution derived from the simplex algorithm were developed in 1947. The transportation problem can be modeled as a standard linear programming problem, which can be solved by simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to transportation problem can be made quit efficient in terms of how to evaluate the necessary simplex-method information (variable to enter the basis, variable to leave the basis and optimality conditions).
Charns and Cooper [1] developed the Stepping Stone Method which provides an alternative way to determining the simplex-method information in 1954.
Dantzig [2] used the simplex method to the transportation problem as the Primal simplex transportation method in 1963 . An initial basic feasible solution for the transportation problem can be obtained by using the North west corner rule, Row minima, Column minima, Matrix minima, or the Vogel approximation method. The Modified distribution method is useful for finding the optimal solution for the transportation problem.
In many real life situations, the commodity does vary in some characteristics according to its source and the final commodity mixture reaching at destinations, may then be required to have known specifications. TP with additional impurity restrictions was stated by Haley [10] in 1999.

Chandra et al. [6] developed a method for solvig time minimizing TP with impurities in 1984. Interval transportation problem P. Pandian and D. Anuradha [14] (ITP) is a generalization of the TP in which input data are expressed as intervals instead of fixed values in 1979. This problem can arise when uncertainty exists in data problem and decision makers are more comfortable expressing it as intervals. Many researchers [4,7,11,12] have proposed fuzzy and interval programming techniques for solving them. Chanas et al. [5] developed an algorithm determining the optimal integer solution of a more general fuzzy transportation problem in 1963. Das et al. [8] introduced a method, called fuzzy technique to solve ITP by considering the right bound and the midpoint of the interval in 1999. Sengupta and Pal [16] proposed a new fuzzy oriented method to solve ITP by considering the midpoint and width of the interval in the objective function 2003. Singh and Saxena [15] proposed a method for solving multiobjective time TP with additional impurity restrictions 2003. A procedure for finding an optimal solution to fully interval integer TP was presented by Pandian and Natarajan [13]. Dutta et al. [9] introduced a linear fractional programming method for solving a fuzzy TP with additional restrictions in which transportation costs are intervals. Pandian and Anuradha [14] have proposed a floating point method for solving TP with additional constraints 2011.

In this paper, we present the N-vehicle cost varying transportation problem which is a Bi-level Mathematical programming model. To solve this model, use north west corner rule for determining initial basic feasible solution and then set up unit cost (which varies in each iteration) for each cost matrix corresponding to each objective by proper choice of vehicles with our proposed algorithm. Apply optimality test for determining optimal solution.

2 Mathematical Formulation

2.1 Preliminaries

A transportation problem can be stated as **Model 1**

Model 1

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{subject to} \quad \sum_{i=1}^{m} x_{ij} &= a_i, \quad i=1,\ldots,m \qquad (1) \\ \sum_{j=1}^{n} x_{ij} &= b_j, \quad j=1,\ldots,n \qquad (2) \\ \sum_{i=1}^{m} a_i &= \sum_{j=1}^{n} b_j \\ x_{ij} &\geq 0 \;\; \forall i, \;\; \forall j \end{aligned}$$

where a_i is the quantity of material available at source $O_i, i = 1, \ldots, m$
b_j is the quantity of material required at destination $D_j, j = 1, \ldots, n$
c_{ij} is the unit cost of transportation from st source O_i to destination D_j.
The following terms are to be defined with reference to the transportation problems.

Definition 2.1 *Feasible Solution (F.S.): A set of non-negative allocations* $x_{ij} \geq 0$ *which satisfies* $(1), (2)$ *is known as feasible solution.*

Definition 2.2 *Basic Feasible Solution (B.F.S.): A feasible solution to a m-origin and n-destination problem is said to be basic feasible solution if number of positive allocations are* $(m + n - 1)$.
If the number of allocations in a basic feasible solutions are less than (m+n-1), it is called degenerate basic feasible solution (DBFS) otherwise non-degenerate basic feasible solution (NDBFS).

Definition 2.3 *Optimal Solution: A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.*

Theorem 2.4 *The number of basic variables in a Transportation Problem(T.P.) is at most* $(m + n - 1)$

Theorem 2.5 *There exits a F.S. in each Transportation Problem(T.P.)*

Theorem 2.6 *In each T.P. there exits at least one B.F.S. which makes the objective function a minimum*

Theorem 2.7 *The solution of a T.P. is never unbounded*

Definition 2.8 *Loop: In the Transportation table, a sequence of cells is said to form a loop, if*
(i) each adjacent pair of cells either lies in the same column or in the same row;
(ii) not more than two consecutive cells in the sequence lie in the same row or in the same column;
(iii) the first and the last cells in the sequence lie either in the same row or in the same column;
(iv) the sequence must involve at least two rows or two columns of the table.

Theorem 2.9 *A sub-set of the columns of the coefficient matrix of a T.P. are linearly dependent, iff, the corresponding cells or a sub-set of them can be sequenced to form a loop.*

2.1.1 North-West corner rule

Step 1. Compute min (a_1, b_1). If $a_1 < b_1$, min $(a_1, b_1) = a_1$ and if $a_1 > b_1$, min $(a_1, b_1) = b_1$. Select $x_{11} =$ min (a_1, b_1) allocate the value of x_{11} in the cell $(1, 1)$.

Step 2. If $a_1 < b_1$, compute min $(a_2, b_1 - a_1)$. Select $x_{21} =$ min $(a_2, b_1 - a_1)$ and allocate the value of x_{21} in the cell $(2, 1)$.
If $a_1 > b_1$, compute min $(a_1 - b_1, b_2)$. Select $x_{12} =$ min $(a_1 - b_1, b_2)$ and allocate the value of x_{12} in the cell $(1, 2)$.
Let us now make an assumption that $a_1 - b_1 < b_2$. With this assumption the next cell for which some allocation is to made, is the cell $(2, 2)$.
If $a_1 = b_1$ then allocate 0 only in one of two cells $(2, 1)$ or $(1, 2)$. The next allocation is to be made cell $(2, 2)$.
In general, if an allocation is made in the cell $(i + 1, j)$ in the current step, the next allocation will be made either in cell (i, j) or $(i, j + 1)$.
The feasible solution obtained by this away is always a B.F.S.

2.1.2 Optimality test:

In order to test for optimality we should follow the procedure as given bellow:
Step 1. Start with B.F.S. consisting of $m + n - 1$ allocation in independent positions.
Step 2. Determine a set of $m + n$ numbers $u_i, i = 1, \ldots, m$ and $v_j, j = 1, \ldots, n$ such that in each cell (i, j) $c_{ij} = u_i + v_j$
Step 3. Calculate cell evaluations (unit cost difference) d_{ij} for each empty cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
Step 4. Examine the matrix of cell evaluation d_{ij} for negative entries and conclude that
(i) If all $d_{ij} > 0$, then Solution is optimal and unique.
(ii) If all $d_{ij} \geq 0$ and at least one $d_{ij} = 0$, then solution is optimal and alternative solution also exists.
(iii) If at least one $d_{ij} < 0$, then solution is not optimal.
If it is so, further improvement is required by repeating the above process after Step 5 and onwards.

Step 5. (i) See the most negative cell in the matrix $[d_{ij}]$.
(ii) Allocate θ to this empty cell in the final allocation table. Subtract and add the amount of this allocation to other corners of the loop in order to restore feasibility.
(iii) This value of θ, in general is obtained by equating to zero the minimum of the allocations containing $-\theta$ (not $+\theta$) only at the corners of the closed loop.
(iv) Substitute the value of θ and find a fresh allocation table.
Step 6. Again, apply the above test for optimality till we find $d_{ij} \geq 0$.

2.2 N-Vehicle Cost Varying Transportation Problem

Suppose there are N-types off vehicles $V_l, l = 1, \ldots, N$ from each source to each destination. Let $C_l, l = 1, \ldots, N$ are the capacities(in unit) of the vehicles $V_l, l = 1, \ldots, N$ respectively, where $C_1 < C_2 < \ldots < C_N$. $R_{ij} = (R_{ij}^1, \ldots, R_{ij}^N)$ represent transportation cost for each cell (i, j); where R_{ij}^l is the transportation cost from source $O_i, i = 1, \ldots, m$ to the destination $D_j, j = 1, \ldots, n$ by the vehicle $V_l, l = 1, \ldots, N$. So, cost varying transportation problem can be represent in the following tabulated form.

	D_1	D_2	..	D_n	*stock*
O_1	$R_{11}^1, \ldots, R_{11}^N$	$R_{12}^1, \ldots, R_{12}^N$	....	$R_{1n}^1, \ldots, R_{1n}^N$	a_1
O_2	$R_{21}^1, \ldots, R_{21}^N$	$R_{22}^1, \ldots, R_{22}^N$	....	$R_{2n}^1, \ldots, R_{2n}^N$	a_2
....	....	....	....	....	....
O_m	$R_{m1}^1, \ldots, R_{m1}^N$	$R_{m2}^1, \ldots, R_{m2}^N$	....	$R_{mn}^1, \ldots, R_{mn}^N$	a_m
Demand	b_1	b_2	....	b_n	

Table: Tabular representation of N-vehicle cost varying transportation problem.

2.3 Solution procedure of cost varying transportation problem

2.3.1 Determination of c_{ij}

To solve this problem, apply our proposed **Algorithms** stated as follows:

Case 1:

$$\text{When} \quad \max_i \; a_i \leq C_N$$

To solve this problem, apply our proposed algorithm stated as follows:

Step 1. Since unit cost is not determined (because it depends on quantity of transport), so North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S.

Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{R_{ij}}{x_{ij}} & \text{if} \quad C_{l-1} \leq x_{ij} \leq C_l, l = 2, \ldots, N, \\ \\ 0 & \text{if} \;\; x_{ij} = 0 \end{cases} \tag{3}$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij}, then for non-basic cell c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{R_{ij}}{x_{ij}} & \text{if} \quad C_{l-1} \leq x_{ij} \leq C_l, l = 2, \ldots, N, \\ \\ 0 & \text{if} \;\; x_{ij} = 0 \end{cases} \tag{4}$$

In this manner we convert cost varying transportation problem to a usual transportation problem but c_{ij} is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix c_{ij} by **Step 2** and for non-basic we fix c_{ij} by **Step 3**.

Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

Case 2: In general,

$$\text{When} \qquad \max_i \ a_i \text{is finite}$$

To solve this problem, apply our proposed algorithm stated as follows:

2.3.2 Algorithm(TP2)

Step 1. Since unit cost is not determined (because it depends on quantity of transport), so North-west corner rule (because it does not depend on unit transportation cost) is applicable to allocate initial B.F.S.

Step 2. After the allocate x_{ij} by North-west corner rule, for basic cell we determine c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{\sum t_l R_{ij}(l)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

$$\begin{aligned} \text{where} \qquad & t_l \text{ are integer solution of} \\ \min \qquad & \sum t_l R_{ij}(l) \\ \text{s.t. } x_{ij} \leq \; & \sum t_l C_l \end{aligned}$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij}, then for non-basic cell c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{\sum t_l R_{ij}(l)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

$$\begin{aligned} \text{where} \quad & t_l \text{ are integer solution of} \\ \min \quad & \sum t_l R_{ij}(l) \\ \text{s.t. } x_{ij} \quad \leq \quad & \sum t_l C_l \end{aligned}$$

In this manner we convert cost varying transportation problem to a usual transportation problem but c_{ij} is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix c_{ij} by **Step 2** and for non-basic we fix c_{ij} by **Step 3**.

Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

2.3.3 Bi-level Mathematical Programming for N-Vehicle Multi-objective Cost Varying Transportation Problem

The Bi-level mathematical programming for N-vehicle multi-objective cost varying transportation problem is formulated in **Model 2** as follows:

Model 2

$$\min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{5}$$

$$\begin{aligned} \text{where,} \quad & c_{ij} \text{ is determined by following mathematical programming} \\ c_{ij} \quad = \quad & \begin{cases} \frac{\sum t_l R_{ij}(l)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ \\ 0 & \text{if } x_{ij} = 0 \end{cases} \\ \text{where} \quad & t_l, l = 1, \ldots, N \text{ are integer solution of} \\ \min \quad & \sum t_l R_{ij}(l) \\ \text{s.t. } x_{ij} \quad \leq \quad & \sum t_l C_l \\ \sum_{i=1}^{m} x_{ij} \quad = \quad & a_i, \quad i = 1, \ldots, m; \sum_{j=1}^{n} x_{ij} = b_j, \quad j = 1, \ldots, n \\ \sum_{j=1}^{n} x_{ij} \quad = \quad & b_j, \quad j = 1, \ldots, n; \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j; x_{ij} \geq 0 \quad \forall i, \quad \forall j \end{aligned}$$

3 Numerical Example

Example 1:

Consider a 2-vehicle cost varying transportation problem as

	D_1	D_2	D_3	*stock*
O_1	$5, 10$	$8, 12$	$6, 9$	15
O_2	$6, 8$	$12, 15$	$15, 18$	12
O_3	$4, 6$	$8, 16$	$5, 10$	3
Demand	10	10	10	

The capacities of vehicles of V_1 and V_2 are respectively, $C_1 = 10$ and $C_2 = 20$.

Step 1. By North-west corner rule initial B.F.S. is

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 10$ $5, 10$	$x_{12} = 5$ $8, 12$	$6, 9$	15
O_2	$6, 8$	$x_{22} = 5$ $12, 15$	$x_{23} = 7$ $15, 18$	12
O_3	$4, 6$	$8, 16$	$x_{33} = 3$ $5, 10$	3
Demand	10	10	10	

Step 2. Using (3), we determine $c_{11} = \frac{5}{10}$, $c_{12} = \frac{8}{5}$, $c_{22} = \frac{12}{5}$, $c_{23} = \frac{15}{7}$, $c_{33} = \frac{5}{3}$,

Step 3. Using (4), we determine $C_{13} = \frac{6}{5}$, $C_{21} = \frac{6}{5}$, $C_{31} = \frac{4}{3}$ $C_{32} = \frac{8}{3}$

With these c_{ij} the transportation problem converted to

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 10$ $c_{11} = \frac{5}{10}$ $5, 10$	$x_{12} = 5$ $c_{12} = \frac{8}{5}$ $8, 12$	$C_{13} = \frac{6}{5}$ $6, 9$	15
O_2	$C_{21} = \frac{6}{5}$ $6, 8$	$x_{22} = 5$ $c_{22} = \frac{12}{5}$ $12, 15$	$x_{23} = 7$ $c_{23} = \frac{15}{7}$ $15, 18$	12
O_3	$C_{31} = \frac{4}{3}$ $4, 6$	$C_{32} = \frac{8}{3}$ $8, 16$	$x_{33} = 3$ $c_{33} = \frac{5}{3}$ $5, 10$	3
Demand	10	10	10	

optimality test

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 10$ $c_{11} = \frac{5}{10}$ $5, 10$	$x_{12} = 5$ $c_{12} = \frac{8}{5}$ $8, 12$	$C_{13} = \frac{6}{5}$ $6, 9$ $d_{13} = -\frac{1}{7}$	0
O_2	$C_{21} = \frac{6}{5}$ $6, 8$ $d_{21} = -\frac{1}{10}$	$x_{22} = 5$ $c_{22} = \frac{12}{5}$ $12, 15$	$x_{23} = 7$ $c_{23} = \frac{15}{7}$ $15, 18$	$\frac{4}{5}$
O_3	$C_{31} = \frac{4}{3}$ $4, 6$ $d_{31} = \frac{107}{210}$	$C_{32} = \frac{8}{3}$ $8, 16$ $d_{32} = \frac{78}{105}$	$x_{33} = 3$ $c_{33} = \frac{5}{3}$ $5, 10$	$\frac{34}{105}$
v_j	$\frac{5}{10}$	$\frac{8}{5}$	$\frac{47}{35}$	

Since $d_{13} = -\frac{1}{7} < 0$ and $d_{21} = -\frac{1}{10} < 0$ so, solution is not optimal. So a loop occurred in cells$(1, 1), (1, 2), (2, 1), (2, 2)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 5$ $c_{11} = \frac{5}{5}$ $5, 10$	$x_{12} = 10$ $c_{12} = \frac{8}{10}$ $8, 12$	$C_{13} = \frac{6}{5}$ $6, 9$	15
O_2	$x_{21} = 5$ $c_{21} = \frac{6}{5}$ $6, 8$	$c_{22} = \frac{12}{5}$ $12, 15$	$x_{23} = 7$ $c_{23} = \frac{15}{7}$ $15, 18$	12
O_3	$C_{31} = \frac{4}{3}$ $4, 6$	$C_{32} = \frac{8}{3}$ $8, 16$	$x_{33} = 3$ $c_{33} = \frac{5}{3}$ $5, 10$	3
Demand	10	10	10	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 5$ $c_{11} = \frac{5}{5}$ $5, 10$	$x_{12} = 10$ $c_{12} = \frac{8}{10}$ $8, 12$	$C_{13} = \frac{6}{5}$ $6, 9$ $d_{13} = -\frac{32}{35}$	0
O_2	$x_{21} = 5$ $C_{21} = \frac{6}{5}$ $6, 8$	$c_{22} = \frac{12}{5}$ $12, 15$ $d_{22} = \frac{7}{5}$	$x_{23} = 7$ $c_{23} = \frac{15}{7}$ $15, 18$	$\frac{1}{5}$
O_3	$C_{31} = \frac{4}{3}$ $4, 6$ $d_{31} = \frac{82}{105}$	$C_{32} = \frac{8}{3}$ $8, 16$ $d_{32} = \frac{143}{105}$	$x_{33} = 3$ $c_{33} = \frac{5}{3}$ $5, 10$	$-\frac{47}{105}$
v_j	1	$\frac{8}{10}$	$\frac{74}{35}$	

Since $d_{13} = -\frac{32}{35} < 0$ so, solution is not optimal. So a loop occurred in cells$(1, 1), (1, 3), (2, 1), (2, 3)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*Stock*
O_1	$c_{11} = \frac{5}{5}$ $5, 10$	$x_{12} = 10$ $c_{12} = \frac{8}{10}$ $8, 12$	$x_{13} = 5$ $c_{13} = \frac{6}{5}$ $6, 9$	15
O_2	$x_{21} = 10$ $c_{21} = \frac{6}{10}$ $6, 8$	$c_{22} = \frac{12}{2}$ $12, 15$	$x_{23} = 2$ $c_{23} = \frac{15}{2}$ $15, 18$	12
O_3	$C_{31} = \frac{4}{3}$ $4, 6$	$C_{32} = \frac{8}{3}$ $8, 16$	$x_{33} = 3$ $c_{33} = \frac{5}{3}$ $5, 10$	3
Demand	10	10	10	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$c_{11} = \frac{5}{5}$ $5, 10$ $d_{11} > 0$	$x_{12} = 10$ $c_{12} = \frac{8}{10}$ $8, 12$	$x_{13} = 5$ $c_{13} = \frac{6}{5}$ $6, 9$	$\frac{6}{5}$
O_2	$x_{21} = 10$ $c_{21} = \frac{6}{10}$ $6, 8$	$c_{22} = \frac{12}{2}$ $12, 15$ $d_{22} < 0$	$x_{23} = 2$ $c_{23} = \frac{15}{2}$ $15, 18$	$\frac{15}{2}$
O_3	$C_{31} = \frac{4}{3}$ $4, 6$ $d_{31} > 0$	$C_{32} = \frac{8}{3}$ $8, 16$ $d_{32} > 0$	$x_{33} = 3$ $c_{33} = \frac{5}{3}$ $5, 10$	$\frac{5}{3}$
v_j	$-\frac{69}{10}$	$-\frac{2}{5}$	0	

Since $d_{22} < 0$ so, solution is not optimal. So a loop occurred in cells$(1, 2), (2, 2), (1, 3), (2, 3)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table. (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*Stock*
O_1	$c_{11} = \frac{5}{5}$ $5, 10$	$x_{12} = 8$ $c_{12} = \frac{8}{8}$ $8, 12$	$x_{13} = 7$ $c_{13} = \frac{6}{7}$ $6, 9$	15
O_2	$x_{21} = 10$ $c_{21} = \frac{6}{10}$ $6, 8$	$x_{22} = 2$ $c_{22} = \frac{12}{2}$ $12, 15$	$c_{23} = \frac{15}{2}$ $15, 18$	12
O_3	$C_{31} = \frac{4}{3}$ $4, 6$	$C_{32} = \frac{8}{3}$ $8, 16$	$x_{33} = 3$ $c_{33} = \frac{5}{3}$ $5, 10$	3
Demand	10	10	10	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$c_{11} = \frac{5}{5}$ $5, 10$ $d_{11} > 0$	$x_{12} = 8$ $c_{12} = \frac{8}{8}$ $8, 12$	$x_{13} = 7$ $c_{13} = \frac{6}{7}$ $6, 9$	-5
O_2	$x_{21} = 10$ $c_{21} = \frac{6}{10}$ $6, 8$	$x_{22} = 2$ $c_{22} = \frac{12}{2}$ $12, 15$	$c_{23} = \frac{15}{2}$ $15, 18$ $d_{23} > 0$	0
O_3	$C_{31} = \frac{4}{3}$ $4, 6$ $d_{31} > 0$	$C_{32} = \frac{8}{3}$ $8, 16$ $d_{32} > 0$	$x_{33} = 3$ $c_{33} = \frac{5}{3}$ $5, 10$	$-\frac{88}{21}$
v_j	$\frac{6}{10}$	6	$\frac{41}{7}$	

Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution. $x_{12} = 8, x_{13} = 7, x_{21} = 10, x_{22} = 2, x_{33} = 3$. Minimum cost $= 8 + 6 + 6 + 10 + 5 = 25$ unit(Rs.)

Example 2:

Consider a 2-vehicle cost varying transportation problem as

	D_1	D_2	D_3	*stock*
O_1	$4, 8$	$5, 10$	$10, 20$	48
O_2	$2, 3$	$8, 16$	$6, 12$	52
O_3	$7, 14$	$3, 6$	$9, 18$	25
Demand	75	30	20	

The capacities of vehicles of V_1 and V_2 are respectively, $C_1 = 6$ and $C_2 = 18$.

Step 1. By North-west corner rule initial B.F.S. is

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 48$ $4, 8$	$5, 10$	$10, 20$	48
O_2	$x_{21} = 27$ $2, 3$	$x_{22} = 25$ $8, 16$	$6, 12$	52
O_3	$7, 14$	$x_{32} = 5$ $3, 6$	$x_{33} = 20$ $9, 18$	25
Demand	75	30	20	

Step 2. Using (5), we determine $c_{11} = \frac{24}{48}$, $C_{21} = \frac{6}{27}$, $c_{22} = \frac{32}{25}$,$C_{32} = \frac{3}{5}$ $c_{33} = \frac{27}{20}$,

Step 3. Using (6), we determine$c_{12} = \frac{20}{25}$, $c_{23} = \frac{18}{20}$, $C_{13} = \frac{30}{20}$, $C_{31} = \frac{7}{5}$

With these c_{ij} the transportation problem converted to

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 48$ $c_{11} = \frac{24}{48}$ $4, 8$	$c_{12} = \frac{20}{25}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 27$ $c_{21} = \frac{6}{27}$ $2, 3$	$x_{22} = 25$ $c_{22} = \frac{32}{25}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{7}{5}$ $7, 14$	$x_{32} = 5$ $c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20$ $c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

optimality test

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,

each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 48$ $c_{11} = \frac{24}{48}$ $4, 8$	$c_{12} = \frac{20}{25}$ $5, 10$ $d_{12} < 0$	$c_{13} = \frac{30}{20}$ $10, 20$ $d_{13} > 0$	0
O_2	$x_{21} = 27$ $c_{21} = \frac{6}{27}$ $2, 3$	$x_{22} = 25$ $c_{22} = \frac{32}{25}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$ $d_{23} > 0$	$-\frac{5}{18}$
O_3	$c_{31} = \frac{7}{5}$ $7, 14$ $d_{31} < 0$	$x_{32} = 5$ $c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20$ $c_{33} = \frac{27}{20}$ $9, 18$	$\frac{2077}{900}$
v_j	$\frac{1}{2}$	$\frac{701}{450}$	$-\frac{431}{450}$	

Since $d_{12} < 0$ so, solution is not optimal. So a loop occurred in cells$(1, 1), (1, 2), (2, 2), (2, 1), (1, 1)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 23$ $c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 25$ $c_{12} = \frac{20}{25}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 52$ $c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{32}{25}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{7}{5}$ $7, 14$	$x_{32} = 5$ $c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20$ $c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,

each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 23$ $c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 25$ $c_{12} = \frac{20}{25}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$ $d_{13} < 0$	0
O_2	$x_{21} = 52$ $c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{32}{25}$ $8, 16$ $d_{22} > 0$	$c_{23} = \frac{18}{20}$ $6, 12$ $d_{23} < 0$	$-\frac{417}{1196}$
O_3	$c_{31} = \frac{7}{5}$ $7, 14$ $d_{31} > 0$	$x_{32} = 5$ $c_{32} = \frac{3}{5}$ $3, 6$	$x_{33} = 20$ $c_{33} = \frac{27}{20}$ $9, 18$	$-\frac{1}{5}$
v_j	$\frac{12}{23}$	$\frac{20}{25}$	$\frac{31}{20}$	

Since $d_{13} < 0$ (i.e, most negative) so, solution is not optimal. So a loop occurred in cells$(1, 2), (3, 2), (3, 3), (1, 3), (1, 2)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 23$ $c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 5$ $c_{12} = \frac{5}{5}$ $5, 10$	$x_{13} = 20$ $c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 52$ $c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16$	$c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{7}{5}$ $7, 14$	$x_{32} = 25$ $c_{32} = \frac{6}{25}$ $3, 6$	$x_{33} = 20$ $c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,

each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 23$ $c_{11} = \frac{12}{23}$ $4, 8$	$x_{12} = 5$ $c_{12} = \frac{5}{5}$ $5, 10$	$x_{13} = 20$ $c_{13} = \frac{30}{20}$ $10, 20$	0
O_2	$x_{21} = 52$ $c_{21} = \frac{9}{52}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16$ $d_{22} > 0$	$c_{23} = \frac{18}{20}$ $6, 12$ $d_{23} < 0$	$-\frac{417}{1196}$
O_3	$c_{31} = \frac{7}{5}$ $7, 14$ $d_{31} > 0$	$x_{32} = 25$ $c_{32} = \frac{6}{25}$ $3, 6$	$c_{33} = \frac{27}{20}$ $9, 18$ $d_{33} > 0$	$-\frac{19}{25}$
v_j	$\frac{12}{23}$	$\frac{25}{25}$	$\frac{3}{2}$	

Since $d_{23} < 0$ (i.e, most negative) so, solution is not optimal. So a loop occurred in cells$(1, 1), (1, 3), (2, 3), (2, 1), (1, 1)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 43$ $c_{11} = \frac{28}{43}$ $4, 8$	$x_{12} = 5$ $c_{12} = \frac{5}{5}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$	48
O_2	$x_{21} = 32$ $c_{21} = \frac{6}{32}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16$	$x_{23} = 20$ $c_{23} = \frac{18}{20}$ $6, 12$	52
O_3	$c_{31} = \frac{28}{25}$ $7, 14$	$x_{32} = 25$ $c_{32} = \frac{6}{25}$ $3, 6$	$c_{33} = \frac{27}{20}$ $9, 18$	25
Demand	75	30	20	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 43$ $c_{11} = \frac{28}{43}$ $4, 8$	$x_{12} = 5$ $c_{12} = \frac{5}{5}$ $5, 10$	$c_{13} = \frac{30}{20}$ $10, 20$ $d_{13} > 0$	 0
O_2	$x_{21} = 32$ $c_{21} = \frac{6}{32}$ $2, 3$	$c_{22} = \frac{8}{5}$ $8, 16$ $d_{22} > 0$	$x_{23} = 20$ $c_{23} = \frac{18}{20}$ $6, 12$	 $-\frac{319}{688}$
O_3	$c_{31} = \frac{28}{25}$ $7, 14$ $d_{31} > 0$	$x_{32} = 25$ $c_{32} = \frac{6}{25}$ $3, 6$	$c_{33} = \frac{27}{20}$ $9, 18$ $d_{33} > 0$	 $-\frac{19}{25}$
v_j	$\frac{28}{43}$	$\frac{25}{25}$	1.364	

Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution. $x_{11} = 43, x_{12} = 5, x_{21} = 32, x_{23} = 20, x_{32} = 25$. Minimum cost $= 28 + 5 + 6 + 18 + 6 = 61$ unit(Rs.)

Example 3:

Consider a 3-vehicle cost varying transportation problem as

	D_1	D_2	D_3	*stock*
O_1	$5, 8, 10$	$10, 13, 15$	$7, 10, 12$	75
O_2	$4, 7, 9$	$6, 9, 11$	$8, 11, 13$	60
O_3	$12, 15, 17$	$3, 6, 8$	$5, 8, 10$	40
Demand	50	55	70	

The capacities of vehicles of V_1 and V_2 are respectively, $C_1 = 10$, $C_2 = 15$ and $C_3 = 20$.

Step 1. By North-west corner rule initial B.F.S. is

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 50$ $5, 8, 10$	$x_{12} = 25$ $10, 13, 15$	 $7, 10, 12$	 75
O_2	 $4, 7, 9$	$x_{22} = 30$ $6, 9, 11$	$x_{23} = 30$ $8, 11, 13$	 60
O_3	 $12, 15, 17$	 $3, 6, 8$	$x_{33} = 40$ $5, 8, 10$	 40
Demand	50	55	70	

Step 2. Using (5), we determine $c_{11} = \frac{25}{50}$, $c_{12} = \frac{23}{25}$, $c_{22} = \frac{17}{30}$,$c_{23} = \frac{21}{30}$ $c_{33} = \frac{20}{40}$,

Step 3. Using (6), we determine$c_{21} = \frac{13}{30}$, $c_{13} = \frac{17}{25}$, $c_{31} = \frac{34}{40}$, $c_{32} = \frac{11}{30}$

With these c_{ij} the transportation problem converted to

	D_1	D_2	D_3	*stock*
O_1	$x_{11} = 50$ $c_{11} = \frac{25}{50}$ $5, 8, 10$	$x_{12} = 25$ $c_{12} = \frac{23}{25}$ $10, 13, 15$	$c_{13} = \frac{17}{25}$ $7, 10, 12$	75
O_2	$c_{21} = \frac{13}{30}$ $4, 7, 9$	$x_{22} = 30$ $c_{22} = \frac{17}{30}$ $6, 9, 11$	$x_{23} = 30$ $c_{23} = \frac{21}{30}$ $8, 11, 13$	60
O_3	$c_{31} = \frac{34}{40}$ $12, 15, 17$	$c_{32} = \frac{11}{30}$ $3, 6, 8$	$x_{33} = 40$ $c_{33} = \frac{20}{40}$ $5, 8, 10$	40
Demand	50	55	70	

optimality test

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,

each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 50$ $c_{11} = \frac{25}{50}$ $5, 8, 10$	$x_{12} = 25$ $c_{12} = \frac{23}{25}$ $10, 13, 15$	$c_{13} = \frac{17}{25}$ $7, 10, 12$ $d_{13} < 0$	0
O_2	$c_{21} = \frac{13}{30}$ $4, 7, 9$ $d_{21} < 0$	$x_{22} = 30$ $c_{22} = \frac{17}{30}$ $6, 9, 11$	$x_{23} = 30$ $c_{23} = \frac{21}{30}$ $8, 11, 13$	$-\frac{53}{150}$
O_3	$c_{31} = \frac{34}{40}$ $12, 15, 17$ $d_{31} > 0$	$c_{32} = \frac{11}{30}$ $3, 6, 8$ $d_{32} = 0$	$x_{33} = 40$ $c_{33} = \frac{20}{40}$ $5, 8, 10$	$-\frac{83}{150}$
v_j	$\frac{25}{20}$	$\frac{23}{25}$	$\frac{79}{75}$	

Since $d_{21} < 0$ (most negative), so, solution is not optimal. So a loop occurred in cells$(1, 1), (1, 2), (2, 2), (2, 1)$, $(1, 1)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*Stock*
O_1	$x_{11} = 20$ $c_{11} = \frac{10}{20}$ $5, 8, 10$	$x_{12} = 55$ $c_{12} = \frac{43}{55}$ $10, 13, 15$	$c_{13} = \frac{12}{20}$ $7, 10, 12$	75
O_2	$x_{21} = 30$ $c_{21} = \frac{13}{30}$ $4, 7, 9$	$c_{22} = \frac{17}{30}$ $6, 9, 11$	$x_{23} = 30$ $c_{23} = \frac{21}{30}$ $8, 11, 13$	60
O_3	$c_{31} = \frac{29}{30}$ $12, 15, 17$	$c_{32} = \frac{12}{40}$ $3, 6, 8$	$x_{33} = 40$ $c_{33} = \frac{20}{40}$ $5, 8, 10$	40
Demand	50	55	70	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$x_{11} = 20$ $c_{11} = \frac{10}{20}$ $5, 8, 10$	$x_{12} = 55$ $c_{12} = \frac{43}{55}$ $10, 13, 15$	$c_{13} = \frac{12}{20}$ $7, 10, 12$ $d_{13} < 0$	0
O_2	$x_{21} = 30$ $c_{21} = \frac{13}{30}$ $4, 7, 9$	$c_{22} = \frac{17}{30}$ $6, 9, 11$ $d_{22} < 0$	$x_{23} = 30$ $c_{23} = \frac{21}{30}$ $8, 11, 13$	$-\frac{1}{15}$
O_3	$c_{31} = \frac{29}{30}$ $12, 15, 17$ $d_{31} > 0$	$c_{32} = \frac{12}{40}$ $3, 6, 8$ $d_{32} > 0$	$x_{33} = 40$ $c_{33} = \frac{20}{40}$ $5, 8, 10$	$-\frac{4}{15}$
v_j	$\frac{10}{20}$	$\frac{43}{55}$	$\frac{23}{30}$	

Since $d_{13} < 0$ (i.e, most negative) so, solution is not optimal. So a loop occurred in cells$(1, 3), (2, 3), (2, 1), (1, 1),$ $(1, 3)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*Stock*
O_1	$c_{11} = \frac{10}{20}$ $5, 8, 10$	$x_{12} = 55$ $c_{12} = \frac{43}{55}$ $10, 13, 15$	$x_{13} = 20$ $c_{13} = \frac{12}{20}$ $7, 10, 12$	75
O_2	$x_{21} = 50$ $c_{21} = \frac{19}{50}$ $4, 7, 9$	$c_{22} = \frac{6}{10}$ $6, 9, 11$	$x_{23} = 10$ $c_{23} = \frac{8}{10}$ $8, 11, 13$	60
O_3	$c_{31} = \frac{29}{30}$ $12, 15, 170$	$c_{32} = \frac{12}{40}$ $3, 6, 8$	$x_{33} = 40$ $c_{33} = \frac{20}{40}$ $5, 8, 10$	40
Demand	50	55	70	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,\ \ v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$c_{11} = \frac{10}{20}$ $5, 8, 10$ $d_{11} > 0$	$x_{12} = 55$ $c_{12} = \frac{43}{55}$ $10, 13, 15$	$x_{13} = 20$ $c_{13} = \frac{12}{20}$ $7, 10, 12$	$\frac{12}{20}$
O_2	$x_{21} = 50$ $c_{21} = \frac{19}{50}$ $4, 7, 9$	$c_{22} = \frac{6}{10}$ $6, 9, 11$ $d_{22} < 0$	$x_{23} = 10$ $c_{23} = \frac{8}{10}$ $8, 11, 13$	$\frac{8}{10}$
O_3	$c_{31} = \frac{29}{30}$ $12, 15, 17$ $d_{31} > 0$	$c_{32} = \frac{12}{40}$ $3, 6, 8$ $d_{32} < 0$	$x_{33} = 40$ $c_{33} = \frac{20}{40}$ $5, 8, 10$	$\frac{20}{40}$
v_j	$-\frac{21}{50}$	$\frac{2}{11}$	0	

Since $d_{32} < 0$ (i.e, most negative) so, solution is not optimal. So a loop occurred in cells$(3, 2), (3, 3), (1, 3), (1, 2),$ $(3, 2)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*Stock*
O_1	$c_{11} = \frac{25}{50}$ $5, 8, 10$	$x_{12} = 15$ $c_{12} = \frac{13}{15}$ $10, 13, 15$	$x_{13} = 60$ $c_{13} = \frac{36}{60}$ $7, 10, 12$	75
O_2	$x_{21} = 50$ $c_{21} = \frac{19}{50}$ $4, 7, 9$	$c_{22} = \frac{6}{10}$ $6, 9, 11$	$x_{23} = 10$ $c_{23} = \frac{8}{10}$ $8, 11, 13$	60
O_3	$c_{31} = \frac{34}{40}$ $12, 15, 17$	$x_{32} = 40$ $c_{32} = \frac{12}{40}$ $3, 6, 8$	$c_{33} = \frac{20}{40}$ $5, 8, 10$	40
Demand	50	55	70	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$c_{11} = \frac{25}{50}$ $5, 8, 10$ $d_{11} > 0$	$x_{12} = 15$ $c_{12} = \frac{13}{15}$ $10, 13, 15$	$x_{13} = 60$ $c_{13} = \frac{36}{60}$ $7, 10, 12$	0
O_2	$x_{21} = 50$ $c_{21} = \frac{19}{50}$ $4, 7, 9$	$c_{22} = \frac{6}{10}$ $6, 9, 11$ $d_{22} < 0$	$x_{23} = 10$ $c_{23} = \frac{8}{10}$ $8, 11, 13$	$\frac{1}{5}$
O_3	$c_{31} = \frac{34}{40}$ $12, 15, 17$ $d_{31} > 0$	$x_{32} = 40$ $c_{32} = \frac{12}{40}$ $3, 6, 8$	$c_{33} = \frac{20}{40}$ $5, 8, 10$ $d_{33} > 0$	$-\frac{17}{30}$
v_j	$\frac{9}{50}$	$\frac{13}{15}$	$\frac{36}{60}$	

Since $d_{22} < 0$ (i.e, most negative) so, solution is not optimal. So a loop occurred in cells$(2, 2), (2, 3), (1, 3), (2, 1),$ $(2, 2)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	*Stock*
O_1	$c_{11} = \frac{5}{5}$ $5, 8, 10$	$x_{12} = 5$ $c_{12} = \frac{10}{5}$ $10, 13, 15$	$x_{13} = 70$ $c_{13} = \frac{43}{70}$ $7, 10, 12$	75
O_2	$x_{21} = 50$ $c_{21} = \frac{19}{50}$ $4, 7, 9$	$x_{22} = 10$ $c_{22} = \frac{6}{10}$ $6, 9, 11$	$c_{23} = \frac{8}{10}$ $8, 11, 13$	60
O_3	$c_{31} = \frac{34}{40}$ $12, 15, 170$	$x_{32} = 40$ $c_{32} = \frac{12}{40}$ $3, 6, 8$	$c_{33} = \frac{20}{40}$ $5, 8, 10$	40
Demand	50	55	70	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$c_{11} = \frac{5}{5}$ $5, 8, 10$ $d_{11} < 0$	$x_{12} = 5$ $c_{12} = \frac{10}{5}$ $10, 13, 15$	$x_{13} = 70$ $c_{13} = \frac{43}{70}$ $7, 10, 12$	0
O_2	$x_{21} = 50$ $c_{21} = \frac{19}{50}$ $4, 7, 9$	$x_{22} = 10$ $c_{22} = \frac{6}{10}$ $6, 9, 11$	$c_{23} = \frac{8}{10}$ $8, 11, 13$ $d_{23} > 0$	$-\frac{7}{5}$
O_3	$c_{31} = \frac{34}{40}$ $12, 15, 170$ $d_{31} > 0$	$x_{32} = 40$ $c_{32} = \frac{12}{40}$ $3, 6, 8$	$c_{33} = \frac{20}{40}$ $5, 8, 10$ $d_{33} > 0$	$-\frac{17}{10}$
v_j	$\frac{89}{50}$	2	$\frac{43}{70}$	

Since $d_{11} < 0$ (i.e, most negative) so, solution is not optimal. So a loop occurred in cells$(1, 1), (1, 2), (2, 2), (1, 2),$ $(1, 1)$ and modified basic cell and unit transportation cost (by our proposed algorithm) is represented in the following table.

	D_1	D_2	D_3	$Stock$
O_1	$x_{11} = 5$ $c_{11} = \frac{5}{5}$ $5, 8, 10$	$c_{12} = \frac{10}{5}$ $10, 13, 15$	$x_{13} = 70$ $c_{13} = \frac{43}{70}$ $7, 10, 12$	75
O_2	$x_{21} = 55$ $c_{21} = \frac{25}{50}$ $4, 7, 9$	$x_{22} = 15$ $c_{22} = \frac{9}{15}$ $6, 9, 11$	$c_{23} = \frac{37}{55}$ $8, 11, 13$	60
O_3	$c_{31} = \frac{34}{40}$ $12, 15, 170$	$x_{32} = 40$ $c_{32} = \frac{12}{40}$ $3, 6, 8$	$c_{33} = \frac{20}{40}$ $5, 8, 10$	40
$Demand$	50	55	70	

Determine a set of 6 numbers $u_i, i = 1, 2, 3$ and $v_j, j = 1, 2, 3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,
each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$
So the tabular representation of $u_i, i = 1, 2, 3,$ $v_j, j = 1, 2, 3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	u_i
O_1	$c_{11} = \frac{5}{5}$ $5, 8, 10$	$x_{12} = 5$ $c_{12} = \frac{10}{5}$ $10, 13, 15$ $d_{12} > 0$	$x_{13} = 70$ $c_{13} = \frac{43}{70}$ $7, 10, 12$	$\frac{6}{11}$
O_2	$x_{21} = 50$ $c_{21} = \frac{19}{50}$ $4, 7, 9$	$x_{22} = 10$ $c_{22} = \frac{6}{10}$ $6, 9, 11$	$c_{23} = \frac{8}{10}$ $8, 11, 13$ $d_{23} > 0$	0
O_3	$c_{31} = \frac{34}{40}$ $12, 15, 170$ $d_{31} > 0$	$x_{32} = 40$ $c_{32} = \frac{12}{40}$ $3, 6, 8$	$c_{33} = \frac{20}{40}$ $5, 8, 10$ $d_{33} > 0$	$-\frac{3}{10}$
v_j	$\frac{25}{55}$	$\frac{9}{15}$	$\frac{53}{770}$	

Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution. $x_{11} = 5, x_{13} = 70, x_{21} = 55, x_{22} = 15, x_{32} = 40$. Minimum cost $= 5 + 43 + 25 + 9 + 12 = 94$ unit(Rs.)

4 Conclusion

In this paper we have developed N-vehicle cost varying transportation problem. We transfer this cost varying transportation problem to usual transportation problem by Northwest corner rule and then apply optimality test where unit transportation cost vary from one table to another table. Finally, achieve optimal solution. This problem is more real life problem than usual transportation problem.

References

[1] Charnes, and Cooper, W. W., The Stepping-stone method for explaining linear programming calculation in transportation problem, *Management Science,* 1,1954, 49-69.

[2] Dantzig, G. B., *Linear Programming and Extensions*, Princeton University Press, Princeton, 1963.

[3] Hitchcock, F. L., The distribution of a product from several sources to numerous localities, *Journal of Mathematical Physics,* 20, 1941, 224-230.

[4] S. Chanas , W. Kolosziejezyj and A. Machaj, A fuzzy approach to the transportation problem, *Fuzzy Sets and Systems,* 13, 1984, 211-221.

[5] S. Chanas, D. Kutcha D, Fuzzy integer transportation problem, *Fuzzy Sets and Systems,* 98, 1998, 291-298.

[6] S. Chandra, K. Seth, P.K. Saxena, Time minimizing transportation problem with impurities, *Asia-Pacific Journal of Operational Research,* 4, 1987, 19-27.

[7] J. W. Chinneck and K. Ramadan, Linear programming with interval coefficients, *Journal of the Operational Research Society,* 51, 2000, 209 220.

[8] S. K. Das, A. Goswami, and S. S Alam, Multiobjective transportation problem with interval cost, source and destination parameters, *European Journal of Operational Research,* 117, 1999, 100112.

[9] D. Dutta, and A. Satyanarayana Murthy, Fuzzy transportation problem with additional restrictions, *ARPN Journal of Engineering and Applied Sciences,* 5(2), 2010, 36-40.

[10] K. B. Haley, and A. J. Smith, Transportation problems with additional restrictions, *JSTOR,* 15(2), 1996, 116-127

[11] H. Ishibuchi, and H. Tanaka, Multiobjective programming in optimization of the interval objective function, *European Journal of Operational Research,* 48, 1990, 219225.

[12] R. E. Moore, Method and applications of interval analysis, *SLAM, Philadelphia, PA, 1979.*

[13] P. Pandian and G. Natarajan, A new method for finding an optimal solution of fully interval integer transportation problems, *Applied Mathematical Sciences,* 4, 2010, 1819-1830.

[14] P. Pandian and D. Anuradha, Floating point method for solving transportation problem with additional constraints, *International Mathematical Forum,* 6, 2011, 1983-1992.

[15] Preetvanti Singh and P. K. Saxena, The multiobjective time transportation problem with additional restrictions, *European Journal of Operational Research,* 146, 2003, 460- 476.

[16] Sengupta and T.K. Pal, Interval-valued transportation problem with multiple penalty factors, *VU Journal of Physical Sciences,* 9, 2003, 71-81.