AMO-Advanced Modelling and Optimization, Volume 15, Number 3, 2013

Multi-objective Geometric Programming Problems With Cost Co-efficients as Multiple Parameters.

A. K.Ojha† and Rashmi Ranjan Ota*
† School of Basic Sciences,
Indian Institute of Technology, Bhubaneswar,India
Email: akojha57@yahoo.com; akojha@iitbbs.ac.in
* C.V.Raman college of Engineering, Bhubaneswar, India
Email: otapink@gmail.com

Abstract

Geometric programming(GP) is a powerful Optimization technique specially design to solve non-linear programming problems. It has a wide range of application in various fields of science and engineering for solving certain complex decision making problems. Generally, an engineering design problem has multi-objective functions. Some of these problems can be formulated as multi-objective Geometric programming models. Very often,in real world problem, the co-efficient of the objective functions are not known precisely. Co-efficient may be interpreted as multiple parameters which lead to a multi-objective Geometric programming with multiple parameters. In this paper we have developed a method to solve multi-objective Geometric programming problem where cost co-efficient of objective function as well as co-efficient of constraints are multiple parameters. Especially the multiple parameters are considered in an interval which are the Arithmetic mean(A.M), Geometric mean(G.M) and Harmonic mean(H.M) of the end points of the interval. Subsequently we have solved the multi-objective Geometric programming problem. It has been verified that the objective values so obtained are also maintained in the form of A.M, G.M and H.M in the interval range of the objective values of objective function. Then the result has been verified by the illustrative example.

Keywords : Multi-objective Geometric Programming Problem, Duality theorem, Optimization, Weighted mean method.

1 Introduction

Geometric Programming(G.P) is an optimization technique developed to solve a certain class of algebraic non-linear optimization problems especially found in the field of engineering design and manufacturing.The theory of G.P by Duffin, peterson and Zener[11] put a foundation stone to solve a wide range of engineering design problems. In their work, they have shown that many engineering design problems have an objective function consisting of component cost and can be minimized under certain constraint which are in the form of posynomials. One of the important characteristic of GP is that a problem with highly non-linear constraints can be converted to a problem with linear constraints using its duality theorem, so that the problem will be easy to solve. GP is such a powerful optimization technique whose elegant theoretical concept has led a number of researchers to develop its interesting application in various fields. Several important developments of GP took place in late 70's by Ecker[12], Beightler and Phillips[1]. Subsequently, GP was extended to include more general formulation beyond posynomial tied with convex optimization and Lagrange duality. Many numerical algorithms were proposed and tested by researchers of mechanical engineering, civil engineering and chemical engineering by applying GP technique to their problems. Today, most of the real world decision making problems in the environmental, social, economic and technical areas are multidimensional and multi-objective ones. Generally an engineering design problem has multiple objective functions that are usually non commensurable and conflict with each other.The objective of study of multi-objective optimization is to investigate in finding solutions that are evaluated under several objective functions, typically defined for multi dimensional cost vector. In a multi-objective optimization problem, we are interested not only in finding single optimal solution but also in computing trade off between different objective functions. However, it is possible for the decision maker to state the desirability of achieving an aspiration level in an imprecise interval around it. To solve engineering design problems subject to algebraic linear or non-linear constraint several extension proposed by different authors[13, 14, 15]. A.J.Morris[16] has studied the minimum wight design of statically determinate structure using Geometric programming. Rao[20] has shown the application of complimentary Geometric programming to mechanical design problems. Now-a-days, GP technique has been used in the area integrated circuit design[3, 5, 6], project management[23] and short term or long term profit maximization. Cao[9, 10], the first man to transform Geometric programming problem (GPP) to its corresponding fuzzy state and has shown that Fuzzy programming is a useful method to solve multi-objective optimization problem. Subsequently, Verma[22] studied several objective functions by using GP techniques. Biswal[4] developed fuzzy programming with non-linear membership function approach to multi-objective GP problems. Sensitive analysis of various optimal solution due to Dembo[7], Dinkle and tretter[8]using GP technique simplifies certain engineering design problems, in which some problem parameters estimate the actual value. Beightler and philips[2], Kortanek [18], Peterson[19], Rajgopal and Bricker[21] presented several algorithms for finding the solution of GPP if cost and constraint co-efficient are exact.

We have emphasized on posynomial Geometric programming problems and develop a solution method which will be able to calculate the bounds of objective value for the problems where the cost and the constraint co-efficient are interval parameters. When the cost and the constraint co-efficient are interval parameters,the derived objective value should lie in an interval as well. Liu[17] develops a solution method to calculate the bounds of the objective value in GP with interval parameters. Since the cost and the constraints co-efficient are imprecise, the objective value should be inaccurate as well. The ability of calculating the bounds of objective value is basically developed in this paper that may help researchers in constructing more realistic model in optimization field.

In this paper, we have shown the values of objective function at the multiple parameters such as A.M, G.M and H.M in an interval preserves the same relation. Weighted mean method has been applied to these problems to find the compromise solution which are also preserving the relationship of A.M, G.M and H.M.The Organisation of this paper as follows: following introduction,multi-objective GP, Duality theorem and Weighted mean method has been discussed in sec-2, sec-3 and sec-4 respectively. Mathematical formulation of multi-objective optimization problem using multiple parameter has been presented in sec-5 and it's corresponding numerical example has been incorporated in sec-6. Finally some conclusions are drawn from discussion in sec-7.

2 Multi-objective Geometric Programming Problem(MOGPP):

A Multi-objective Geometric programming problem can be stated as:
Find $x = (x_1, x_2, ...x_n)^T$ so as to

$$\min : f_0^k(x) = \sum_{t=1}^{T_0^k} C_{0t}^k \prod_{j=1}^{n} x_j^{a_{0tj}^k}, \quad k = 1, 2, ...p \tag{2.1}$$

Subject to

$$g_i(x) = \sum_{t=1}^{T_i} C_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1, \quad i = 1, 2, ...m \tag{2.2}$$

$$x_j > 0, \quad j = 1, 2, ...n \tag{2.3}$$

where $C_{0t}^k \geq 0$ for all k and t
$C_{it} \geq 0$ for all i and t
a_{0tj}^k and a_{itj} are real numbers for all i, j, k, t.
T_0^k = number of terms present in the k^{th} objective function $f_0^k(x)$.
T_i= number of terms present in the i^{th} constraint.
In the above multi-objective geometric programming problem there are p number of minimization type objective functions, m number of inequality type constraints and n number of strictly positive decision variables.
The Multi-objective Geometric programming problem defined in (2.1)-(2.3) is considered as a vector-minimization problem. It is assumed that the problem has a optimal compromise solution.

3 Dual form of MOGPP:

The model given by (2.1)-(2.3) is a conventional type Geometric programming problem. The solution procedure for a Geometric programming problem may be categorized as of two types. It is either primal based algorithms that directly solve the non-linear primal problem, or dual based algorithms that solve the equivalent linear constraint dual program[19]. In view of Rajgopal and Bricker[21], the dual program has the desirable features of some linear constrains and having an objective function with attractive structural properties, which enables getting a solution. According to Beightler and Phillips [1] and Duffin et al.[11], one can obtain the corresponding dual program of (2.1)- (2.3) as follows:

Dual Program:

$$max : \prod_{k=1}^{p}\prod_{t=1}^{T_0^k}\left(\frac{C_{0t}^k}{w_{0t}^k}\right)^{w_{0t}^k}\prod_{i=1}^{m}\prod_{t=1}^{T_i}\left(\frac{C_{it}}{w_{it}}\right)^{w_{it}}\prod_{k=1}^{p}\left(\lambda^k\right)^{\lambda^k}\prod_{i=1}^{m}(\lambda^i)^{\lambda^i} \tag{3.1}$$

subject to

$$\sum_{t=1}^{T_0^r} w_{0t}^r = \lambda^r, \qquad r = 1, 2, ..., k-1, k+1, ..., p \tag{3.2}$$

$$\sum_{t=1}^{T_0^k} w_{0t}^k = \lambda^k = 1 \tag{3.3}$$

(normality condition)

$$\sum_{t=1}^{T_i} w_{it} = \lambda_i, \qquad i = 1, 2, ...m \tag{3.4}$$

$$\sum_{k=1}^{p}\sum_{t=1}^{T_0^k} a_{0tj}^k w_{0t}^k + \sum_{i=1}^{m}\sum_{t=1}^{T_i} a_{itj} w_{it} = 0, \quad j = 1, 2, ...m \tag{3.5}$$

$$w_{it} \geq 0 \;\; \forall\; t, i$$

$$w_{0t} \geq 0 \;\; \forall\; k, t$$

This dual problem can be solved by using the duality theory of Geometric programming problem.

4 Weighted Mean Method:

Weighted mean method probably is the simplest method widely used to convert a set of objectives into a single objective by multiplying each objective with weights to find the non-inferior optimal solution of a multi-objective optimization problem within the convex objective space. If $f_0^1(x), f_0^2(x), ...f_0^p(x)$ are 'p' objective functions for any vector $x = (x_1, x_2, ...x_n)^T$, then we can define weighting mean method is as follows.

$$Let \;\; W = \{w : w \in R^n, w_k > 0, \sum_{k=1}^{n} w_k = 1\} \tag{4.1}$$

be the set of non-negative weights. Using weighting method the multi-objective function with constraints can be defined as:

$$Q(w) = \min_{x \in X} \sum_{k=1}^{p} w_k f_0^k(x) \tag{4.2}$$

subject to

$$g_i(x) \leq 1, i = 1, 2, ...m \tag{4.3}$$

$$x_j > 0, j = 1, 2, ...n \tag{4.4}$$

It is necessary that the objective space of original problem should convex. If non-convex, then weighting method may not capable of generating the efficient solutions on the non-convex part of efficient frontier. It must be noted that the optimal solution of a optimization problem using weighting method should not be accepted as the best compromise solution if that do not reflect in decision makers mind.
Based on importance of p number of objective functions defined in(2.1) the weights $w_1, w_2, ...w_p$ are assigned to define a new minimization type objective function $F(x)$ which can be defined as:

$$\min_x : F(x) = \sum_{k=1}^{p} w_k f_0^k(x) = \sum_{k=1}^{p} w_k (\sum_{t=1}^{T_0^k} C_{0t}^k \prod_{j=1}^{n} x_j^{a_{0tj}^k}) = \sum_{k=1}^{p} \sum_{t=1}^{T_0^k} w_k C_{0t}^k \prod_{j=1}^{n} x_j^{a_{0tj}^k} \tag{4.5}$$

$$x_j > 0, j = 1, 2....., n \tag{4.6}$$

where

$$\sum_{k=1}^{p} w_k = 1, w_k > 0, k = 1, 2, ...p \tag{4.7}$$

5 Mathematical Formulation:

As defined in sec(2.1)-(2.3), a general multi-objective optimization problem is given below:

$$\min : f_0^k(x) = \sum_{t=1}^{T_0^k} C_{0t}^k \prod_{j=1}^{n} x_j^{a_{0tj}^k}, \quad k = 1, 2, ...p \tag{5.1}$$

Subject to

$$g_i(x) = \sum_{t=1}^{T_i} C_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1, \quad i = 1, 2, ...m \tag{5.2}$$

$$x_j > 0, \quad j = 1, 2, ...n \tag{5.3}$$

Considering the cost co-efficient of variables in objective functions and constraints as the multiple parameters the problem defined in (2.1) to (2.3)can be reformulated as:

$$\min_x : \sum_{t=1}^{T_0^k} \overline{C}_{0t}^k \prod_{j=1}^{n} x_j^{a_{0tj}^k}, \quad k = 1, 2, ...p \tag{5.4}$$

Subject to

$$\sum_{t=1}^{T_i} \overline{C}_{it} \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1, \quad i = 1, 2, ...m \tag{5.5}$$

$$x_j > 0, \quad j = 1, 2, ...n \tag{5.6}$$

$$Where \ \ \overline{C}_{0t}^k = \{{C_{0t}^k}^L, {C_{0t}^k}^{A.M}, {C_{0t}^k}^{G.M}, {C_{0t}^k}^{H.M}, {C_{0t}^k}^U\}, \tag{5.7}$$

$$\overline{C}_{it} = \{{C_{it}}^L, {C_{it}}^{A.M}, {C_{it}}^{G.M}, {C_{it}}^{H.M}, {C_{it}}^U\} \tag{5.8}$$

and L=Lower bound, A.M=Arithmetic mean, G.M=Geometric mean, H.M=Harmonic mean, U=Upper bound of parameter interval.

Using the multiple parameter as defined above, we can define the optimization problem as:

$${f_0^k}^L = \min_x \sum_{t=1}^{T_0^k} {C_{0t}^k}^L \prod_{j=1}^{n} x_j^{a_{0tj}^k}, \quad k = 1, 2, ...p \tag{5.9}$$

Subject to

$$\sum_{t=1}^{T_i} {C_{it}}^L \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1, \quad i = 1, 2, ...m \tag{5.10}$$

$$x_j > 0, \quad j = 1, 2, ...n \tag{5.11}$$

are the minimum values of objective functions in Lower bound of interval of parameter. Similarly

$${f_0^k}^U = \max_x \sum_{t=1}^{T_0^k} {C_{0t}^k}^U \prod_{j=1}^{n} x_j^{a_{0tj}^k}, \quad k = 1, 2, ...p \tag{5.12}$$

Subject to

$$\sum_{t=1}^{T_i} {C_{it}}^U \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1, \quad i = 1, 2, ...m \tag{5.13}$$

$$x_j > 0, \quad j = 1, 2, ...n \tag{5.14}$$

are the maximum values of objective functions at the upper bound of interval of parameter. The minimum values of objective functions at A.M of Lower bound and Upper bound of interval parameter is given below:

$$f_0^{k^{A.M}} = \min_x \sum_{t=1}^{T_0^k} C_{0t}^{k^{A.M}} \prod_{j=1}^{n} x_j^{a_{0tj}^k}, \quad k = 1, 2, ...p \tag{5.15}$$

Subject to

$$\sum_{t=1}^{T_i} C_{it}^{A.M} \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1, \quad i = 1, 2, ...m \tag{5.16}$$

$$x_j > 0, \quad j = 1, 2, ...n \tag{5.17}$$

Minimum values of objective functions at G.M of interval of parameter can be obtained as follows:

$$f_0^{k^{G.M}} = \min_x \sum_{t=1}^{T_0^k} C_{0t}^{k^{G.M}} \prod_{j=1}^{n} x_j^{a_{0tj}^k}, \quad k = 1, 2, ...p \tag{5.18}$$

Subject to

$$\sum_{t=1}^{T_i} C_{it}^{G.M} \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1, \quad i = 1, 2, ...m \tag{5.19}$$

$$x_j > 0, \quad j = 1, 2, ...n \tag{5.20}$$

Similarly minimum values of objective functions at H.M of interval of parameter can be obtained using the following formulation:

$$f_0^{k^{H.M}} = \min_x \sum_{t=1}^{T_0^k} C_{0t}^{k^{H.M}} \prod_{j=1}^{n} x_j^{a_{0tj}^k}, \quad k = 1, 2, ...p \tag{5.21}$$

Subject to

$$\sum_{t=1}^{T_i} C_{it}^{H.M} \prod_{j=1}^{n} x_j^{a_{itj}} \leq 1, \quad i = 1, 2, ...m \tag{5.22}$$

$$x_j > 0, \quad j = 1, 2, ...n \tag{5.23}$$

6 Illustrative Examples:

The following example illustrate the methodology proposed in this paper for solving a MOGPP with multiple parameters of cost and constraint co-efficient.

Example

Find x_1, x_2, x_3 so as to

$$\min f_1(x) = (2,6,2\sqrt{5},\frac{3}{10},10)x_1^{-1}x_2^{-1} + (20,25,10\sqrt{6},\frac{1}{24},30)x_2x_3 + (12,15,6\sqrt{6},\frac{5}{72},18)x_3^{-1} \tag{6.1}$$

$$\min f_2(x) = (20,25,10\sqrt{6},\frac{1}{24},30)x_1^{-1}x_2^{-1}x_3^{-1} + (10,15,10\sqrt{2},\frac{3}{40},20)x_1^{-2}x_3 + (4,5,2\sqrt{6},\frac{5}{24},6)x_2^{-2} \tag{6.2}$$

subject to

$$(8,10,4\sqrt{6},\frac{5}{48},12)x_1x_2 + (2,3,2\sqrt{2},\frac{3}{8},4)x_2^{-1}x_3^2 \leq 1 \tag{6.3}$$

$$where \ \ x_1, x_2, x_3 \geq 0 \tag{6.4}$$

Case1-Primal solution of $f_1(x)$:

Find x_1, x_2, x_3 so as to

$$\min f_1(x) = (2,6,2\sqrt{5},\frac{3}{10},10)x_1^{-1}x_2^{-1} + (20,25,10\sqrt{6},\frac{1}{24},30)x_2x_3 + (12,15,6\sqrt{6},\frac{5}{72},18)x_3^{-1} \tag{6.5}$$

subject to

$$(8,10,4\sqrt{6},\frac{5}{48},12)x_1x_2 + (2,3,2\sqrt{2},\frac{3}{8},4)x_2^{-1}x_3^2 \leq 1 \tag{6.6}$$

$$where \ \ x_1, x_2, x_3 \geq 0 \tag{6.7}$$

According to model given in (3.1) to (3.5), the problem can be transformed to its corresponding dual program as:

$$f_1^L = \max_w : \left(\frac{2}{w_{01}}\right)^{w_{01}} \left(\frac{20}{w_{02}}\right)^{w_{02}} \left(\frac{12}{w_{03}}\right)^{w_{03}} \left(\frac{8}{w_{11}}\right)^{w_{11}} \left(\frac{2}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.8}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.9}$$

$$-w_{01} + w_{11} = 0 \tag{6.10}$$

$$-w_{01} + w_{02} + w_{11} - w_{12} = 0 \tag{6.11}$$

$$w_{02} - w_{03} + 2w_{12} = 0 \tag{6.12}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.13}$$

Optimal value of above dual problem is f_1^L=61.30129 for $w_{01} = 0.3714305, w_{02} = 0.1571424, w_{03} = 0.4714271, w_{11} = 0.3714305, w_{12} = 0.1571424$ and corresponding primal is obtained for $x_1 = 0.075726, x_2 = 1.159940$ and $x_3 = 0.4152379$.

$$f_1^U = \max_w : \left(\frac{10}{w_{01}}\right)^{w_{01}} \left(\frac{30}{w_{02}}\right)^{w_{02}} \left(\frac{18}{w_{03}}\right)^{w_{03}} \left(\frac{12}{w_{11}}\right)^{w_{11}} \left(\frac{4}{w_{12}}\right)^{w_{12}} (w_{11} + w_{12})^{(w_{11}+w_{12})} \tag{6.14}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.15}$$

$$-w_{01} + w_{11} = 0 \tag{6.16}$$

$$-w_{01} + w_{02} + w_{11} - w_{12} = 0 \tag{6.17}$$

$$w_{02} - w_{03} + 2w_{12} = 0 \tag{6.18}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.19}$$

Optimal value of above dual problem is f_1^U=222.4514 for $w_{01} = 0.621556, w_{02} = 0.0946109, w_{03} = 0.2838330, w_{11} = 0.621556, w_{12} = 0.0946109$ and corresponding primal is obtained for $x_1 = 0.029390, x_2 = 2.460835$ and $x_3 = 0.2850852$.

$$f_1^{A.M} = \max_w : \left(\frac{6}{w_{01}}\right)^{w_{01}} \left(\frac{25}{w_{02}}\right)^{w_{02}} \left(\frac{15}{w_{03}}\right)^{w_{03}} \left(\frac{10}{w_{11}}\right)^{w_{11}} \left(\frac{3}{w_{12}}\right)^{w_{12}} (w_{11} + w_{12})^{(w_{11}+w_{12})} \tag{6.20}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.21}$$

$$-w_{01} + w_{11} = 0 \tag{6.22}$$

$$-w_{01} + w_{02} + w_{11} - w_{12} = 0 \tag{6.23}$$

$$w_{02} - w_{03} + 2w_{12} = 0 \tag{6.24}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.25}$$

Optimal value of above dual problem is $f_1^{A.M}$=133.5810 for $w_{01} = 0.5434873, w_{02} = 0.1141282, w_{03} = 0.3423845, w_{11} = 0.5434873, w_{12} = 0.1141282$ and corresponding primal is obtained for $x_1 = 0.044447,_2 = 1.859368$ and $x_3 = 0.3279688$.

$$f_1^{G.M} = \max_w : \left(\frac{2\sqrt{5}}{w_{01}}\right)^{w_{01}} \left(\frac{10\sqrt{6}}{w_{02}}\right)^{w_{02}} \left(\frac{6\sqrt{6}}{w_{03}}\right)^{w_g 03} \left(\frac{4\sqrt{6}}{w_{11}}\right)^{w_{11}} \left(\frac{2\sqrt{2}}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.26}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.27}$$

$$-w_{01} + w_{11} = 0 \tag{6.28}$$

$$-w_{01} + w_{02} + w_{11} - w_{12} = 0 \tag{6.29}$$

$$w_{02} - w_{03} + 2w_{12} = 0 \tag{6.30}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.31}$$

Optimal value of above dual problem is $f_1^{G.M}$=111.5915 for $w_{01} = 0.4934388, w_{02} = 0.1266403, w_{03} = 0.3799209, w_{11} = 0.4934388, w_{12} = 0.1266403$ and corresponding primal is obtained for $x_1 = 0.048800, x_2 = 1.664275$ and $x_3 = 0.3466590$.

$$f_1^{H.M} = \max_w : \left(\frac{3}{10w_{01}}\right)^{w_{01}} \left(\frac{1}{24w_{02}}\right)^{w_{02}} \left(\frac{5}{72w_{03}}\right)^{w_{03}} \left(\frac{5}{48w_{11}}\right)^{w_{11}} \left(\frac{3}{8w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.32}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.33}$$

$$-w_{01} + w_{11} = 0 \tag{6.34}$$

$$-w_{01} + w_{02} + w_{11} - w_{12} = 0 \tag{6.35}$$

$$w_{02} - w_{03} + 2w_{12} = 0 \tag{6.36}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.37}$$

Similarly Optimal value of above dual problem is $f_1^{H.M}$=0.1571764 for $w_{01} = 0.3096412, w_{02} = 0.1725897, w_{03} = 0.5177691, w_{11} = 0.3096412, w_{12} = 0.1725897$ and corresponding primal is obtained for $x_1 = 8.079317, x_2 = 0.7629575$ and $x_3 = 0.8533249$.

Case2-Primal solution of $f_2(x)$:

Find x_1, x_2, x_3 so as to

$$\min f_2(x) = (20, 25, 10\sqrt{6}, \frac{1}{24}, 30)x_1^{-1}x_2^{-1}x_3^{-1} + (10, 15, 10\sqrt{2}, \frac{3}{40}, 20)x_1^{-2}x_3 + (4, 5, 2\sqrt{6}, \frac{5}{24}, 6)x_2^{-2} \tag{6.38}$$

subject to

$$(8, 10, 4\sqrt{6}, \frac{5}{48}, 12)x_1x_2 + (2, 3, 2\sqrt{2}, \frac{3}{8}, 4)x_2^{-1}x_3^2 \leq 1 \tag{6.39}$$

$$where \;\; x_1, x_2, x_3 \geq 0 \tag{6.40}$$

Similarly according to model given in (3.1) to (3.5), the problem can be transformed to its corresponding dual program as:

$$f_2^L = \max_w : \left(\frac{20}{w_{01}}\right)^{w_{01}} \left(\frac{10}{w_{02}}\right)^{w_{02}} \left(\frac{4}{w_{03}}\right)^{w_{03}} \left(\frac{8}{w_{11}}\right)^{w_{11}} \left(\frac{2}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.41}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.42}$$

$$-w_{01} - 2w_{02} + w_{11} = 0 \tag{6.43}$$

$$-w_{01} - 2w_{03} + w_{11} - w_{12} = 0 \tag{6.44}$$

$$-w_{01} + w_{02} + 2w_{12} = 0 \tag{6.45}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.46}$$

Optimal value of above dual problem is f_2^L=890.0586 for $w_{01} = 0.8203274, w_{02} = 0.1710020, w_{03} = 0.008670, w_{11} = 1.162331, w_{12} = 0.3246627$ and corresponding primal is obtained for $x_1 = 0.1357178, x_2 = 0.7199359$ and $x_3 = 0.2803454$.

$$f_2^U = \max_w : \left(\frac{30}{w_{01}}\right)^{w_{01}} \left(\frac{20}{w_{02}}\right)^{w_{02}} \left(\frac{6}{w_{03}}\right)^{w_{03}} \left(\frac{12}{w_{11}}\right)^{w_{11}} \left(\frac{4}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.47}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.48}$$

$$-w_{01} - 2w_{02} + w_{11} = 0 \tag{6.49}$$

$$-w_{01} - 2w_{03} + w_{11} - w_{12} = 0 \tag{6.50}$$

$$-w_{01} + w_{02} + 2w_{12} = 0 \tag{6.51}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.52}$$

Optimal value of above dual problem is f_2^U=2819.025 for $w_{01} = 0.8270364, w_{02} = 0.1687656, w_{03} = 0.004197, w_{11} = 1.164568, w_{12} = 0.3291354$ and corresponding primal is obtained for $x_1 = 0.0912457, x_2 = 0.712043$ and $x_3 = 0.198051$.

$$f_2^{A.M} = \max_w : \left(\frac{25}{w_{01}}\right)^{w_{01}} \left(\frac{15}{w_{02}}\right)^{w_{02}} \left(\frac{5}{w_{03}}\right)^{w_{03}} \left(\frac{10}{w_{11}}\right)^{w_{11}} \left(\frac{3}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.53}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.54}$$

$$-w_{01} - 2w_{02} + w_{11} = 0 \tag{6.55}$$

$$-w_{01} - 2w_{03} + w_{11} - w_{12} = 0 \tag{6.56}$$

$$-w_{01} + w_{02} + 2w_{12} = 0 \tag{6.57}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.58}$$

Optimal value of above dual problem is $f_2^{A.M}$=1698.114 for $w_{01} = 0.8246890, w_{02} = 0.1695481, w_{03} = 0.005762, w_{11} = 1.163785, w_{12} = 0.3275704$ and corresponding primal is obtained for $x_1 = 0.1091720, x_2 = 0.7147930$ and $x_3 = 0.2287659$.

$$f_2^{G.M} = \max_w : \left(\frac{10\sqrt{6}}{w_{01}}\right)^{w_{01}} \left(\frac{10\sqrt{2}}{w_{02}}\right)^{w_{02}} \left(\frac{2\sqrt{6}}{w_{03}}\right)^{w_{03}} \left(\frac{4\sqrt{6}}{w_{11}}\right)^{w_{11}} \left(\frac{2\sqrt{2}}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.59}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.60}$$

$$-w_{01} - 2w_{02} + w_{11} = 0 \tag{6.61}$$

$$-w_{01} - 2w_{03} + w_{11} - w_{12} = 0 \tag{6.62}$$

$$-w_{01} + w_{02} + 2w_{12} = 0 \tag{6.63}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.64}$$

Optimal value of above dual problem is $f_2^{G.M}$=1583.354 for $w_{01} = 0.8242625, w_{02} = 0.1696903, w_{03} = 0.0060472, w_{11} = 1.163643, w_{12} = 0.3272861$ and corresponding primal is obtained for $x_1 = 0.1113634, x_2 = 0.7152939$ and $x_3 = 0.2356161$.

$$f_2^{H.M} = \max_w : \left(\frac{1}{24w_{01}}\right)^{w_{01}} \left(\frac{3}{40w_{02}}\right)^{w_{02}} \left(\frac{5}{24w_{03}}\right)^{w_{03}} \left(\frac{5}{48w_{11}}\right)^{w_{11}} \left(\frac{3}{8w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.65}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1 \tag{6.66}$$

$$-w_{01} - 2w_{02} + w_{11} = 0 \tag{6.67}$$

$$-w_{01} - 2w_{03} + w_{11} - w_{12} = 0 \tag{6.68}$$

$$-w_{01} + w_{02} + 2w_{12} = 0 \tag{6.69}$$

$$w_{01}, w_{02}, w_{03}, w_{11}, w_{12} \geq 0 \tag{6.70}$$

Similarly Optimal value of above dual problem is $f_2^{H.M}$=0.0275445 for $w_{01} = 0.3570402, w_{02} = 0.3254310, w_{03} = 0.3175288, w_{11} = 1.007902, w_{12} = 0.0158045$ and corresponding primal is obtained for $x_1 = 1.936618, x_2 = 4.880567$ and $x_3 = 0.4482507$.

From above discussion we observe that the value of the objective function in interval of parameters for A.M, G.M and H.M preserve the same relationship. That is the values so obtained for the objective functions are in the form A.M>G.M>H.M in between the values of the function in lower bound and upper bound of the interval.

Case3-Solution of the problem using Weighted mean method:

Using Weighted mean method we can write the given Multi-objective optimization problem as follows:

$$\begin{aligned}\min Z &= w_1((2,6,2\sqrt{5},\frac{3}{10},10)x_1^{-1}x_2^{-1} + (20,25,10\sqrt{6},\frac{1}{24},30)x_2x_3 + \\ &\quad (12,15,6\sqrt{6},\frac{5}{72})x_3^{-1}) + w_2((20,25,10\sqrt{6},\frac{1}{24},30)x_1^{-1}x_2^{-1}x_3^{-1} + \\ &\quad (10,15,10\sqrt{2},\frac{3}{40},20)x_1^{-2}x_3 + (4,5,2\sqrt{6},\frac{5}{24},6)x_2^{-2})\end{aligned} \tag{6.71}$$

subject to

$$(8,10,4\sqrt{6},\frac{5}{48},12)x_1x_2 + (2,3,2\sqrt{2},\frac{3}{8},4)x_2^{-1}x_3^2 \leq 1 \tag{6.72}$$

where

$$w_1 + w_2 = 1 \tag{6.73}$$

$$w_1, w_2, x_1, x_2, x_3 \geq 0 \tag{6.74}$$

As per the condition given in (3.1) to (3.5), we can write its dual program as follows:

$$\begin{aligned}Z^L = \max_w : &\left(\frac{2w_1}{w_{01}}\right)^{w_{01}} \left(\frac{20w_1}{w_{02}}\right)^{w_{02}} \left(\frac{12w_1}{w_{03}}\right)^{w_{03}} \left(\frac{20w_2}{w_{04}}\right)^{w_{04}} \left(\frac{10w_2}{w_{05}}\right)^{w_{05}} \left(\frac{4w_2}{w_{06}}\right)^{w_{06}} \left(\frac{8}{w_{11}}\right)^{w_{11}} \\ &\left(\frac{2}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})}\end{aligned} \tag{6.75}$$

subject to

$$w_1 + w_2 = 1 \tag{6.76}$$

$$w_{01} + w_{02} + w_{03} + w_{04} + w_{05} + w_{06} = 1 \tag{6.77}$$

$$-w_{01} - w_{04} - 2w_{05} + w_{11} = 0 \tag{6.78}$$

$$-w_{01} + w_{02} - w_{04} - 2w_{06} + w_{11} - w_{12} = 0 \tag{6.79}$$

$$w_{02} - w_{03} - w_{04} + w_{05} + 2w_{12} = 0 \quad (6.80)$$

$$w_1, w_2, w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{11}, w_{12} \geq 0 \quad (6.81)$$

Considering different values of w_1 and w_2 and the dual variables,the maximum value of dual objective function is given in Table-1.

Table-1(a)

(Dual solution)

w_1	w_2	w_{01}	w_{02}	w_{03}	w_{04}
0.1	0.9	0.002535	0.000500	0.005291	0.812878
0.2	0.8	0.005650	0.001117	0.111764	0.803755
0.3	0.7	0.009568	0.001896	0.019861	0.792322
0.4	0.6	0.014648	0.002911	0.013028	0.777573
0.5	0.5	0.021495	0.004288	0.044198	0.757822

Table-1(b)

(Dual solution)

w_{05}	w_{06}	w_{11}	w_{12}	Z
0.170204	0.00858	1.155824	0.323732	807.7818
0.169224	0.008488	1.147854	0.322589	725.5003
0.167988	0.008199	1.137868	0.321149	643.2123
0.166384	0.008199	1.124990	0.316758	560.9146
0.164215	0.007980	1.107748	0.316758	478.6018

Considering the primal-dual relationship, the optimal solution of primal is given in following table.

Table-1(c)

(Primal solution)

w_1	w_2	x_1	x_2	x_3	Z
0.1	0.9	0.135559	0.720344	0.280726	807.7818
0.2	0.8	0.135363	0.720855	0.2811965	725.5003
0.3	0.7	0.135113	0.721512	0.281793	643.2123
0.4	0.6	0.134783	0.722390	0.282574	560.9146
0.5	0.5	0.134331	0.723618	0.283642	478.6018

$$Z^U = \max_w : \left(\frac{10w_1}{w_{01}}\right)^{w_{01}} \left(\frac{30w_1}{w_{02}}\right)^{w_{02}} \left(\frac{18w_1}{w_{03}}\right)^{w_{03}} \left(\frac{30w_2}{w_{04}}\right)^{w_{04}} \left(\frac{20w_2}{w_{05}}\right)^{w_{05}}$$
$$\left(\frac{6w_2}{w_{06}}\right)^{w_{06}} \left(\frac{12}{w_{11}}\right)^{w_{11}} \left(\frac{4}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \quad (6.82)$$

subject to

$$w_1 + w_2 = 1 \quad (6.83)$$

$$w_{01} + w_{02} + w_{03} + w_{04} + w_{05} + w_{06} = 1 \quad (6.84)$$

$$-w_{01} - w_{04} - 2w_{05} + w_{11} = 0 \quad (6.85)$$

$$-w_{01} + w_{02} - w_{04} - 2w_{06} + w_{11} - w_{12} = 0 \quad (6.86)$$

$$w_{02} - w_{03} - w_{04} + w_{05} + 2w_{12} = 0 \quad (6.87)$$

$$w_1, w_2, w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{11}, w_{12} \geq 0 \quad (6.88)$$

Considering different values of w_1 and w_2 and the dual variables, the maximum value of dual objective function is given in Table-2.

Table-2(a)

(Dual solution)

w_1	w_2	w_{01}	w_{02}	w_{03}	w_{04}
0.1	0.9	0.006004	0.000165	0.003546	0.818514
0.2	0.8	0.013341	0.000368	0.007883	0.808099
0.3	0.7	0.022509	0.000624	0.013305	0.795080
0.4	0.6	0.034289	0.000956	0.020278	0.778342
0.5	0.5	0.049985	0.001403	0.029581	0.756027

Table-2(b)

(Dual solution)

w_{05}	w_{06}	w_{11}	w_{12}	Z
0.167626	0.004141	1.159772	0.327314	2562.022
0.166233	0.004073	1.153909	0.324689	2305.010
0.164492	0.003987	1.146575	0.321634	2047.985
0.162253	0.003878	1.137140	0.317705	1790.942
0.159266	0.003734	1.124547	0.312469	1533.868

Considering the primal-dual relationship, the optimal solution of primal is given in following table.

Table-2(c)

(Primal solution)

w_1	w_2	x_1	x_2	x_3	Z
0.1	0.9	0.091116	0.713363	0.198082	2562.022
0.2	0.8	0.090955	0.715002	0.198122	2305.010
0.3	0.7	0.090750	0.717103	0.198246	2047.985
0.4	0.6	0.090480	0.719880	0.198246	1790.942
0.5	0.5	0.090107	0.723728	0.198349	1533.868

$$Z^{A.M} = \max_{w} : \left(\frac{6w_1}{w_{01}}\right)^{w_{01}} \left(\frac{25w_1}{w_{02}}\right)^{w_{02}} \left(\frac{15w_1}{w_{03}}\right)^{w_{03}} \left(\frac{25w_2}{w_{04}}\right)^{w_{04}} \left(\frac{15w_2}{w_{05}}\right)^{w_{05}} \left(\frac{5w_2}{w_{06}}\right)^{w_{06}} \left(\frac{10}{w_{11}}\right)^{w_{11}} \left(\frac{3}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.89}$$

subject to

$$w_1 + w_2 = 1 \tag{6.90}$$

$$w_{01} + w_{02} + w_{03} + w_{04} + w_{05} + w_{06} = 1 \tag{6.91}$$

$$-w_{01} - w_{04} - 2w_{05} + w_{11} = 0 \tag{6.92}$$

$$-w_{01} + w_{02} - w_{04} - 2w_{06} + w_{11} - w_{12} = 0 \tag{6.93}$$

$$w_{02} - w_{03} - w_{04} + w_{05} + 2w_{12} = 0 \tag{6.94}$$

$$w_1, w_2, w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{11}, w_{12} \geq 0 \tag{6.95}$$

Considering different values of w_1 and w_2 and the dual variables, the maximum value of dual objective function is given in Table-3.

Table-3(a)

(Dual solution)

w_1	w_2	w_{01}	w_{02}	w_{03}	w_{04}
0.1	0.9	0.004982	0.000265	0.004247	0.816308
0.2	0.8	0.011078	0.000591	0.009437	0.806061
0.3	0.7	0.018706	0.001002	0.015922	0.793245
0.4	0.6	0.028529	0.001536	0.024258	0.776756
0.5	0.5	0.041651	0.002257	0.035366	0.754752

Table-3(b)

(Dual solution)

w_{05}	w_{06}	w_{11}	w_{12}	Z
0.168504	0.005691	1.158301	0.325892	1542.956
0.167227	0.005603	1.151595	0.323839	1387.792
0.165627	0.005494	1.143207	0.321268	1232.623
0.163564	0.005354	1.132415	0.317957	1077.443
0.160804	0.005168	1.118012	0.313529	922.248

Considering the primal-dual relationship, the optimal solution of primal is given in following table.

Table-3(c)

(Primal solution)

w_1	w_2	x_1	x_2	x_3	Z
0.1	0.9	0.109019	0.715854	0.228899	1542.956
0.2	0.8	0.108831	0.717177	0.229064	1387.792
0.3	0.7	0.108590	0.718870	0.229275	1232.623
0.4	0.6	0.108273	0.721114	0.229554	1077.443
0.5	0.5	0.107836	0.724231	0.229940	922.248

$$\begin{aligned} Z^{G.M} \quad = \quad & \max_{w} : \left(\frac{2\sqrt{5}w_1}{w_{01}}\right)^{w_{01}} \left(\frac{10\sqrt{6}w_1}{w_{02}}\right)^{w_{02}} \left(\frac{6\sqrt{6}w_1}{w_{03}}\right)^{w_{03}} \left(\frac{10\sqrt{6}w_2}{w_{04}}\right)^{w_{04}} \left(\frac{10\sqrt{2}w_2}{w_{05}}\right)^{w_{05}} \\ & \left(\frac{2\sqrt{6}w_2}{w_{06}}\right)^{w_{06}} \left(\frac{4\sqrt{6}}{w_{11}}\right)^{w_{11}} \left(\frac{2\sqrt{2}}{w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \end{aligned} \tag{6.96}$$

subject to

$$w_1 + w_2 = 1 \tag{6.97}$$

$$w_{01} + w_{02} + w_{03} + w_{04} + w_{05} + w_{06} = 1 \quad (6.98)$$

$$-w_{01} - w_{04} - 2w_{05} + w_{11} = 0 \quad (6.99)$$

$$-w_{01} + w_{02} - w_{04} - 2w_{06} + w_{11} - w_{12} = 0 \quad (6.100)$$

$$w_{02} - w_{03} - w_{04} + w_{05} + 2w_{12} = 0 \quad (6.101)$$

$$w_1, w_2, w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{11}, w_{12} \geq 0 \quad (6.102)$$

Considering different values of w_1 and w_2 and the dual variables,the maximum value of dual objective function is given in Table-4.

Table-4(a)

(Dual solution)

w_1	w_2	w_{01}	w_{02}	w_{03}	w_{04}
0.1	0.9	0.003906	0.000287	0.004336	0.816672
0.2	0.8	0.008698	0.000642	0.009644	0.807373
0.3	0.7	0.014715	0.001089	0.016290	0.795714
0.4	0.6	0.022495	0.001672	0.024855	0.780666
0.5	0.5	0.032947	0.002462	0.036307	0.760500

Table-4(b)

(Dual solution)

w_{05}	w_{06}	w_{11}	w_{12}	Z
0.168814	0.005981	1.158209	0.325953	1437.281
0.167739	0.005901	1.151552	0.324318	1291.204
0.166388	0.005801	1.143207	0.322263	1145.121
0.164638	0.005671	1.132439	0.319605	999.029
0.162283	0.005498	1.118014	0.316031	852.922

Considering the primal-dual relationship, the optimal solution of primal is given in following table.

Table-4(c)

(Primal solution)

w_1	w_2	x_1	x_2	x_3	Z
0.1	0.9	0.111219	0.716123	0.235807	1437.281
0.2	0.8	0.111041	0.717157	0.236045	1291.204
0.3	0.7	0.110814	0.718482	0.236348	1145.121
0.4	0.6	0.110514	0.720242	0.236746	999.029
0.5	0.5	0.110102	0.722692	0.237294	852.922

$$Z^{H.M} = \max_{w} : \left(\frac{3w_1}{10w_{01}}\right)^{w_{01}} \left(\frac{w_1}{24w_{02}}\right)^{w_{02}} \left(\frac{5w_1}{72w_{03}}\right)^{w_{03}} \left(\frac{w_2}{24w_{04}}\right)^{w_{04}} \left(\frac{3w_2}{40w_{05}}\right)^{w_{05}} \left(\frac{5w_2}{24w_{06}}\right)^{w_{06}} \left(\frac{5}{48w_{11}}\right)^{w_{11}} \left(\frac{3}{8w_{12}}\right)^{w_{12}} (w_{11}+w_{12})^{(w_{11}+w_{12})} \tag{6.103}$$

subject to

$$w_1 + w_2 = 1 \tag{6.104}$$

$$w_{01} + w_{02} + w_{03} + w_{04} + w_{05} + w_{06} = 1 \tag{6.105}$$

$$-w_{01} - w_{04} - 2w_{05} + w_{11} = 0 \tag{6.106}$$

$$-w_{01} + w_{02} - w_{04} - 2w_{06} + w_{11} - w_{12} = 0 \tag{6.107}$$

$$w_{02} - w_{03} - w_{04} + w_{05} + 2w_{12} = 0 \tag{6.108}$$

$$w_1, w_2, w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{11}, w_{12} \geq 0 \tag{6.109}$$

Considering different values of w_1 and w_2 and the dual variables, the maximum value of dual objective function is given in Table-5.

Table-5(a)

(Dual solution)

w_1	w_2	w_{01}	w_{02}	w_{03}	w_{04}
0.1	0.9	0.0635591	0.192098	0.233077	0.135488
0.2	0.8	0.091500	0.261443	0.293592	0.077081
0.3	0.7	0.110257	0.291497	0.320995	0.05003
0.4	0.6	0.125874	0.304694	0.337091	0.034475
0.5	0.5	0.140799	0.308838	0.348797	0.024368

Table-5(b)

(Dual solution)

w_{05}	w_{06}	w_{11}	w_{12}	Z
0.143916	0.231827	0.486912	0.016275	0.050784
0.076874	0.199507	0.322331	0.016177	0.071733
0.045044	0.182170	0.250381	0.017244	0.090884
0.027659	0.170204	0.215669	0.019606	0.108333
0.017270	0.159926	0.199708	0.023527	0.124047

Considering the primal-dual relation, the optimal solution of primal is given in following table.

Table-5(c)

(Primal solution)

w_1	w_2	x_1	x_2	x_3	Z
0.1	0.9	2.32775	3.99075	0.586690	0.050784
0.2	0.8	2.678681	3.412577	0.659477	0.071733
0.3	0.7	3.02622	2.96785	0.714118	0.090884
0.4	0.6	3.379820	2.603689	0.760655	0.108333
0.5	0.5	3.747924	2.291461	0.802505	0.124047

From the above solution it is observed that the compromise solution of the multi-objective functions using Weighted mean method are in form of A.M>G.M>H.M.

7 Conclusion:

In many real world Geometric programming problem, the parameters may not be known precisely which leads to the formulation of mathematical programming problem with multiple parameters. This paper considers the interval of cost as well as constraint co-efficient such as A.M, G.M and H.M of the end points of certain interval as multiple parameters for finding the optimal solution of objective functions. The idea is to find the upper bound and lower bound of the objective function within the interval of parameters as well as to compute the optimal values of the objectives at the indicated points such as A.M, G.M and H.M of the interval. This paper employs GP technique to derive the objective value.The solution of dual variable not only gives the primal optimal solution but also provides a relationship between maximum and minimum value of the objective function. The same results have been shown converting multi-objective Geometric programming problem to a single objective problem using Weighted mean method for verification. Finally we acquired the derived result in range and they are in order A.M>G.M>H.M. With this ability of calculating the bounds of objective value developed in this paper, it may help the researchers for wider application in the field of engineering problems.

References

[1] C.S. Beightler and D.T. Phillips, Applied Geometric programming, John Wily and Sons, New York, 1976.

[2] C.S. Beightler, D.T. Phillips and D.J.Wilde, Foundation of Optimization, Prentice-Hall,New Jersy,1979.

[3] C.Chu and D.F.Wong, VLSI Circuit performance optimization by Geometric programming,Annals of Operation Research, 105, p37-60, 2001

[4] M.P.Biswal, Fuzzy Programming technique to solve Multi-objective Geometric programming problems, Fuzzy Sets and Systems, 51(1991), p.67-71

[5] M.D.Hershenson, S.P.Boyed and T.H.Lee, Optimal design of a CMOS Op-amp via Geometric programming.IEEE Transaction on Computer Aided Design of Integrated circuits and Systems, 20, p1-21, 2001.

[6] P.P.Mandal and V.Visvanathan,FCMOS Op-amp sizing using a Geometric programming function, IEEE Transaction on Computer Aided Design of Integrated Circuits and Systems, 20, p2238, 2001.

[7] R.S.Dembo, The sensitivity of optimal engineering designs using Geometric programming, Engineering Optimization, 5, p27-40, 1980.

[8] J.J.Dinkel and M.J.Tretter, An interval arithmatic approach to sensitivity analysis in Geometric programming, Operation Research, 35, p866-869, 1987.

[9] B.Y.Cao, The further study of posynomial GP with Fuzzy co-efficient, Mathematics Applicata, vol-5, no.4, p119-120(1992).

[10] B.Y.Cao, Extended Fuzzy GP, Journal of Fuzzy Mathematics, vol-1, no.2, p285-293(1993).

[11] R.J.Duffin, E.L.Peterson and C.M.Zener, Geometric Programming Theory and Application, Jhon Wiley and sons, New York, 1967.

[12] J.G.Ecker, Geometric programming methods, Computation and Application, SIAM Review 22, p338-362, 1980.

[13] A.K.Bit, Multi-objective Geometric programming problem: Fuzzy programming with hyperbolic membership function, Journal of Fuzzy mathemetics, vol-6, no.1, p 27-32(1998).

[14] B.Y.Cao, Fuzzy Geometric Programming(1), Fuzzy Sets and Systems, vol-53, p135-153(1993).

[15] B.Y.Cao, Solution and theory of question for a kind of Fuzzy Geometric Program,proc, 2nd IFSA Congress, Tokyo, 1(1987), 205-208..

[16] A.J.Morris, A Transformation for Geometric programming Applied to the Minimum Weight Design of Statically Determinate Structure, International Journal of Mechanical Science, 17, p395-396, 1975.

[17] S.T.Liu, Posynomial Geometric programming with interval exponents and co-efficients, Europian Journal of Opertation Research, 168(2006), no.2, 345-353.

[18] K.Kortanek, X.Xu and Y.Ye, An infeasible interior point algorithm for solving Primal and Dual programs,Mathematical Programming, 76, p155-181, 1997.

[19] E.L.Peterson, The fundamental relations between Geometric programming duality, Parametric programming duality and ordinary Lagrangian duality, Annals of Operations Research, 105, P109-153(2001).

[20] S.S.Rao, Application of Complimentary Geometric Programming to Mechanical Design problems, International Journal of Mechanical Engineering Education, 13, 1, p19-29, 1985.

[21] J.Rajgopal, D.L.Bricker, Solving posynomial Geometric programming problems via generalized linear programming, Computational Optimization and Applications 21, 95-109(2002).

[22] R.K.Verma, Geometric programming with Several objective functions, Fuzzy Sets and Systems, vol.35, p115-120, 1990.

[23] C.H.Scott and T.R.Jefferson, Allocation of resources in project management, International Journal of systems Science, 26, p.413-420(1995).