Cost Benefit Analysis of a 2-Out-of-3 Induced Draft Fans System with Priority for Operation to Cold Standby over Working at Reduced Capacity

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Abstract:

This paper analyses the reliability of a system involving Induced Draft fans installed in boilers of thermal power plants. Induced Draft (ID) fans are used to force the flue gases out of the furnace into the gas stack in order to maintain negative pressure. The boilers used in thermal power plant under study have three identical ID fans, two of them are operative while the third one acts as a cold standby. In case of failure of one of the operative ID fans, after some activation time, the ID fan used as cold standby starts functioning instantaneously. The system may work with low power production when one ID fan is working, if certain parameters are controlled in specific time. Preference is given to make the cold standby operative over working at reduced capacity. The graphical behavior of these measures has also been discussed.

Keywords: 2-out-of-3 system, Induced Draft fan, semi-Markov processes, Cost-Benefit Analysis.

2000 Mathematics Subject Classification: 90B25 and 60K10.

1. Introduction

In developing countries, power sector is the fastest growing sector which affects the economic growth of the country. So, there is a need of improving the performance of this sector by developing new models and techniques. Reliability and availability play a major role in power plants. Reliability of power plant depends upon many parameters such as fuel type, temperature inside the boiler, design of subsystems, maintenance etc. To get high reliability and availability, one should keep these factors in mind.

Many real engineering systems with different parameters and operational conditions and situations have been analyzed by number of researchers. [Singh, Minocha and Taneja, 2007] studied 2-out-of-3 unit system for an ash handling plant where situation of system failure did not arise. [Parashar and Taneja, 2007] discussed the reliability analysis of PLC hot standby system based on Master Slave concept and two types of repair facilities. [Goyal, Taneja and Singh, 2010] did comparison of the two models for sulphated juice pump systems working seasonally and having different configurations for cost effectiveness. [Sharma and Taneja, 2011] analyzed two

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standby oil delivering systems with a provision of switching over to another system at need to increase the availability. [Mathew, Rizwan and Majumdar, 2010] did comparative analysis of Continuous Casting Plant. [Alawi and Mathew, 2012] did reliability modelling of GIV Gulfstream Aircraft. But system consisting of ID fans is still not much explored.

[Bhatia, Naithani, Parashar and Taneja, 2012] did reliability modelling of a 3-unit induced draft fan cold standby system working at full/reduced capacity where priority was given to controlling the parameters to make the system compatible to work at reduced capacity and hence they made the standby unit activated for operation only after controlling the parameters to work at reduced capacity. This may lead to a decrease in the availability of system at full capacity. However, system may work with higher availability at full capacity and hence may be more beneficial if priority is given to activate the standby unit first rather than to control the parameters to make the system compatible to work at reduced capacity.

Thus, in the present paper, cost benefit analysis of a 2-out-of-3 induced draft fans system is done where priority is given to operation of cold standby over controlling the parameters. There are three identical induced draft (ID) fans out of which two are operative whereas one is cold standby. If one of the operative ID fan fails, cold standby ID fan starts working. Some activation time is needed to make cold standby operative. If certain parameters are controlled in a stipulated time after failure of one ID fan then system remains operative but at reduced capacity whereas if parameters are not controlled then system goes to down state.

The present model is analyzed using semi-Markov process and regenerative point technique, and the following reliability indices pertaining to the power plant efficiency are obtained:

- a) Mean Time to System Failure (MTSF)
- b) Availability at Full Capacity
- c) Availability at Reduced Capacity
- d) Busy Period of Repairman
- e) Down Time of the System.

The profit incurred to the system is calculated and graphical study is also done.

2. Notations

0	:	operative state
Cs	:	cold standby
F	:	failed state
Fr	:	failed unit under repair
F _R	:	unit is under repair from previous state
F _{ra}	:	repair of unit is kept in abeyance
F _w	:	failed unit is waiting for repair
S_0	:	switching over is taking place
D	:	down state
λ	:	constant rate of failure of operative unit
η	:	constant rate of going from upstate to down state when only one unit is operative
γ_1	:	constant rate of allowed time to get the parameters changed so that system work at reduced capacity
γ_2	:	constant rate of allowed time to change mode of working from reduced to full capacity
β_1	:	activation rate
p	:	probability that parameters get changed to make the system to work at reduced capacity
q	:	probability that parameters are not changed to make the system to work at reduced capacity
g(t), G(t)	:	p.d.f. and c.d.f. of repair time of the unit
©, \$:	Laplace convolution, Laplace Stieltjes convolution
C_0	:	revenue per unit up time when system works at full capacity
C_1	:	revenue per unit up time when system works at reduced capacity
C ₂	:	cost per unit time for which the repairman is busy in repairing the unit
C ₃	:	cost per unit up time when system is down
C_4	:	loss per unit time due to low power generation

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C		
C_5	:	payment per unit time made to repairman
AF_0	:	probability that the system will be able to operate at full capacity given it that entered
		state O at t=0
AR_0	:	this is the probability that the system will be able to operate at reduced capacity given
		that it entered state O at $t=0$
\mathbf{B}_0	:	busy period of the repairman for repair
DT_0	:	expected downtime, the total fraction of time when system is down

3. System Description and Assumptions

A probabilistic model has been developed which consists of four possible states: full working state, reduced capacity state, down state and failed state. The state transition diagram is shown in Fig. 1 covering all the possibilities of the model under consideration. The state 0,1,6,8 are up states. States 2, 3 and 5 are down states whereas states 4 and 7 are the states where the system works at reduced capacity. States 9 and 10 are states with complete failure.

The assumptions used in developing the probabilistic model are:

- a) Initially the system is operative at full capacity.
- b) Initially two ID fans are operative and one is used as cold standby.
- c) All the three ID fans are identical and constitute a parallel system.
- d) Time period for switching over the cold standby unit to operative mode is called activation time.
- e) With one unit operative and standby under activation, the system goes from up to down state prior to the completion of activation time for the standby unit to become operative, e.g., this situation can be seen when system goes from State 1 to State 2 in the transition diagram shown in Fig. 1.
- f) The cold standby unit cannot fail while switching over is taking place.
- g) The repairman is readily available for repair.
- h) Repair is kept in abeyance when repairman is busy in controlling the parameters.
- i) Failure times are assumed to follow an exponential distribution.
- j) The repaired unit works as good as new one.



Fig. 1. State transition diagram

For the present study, rates, costs, probabilities etc are estimated on the basis of the data/information provided by the thermal power industries:

- Estimated value of failure rate of operative unit (λ) = 0.0001 per hour •
- Estimated value of repair rate $(\alpha) = 0.1$ per hour •
- Estimated value of rate of allowed time to get parameters changed so that system works at full capacity $(\gamma_1) =$ 0.6 per hour
- Estimated value of constant rate of allowed time to change mode of working from reduced capacity to full capacity (γ_2) = 0.6 per hour
- Estimated value of rate of going from up state to down state when only one unit is in operation (η) = 0.5 per hour
- Estimated value of activation rate $(\beta_1) = 0.3$ per hour
- Probability that parameters get changed to make the system work at reduced capacity (p) = 0.8
- Probability that parameters do not get changed to make the system work at reduced capacity (q) = 0.2٠

4. Transition Probabilities and Mean Sojourn Time

The various transition probabilities are given below:

$$\begin{array}{ll} q_{01}(t) &= 2\lambda e^{-2\lambda t} & q_{68}(t) &= 2\lambda e^{-2\lambda t} \overline{G}(t) \\ q_{12}(t) &= \eta e^{-(\eta + \lambda)t} & q_{74}(t) &= e^{-\lambda t} g(t) \\ q_{13}(t) &= \lambda e^{-(\eta + \lambda)t} & q_{79}(t) &= \lambda e^{-\lambda t} \overline{G}(t) \\ q_{24}(t) &= \beta_1 e^{-\beta_1 t} & q_{7,7}^9(t) &= (\lambda e^{-\lambda t} \mathbb{O}I)g(t) \\ q_{35}(t) &= \beta_1 e^{-\beta_1 t} & q_{85}(t) &= q\gamma_1 e^{-(\lambda + \gamma_1)t} \\ q_{46}(t) &= \gamma_2 e^{-\gamma_2 t} & q_{87}(t) &= p\gamma_1 e^{-(\lambda + \gamma_1)t} \\ q_{57}(t) &= \gamma_1 e^{-\gamma_1 t} & q_{8,10}(t) &= \lambda e^{-(\lambda + \gamma_1)t} \\ q_{60}(t) &= e^{-2\lambda t} g(t) & q_{10,7}(t) &= g(t) \end{array}$$

The non-zero element p_{ij} can be obtained as

$$p_{ij} = \lim_{s \to 0} [q_{ij}(s)]$$

$$p_{01} = p_{24} = p_{35} = p_{46} = p_{57} = p_{10,7} = 1$$

$$p_{12} = \frac{\eta}{\lambda + n} \qquad \qquad p_{7,7}^9 = p_{79} = 1 - g^*(\lambda)$$

$$p_{13} = \frac{\lambda}{\lambda + \eta} \qquad p_{85} = \frac{q\gamma_1}{\lambda + \gamma_1}$$

$$p_{60} = g^*(2\lambda) \qquad p_{87} = \frac{p\gamma_1}{\lambda + \gamma_1}$$

$$p_{68} = 1 - g^*(2\lambda) \qquad p_{8,10} = \frac{\lambda}{\lambda + \gamma_1}$$

$$p_{74} = g^*(\lambda)$$

It can be verified that

 $p_{12} = \frac{\eta}{\lambda + \eta}$

 $p_{12} + p_{13} = 1, p_{60} + p_{68} = 1, p_{74} + p_{77}^9 = 1, p_{74} + p_{79} = 1, p_{85} + p_{87} + p_{810} = 1$ The mean sojourn time (μ_i) in the regenerative state i is defined as the time of stay in that state before transition to any other state.

$$\mu_0 = \frac{1}{2\lambda} \qquad \qquad \mu_5 = \frac{1}{\gamma_1}$$

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$$\mu_{1} = \frac{1}{\lambda + \eta} \qquad \qquad \mu_{6} = \frac{1 - g^{*}(2\lambda)}{2\lambda}$$

$$\mu_{2} = \frac{1}{\beta_{1}} \qquad \qquad \mu_{7} = \frac{1 - g^{*}(\lambda)}{\lambda}$$

$$\mu_{3} = \frac{1}{\beta_{1}} \qquad \qquad \mu_{8} = \frac{1}{\lambda + \gamma_{1}}$$

$$\mu_{4} = \frac{1}{\gamma_{2}} \qquad \qquad \mu_{9} = \mu_{10} = \int_{0}^{\infty} tg(t)dt$$

The unconditional mean time taken by the system to transit to any regenerative state j when time is counted from the epoch of entrance into state i is mathematically stated as

$$m_{ij} = \int_{0}^{\infty} t dQ_{ij}(t) = -q_{ij}^{*'}(0)$$

It can be verified that

$$\begin{split} m_{01} &= \mu_0 & m_{60} + m_{68} &= \mu_6 \\ m_{12} + m_{13} &= \mu_1 & m_{76} + m_{79} &= \mu_7 \\ m_{24} &= \mu_2 & m_{85} + m_{87} + m_{8,10} &= \mu_8 \\ m_{35} &= \mu_3 & m_{74} + m_{7,7}^9 &= \mu_9 \\ m_{46} &= \mu_4 & m_{10,7} &= \mu_{10} \\ m_{57} &= \mu_5 & \end{split}$$

5. Mean Time to System Failure (MTSF)

Taking failed state of the system as absorbing state, Mean Time to System Failure (MTSF) of the system can be determined. Let $\phi_i(t)$ be the cumulative distribution function of first passage time from ith state to a failed state. Then,

$$\begin{split} \phi_0(t) &= Q_{01}(t) \,\, \textcircled{S} \,\, \phi_1(t) \\ \phi_1(t) &= Q_{12}(t) \,\, \textcircled{S} \,\, \phi_2(t) + Q_{13}(t) \,\, \textcircled{S} \,\, \phi_3(t) \\ \phi_2(t) &= Q_{24}(t) \,\, \textcircled{S} \,\, \phi_2(t) + Q_{13}(t) \,\, \textcircled{S} \,\, \phi_3(t) \\ \phi_3(t) &= Q_{24}(t) \,\, \textcircled{S} \,\, \phi_4(t) \\ \phi_3(t) &= Q_{35}(t) \,\, \textcircled{S} \,\, \phi_5(t) \\ \phi_4(t) &= Q_{46}(t) \,\, \textcircled{S} \,\, \phi_5(t) \\ \phi_5(t) &= Q_{57}(t) \,\, \textcircled{S} \,\, \phi_7(t) \\ \phi_6(t) &= Q_{60}(t) \,\, \textcircled{S} \,\, \phi_0(t) + Q_{68}(t) \,\, \textcircled{S} \,\, \phi_8(t) \\ \phi_7(t) &= Q_{79}(t) + Q_{74}(t) \,\, \textcircled{S} \,\, \phi_4(t) \\ \phi_8(t) &= Q_{8,10}(t) + Q_{85}(t) \,\, \textcircled{S} \,\, \phi_5(t) + Q_{87}(t) \,\, \textcircled{S} \,\, \phi_7(t) \end{split}$$

Taking Laplace Stieltjes Transform on both sides and solving we get,

$$\phi_{0}^{**}(s) = \frac{N_{0}(s)}{D_{0}(s)} \text{ where } \phi_{0}^{**}(s) \text{ is Laplace Stieltjes Transform of } \phi_{0}(s)$$

$$MTSF = \lim_{s \to 0} \frac{1 - \phi_{0}^{**}(s)}{s} = \frac{D_{0}^{'}(0) - N_{0}^{'}(0)}{D_{0}(0)} = \frac{N_{1}}{D_{1}}$$

where,

$$\begin{split} N_{1} &= (\mu_{0} + \mu_{1})(1 - p_{68}p_{74} + p_{8,10}p_{68}p_{74}) + \mu_{2}(p_{12}p_{79} + p_{12}p_{60}p_{74} + p_{12}p_{8,10}p_{68}p_{74}) + \mu_{3}(p_{13} - p_{74}p_{13}p_{68} + p_{74}p_{13}p_{68}p_{8,10}) + \mu_{4}(p_{12} + p_{74}p_{13}) + \mu_{5}(p_{85}p_{68} + p_{13}p_{60} + p_{13}p_{68}p_{8,10} - p_{79}p_{68}p_{8,10}) + \mu_{6}(1 - p_{79}p_{13}) + \mu_{7}(p_{13} + p_{12}p_{68} - p_{12}p_{68}p_{8,10}) + \mu_{8}(p_{74}p_{68} + p_{12}p_{68}p_{79}) \\ D_{1} &= p_{79} - p_{60}p_{12}p_{79} + p_{74}p_{68}p_{8,10} \end{split}$$

6. Availability Analysis at Full Capacity (AF₀)

Let $AF_i(t)$ be the probability that the system is working at full capacity at instant t, given that the system entered regenerative state i at t=0. Then,

$$\begin{split} AF_{0}(t) &= M_{0}(t) + q_{01}(t) \textcircled{\circ} AF_{1}(t) \\ AF_{1}(t) &= M_{1}(t) + q_{12}(t) \textcircled{\circ} AF_{2}(t) + q_{13}(t) \textcircled{\circ} AF_{3}(t) \\ AF_{2}(t) &= q_{24}(t) \textcircled{\circ} AF_{4}(t) \\ AF_{2}(t) &= q_{24}(t) \textcircled{\circ} AF_{4}(t) \\ AF_{3}(t) &= q_{35}(t) \textcircled{\circ} AF_{5}(t) \\ AF_{3}(t) &= q_{35}(t) \textcircled{\circ} AF_{5}(t) \\ AF_{4}(t) &= q_{46}(t) \textcircled{\circ} AF_{6}(t) \\ AF_{5}(t) &= q_{57}(t) \textcircled{\circ} AF_{6}(t) \\ AF_{5}(t) &= M_{6}(t) + q_{68}(t) \textcircled{\circ} AF_{8}(t) + q_{60}(t) \textcircled{\circ} AF_{0}(t) \\ AF_{6}(t) &= M_{6}(t) + q_{68}(t) \textcircled{\circ} AF_{8}(t) + q_{60}(t) \textcircled{\circ} AF_{0}(t) \\ AF_{7}(t) &= q_{74}(t) \textcircled{\circ} AF_{4}(t) + q_{7,7}^{9}(t) \textcircled{\circ} AF_{7}(t) \\ AF_{8}(t) &= M_{8}(t) + q_{85}(t) \textcircled{\circ} AF_{5}(t) + q_{87}(t) \textcircled{\circ} AF_{7}(t) + q_{8,10}(t) \textcircled{\circ} AF_{10}(t) \\ AF_{10}(t) &= q_{10,7}(t) \textcircled{\circ} AF_{7}(t) \\ where M_{0}(t) &= e^{-2\lambda t} , M_{1}(t) &= e^{-(\lambda + \eta)t} , M_{6}(t) &= e^{-2\lambda t} \overline{G}(t) . M_{\circ}(t) &= e^{-(\lambda + \gamma_{1})t} \\ \end{split}$$

Taking Laplace Transforms on both sides and solving we get:

 $AF_0^*(s) = \frac{N_2(s)}{D_2(s)} \text{ where } AF_0^*(s) \text{ is Laplace Transform of } AF_0(s)$ $AF_0 = \lim_{s \to 0} sAF_0^*(s) = \frac{N_2(0)}{D_2(0)} = \frac{N_2}{D_2}$ where, $N_2 = (\mu_0 + \mu_1)p_{74}p_{60} + (\mu_6 + \mu_8 p_{68})p_{74}$

 $D_2 = \mu_0 p_{60} p_{74} + \mu_1 p_{60} p_{74} + \mu_2 p_{60} p_{12} p_{74} + \mu_3 p_{74} p_{60} p_{13} + \mu_4 p_{74} + \mu_5 (p_{74} p_{60} p_{13} + p_{74} p_{68} p_{85}) + \mu_6 p_{74} + \mu_9 (1 - p_{60} p_{12}) + \mu_8 p_{74} p_{68} + \mu_{10} p_{74} p_{68} p_{8,10}$

7. Availability Analysis at Reduced Capacity (AR₀)

Let $AR_i(t)$ be the probability that the system is working at reduced capacity at instant t, given that the system entered regenerative state i at t=0. Then,

$$\begin{split} AR_{0}(t) &= q_{01}(t) © AR_{1}(t) \\ AR_{1}(t) &= q_{12}(t) © AR_{2}(t) + q_{13}(t) © AR_{3}(t) \\ AR_{2}(t) &= q_{24}(t) © AR_{4}(t) \\ AR_{2}(t) &= q_{24}(t) © AR_{4}(t) \\ AR_{3}(t) &= q_{35}(t) © AR_{5}(t) \\ AR_{4}(t) &= M_{4}(t) + q_{46}(t) © AR_{6}(t) \\ AR_{5}(t) &= q_{57}(t) © AR_{7}(t) \\ AR_{6}(t) &= q_{68}(t) © AR_{8}(t) + q_{60}(t) © AR_{0}(t) \\ AR_{7}(t) &= M_{7}(t) + q_{74}(t) © AR_{4}(t) + q_{7,7}^{9}(t) © AR_{7}(t) \\ AR_{8}(t) &= q_{85}(t) © AR_{5}(t) + q_{87}(t) © AR_{7}(t) + q_{8,10}(t) © AR_{10}(t) \end{split}$$

$$\begin{split} AR_{10}(t) &= q_{10,7}(t) \circledcirc AR_7(t) \\ \text{where,} \quad M_4(t) &= e^{-\gamma_2 t} \text{,} \quad M_7(t) = -e^{-\lambda t} \, \overline{G}(t) \end{split}$$

Taking Laplace transforms on both sides and solving we get:

$$AR_0^*(s) = \frac{N_3(s)}{D_2(s)}$$
 where $AR_0^*(s)$ is Laplace Transform of $AR_0(s)$

$$AR_{0} = \lim_{s \to 0} sAR_{0}^{*}(s) = \frac{N_{3}(0)}{D_{2}(0)} = \frac{N_{3}}{D_{2}}$$

where,

 $N_3 \ = \ \mu_4 p_{12} p_{74} p_{60} + (\mu_7 + \mu_4 p_{74}) (p_{12} p_{68} + p_{13})$

8. Busy Period Analysis (B₀)

Let $B_i(t)$ be the probability that the repairman is busy in the repair at instant t, given that the system entered regenerative state i at t=0. Then, $P_i(t) = \alpha_i(t) \otimes P_i(t)$

$$\begin{split} B_{0}(t) &= q_{01}(t) © B_{1}(t) \\ B_{1}(t) &= W_{1}(t) + q_{12}(t) © B_{2}(t) + q_{13}(t) © B_{3}(t) \\ B_{2}(t) &= W_{2}(t) + q_{24}(t) © B_{4}(t) \\ B_{3}(t) &= W_{3}(t) + q_{35}(t) © B_{5}(t) \\ B_{4}(t) &= q_{46}(t) © B_{6}(t) \\ B_{5}(t) &= W_{5}(t) + q_{57}(t) © B_{7}(t) \\ B_{6}(t) &= W_{6}(t) + q_{68}(t) © B_{8}(t) + q_{60}(t) © B_{0}(t) \\ B_{7}(t) &= W_{7}(t) + q_{74}(t) © B_{4}(t) + q_{7,7}^{9}(t) © B_{7}(t) \\ B_{8}(t) &= W_{8}(t) + q_{85}(t) © B_{5}(t) + q_{87}(t) © B_{7}(t) + q_{8,10}(t) © B_{10}(t) \\ B_{10}(t) &= W_{10}(t) + q_{10,7}(t) © B_{7}(t) \\ \end{split}$$

where, $W_1(t) = e^{-(\lambda+\eta)t}$, $W_2(t) = e^{-\beta_1 t}$, $W_3(t) = e^{-\beta_1 t}$, $W_5(t) = e^{-\gamma_1 t}$, $W_6(t) = \overline{G}(t)$, $W_7(t) = \overline{G}(t)$, $W_8(t) = e^{-(\lambda+\gamma_1)t}$, $W_{10}(t) = \overline{G}(t)$

Taking Laplace transforms on both sides and solving we get:

$$B_{0}^{*}(s) = \frac{N_{4}(s)}{D_{2}(s)} \text{ where } B_{0}^{*}(s) \text{ is Laplace Transform of } B_{0}(s)$$
$$B_{0} = \lim_{s \to 0} sB_{0}^{*}(s) = \frac{N_{4}(0)}{D_{2}^{'}(0)} = \frac{N_{4}}{D_{2}}$$

where,

 $N_4 = (\mu_1 + \mu_2 p_{12} + \mu_3 p_{13} + \mu_5 p_{13}) p_{74} p_{60} + \mu_8 p_{74} p_{68} + \mu_{10} (p_{12} p_{74} p_{60} + p_{13} + p_{12} p_{68} + p_{8,10} p_{74} p_{69})$

9. Down Time of the System (DT₀)

Let $DT_i(t)$ be the probability that system is down at instant t, given that the system entered regenerative state i at t=0. Then,

$$\begin{split} DT_0(t) &= q_{01}(t) © DT_1(t) \\ DT_1(t) &= q_{12}(t) © DT_2(t) + q_{13}(t) © DT_3(t) \\ DT_2(t) &= W_2(t) + q_{24}(t) © DT_4(t) \\ DT_3(t) &= W_3(t) + q_{35}(t) © DT_5(t) \\ DT_4(t) &= q_{46}(t) © DT_6(t) \end{split}$$

$$\begin{split} DT_5(t) &= W_5(t) + q_{57}(t) © DT_7(t) \\ DT_6(t) &= q_{68}(t) © DT_8(t) + q_{60}(t) © DT_0(t) \\ DT_7(t) &= q_{74}(t) © DT_4(t) + q_{7,7}^9(t) © DT_7(t) \\ DT_8(t) &= q_{85}(t) © DT_5(t) + q_{87}(t) © DT_7(t) + q_{8,10}(t) © DT_{10}(t) \\ DT_{10}(t) &= q_{10,7}(t) © DT_7(t) \\ where, W_2(t) &= e^{-\beta_1 t}, W_3(t) = e^{-\beta_1 t}, W_5(t) = e^{-\gamma_1 t} \end{split}$$

Taking Laplace transforms on both sides and solving we get:

$$DT_0^*(s) = \frac{N_5(s)}{D_2(s)} \text{ where } DT_0^*(s) \text{ is Laplace Transform of } DT_0(s)$$
$$DT_0 = \lim_{s \to 0} sDT_0^*(s) = \frac{N_5(0)}{D_2'(0)} = \frac{N_5}{D_2}$$

where,

 $N_5 = (\mu_2 p_{12} + \mu_3 p_{13} + \mu_5 p_{13}) p_{74} p_{60}$

10. Profit Analysis

The expected profit per unit time incurred to the system is given by:

$$P = C_0(AF_0) + C_1(AR_0) - C_2(B_0) - C_3(DT_0) - C_4 - C_5$$
 where $C_1 < C_0$

11. Particular Case

Consider $g(t) = \alpha e^{-\alpha t}$ $g_1^{*'}(0) = -\frac{1}{\alpha}$

Using the values estimated from the data collected i.e. (p = 0.8, q = 0.2, $\beta_1 = 0.3$, $\alpha = 0.1$, $\eta = 0.5$, $\gamma_1 = 0.6$, $\gamma_2 = 0.6$, $\lambda = 0.0001$, $C_0 = 100000$, $C_1 = 60000$, $C_2 = 20000$, $C_3 = 40000$, $C_4 = 30000$, $C_5 = 500$) the following values of various measures of system effectiveness are obtained :

- 1) Mean Time To System Failure (MTSF) = 1982608384 hours
- 2) Availability when System works at Full Capacity (AF_0) = 0.998998
- 3) Availability when System works at Reduced Capacity(AR_0) = 0.000337
- 4) Busy period of Repairman (B_0) = 0.003057
- 5) Expected Down time $(DT_0) = 0.000664$
- 6) Profit incurred to the system (P) = 69332.3359INR

12. Graphical Analysis

Fig. 2 shows the behavior of profit with respect to Revenue per unit time when system is working at Reduced Capacity (C₁) for different values of γ_2 . It can be concluded that profit increases with the increase in the value of C₁ and has higher values for higher γ_2 . If $\gamma_2=0.4$ then P > or = or < 0 accordingly as C₁> or = or < 28706.693359. So, for the model to be beneficial for $\gamma_2=0.4$, the C₁ should be greater than 28706.693359. If $\gamma_2=0.5$ and $\gamma_2=0.6$, the values for C₁ should be greater than 28264.603516 and 27821.828125 respectively. The user of the system should fix the prices in such a way so as to get the revenue per unit time when system is working at reduced capacity not less than that comes out to be at cut off point.

Fig. 3 shows the behavior of profit with respect to Loss per unit time due to low power generation (C₄) for different values of γ_2 . It can be concluded that profit decreases with the increase in the value of C₄ and has higher values for higher γ_2 . If $\gamma_2=0.2$ then P > or = or < 0 accordingly as C₄ < or = or > 99305.828125. So, for the model to be beneficial for $\gamma_2 = 0.2$, the C₄ should be less than 99305.828125. If $\gamma_2 = 0.4$ and $\gamma_2 = 0.6$, the values for C₄ should be less than 99325.703125 and 99332.335938 respectively. The user of the system should fix the prices in such a way so as to get the losses not more than that comes out to be at cut off point.



Profit Vs Revenue Per Unit time for different values of γ_2 when system is working at Reduced Capacity

Fig. 2. Profit (P) vs. Revenue per unit time when system is working at reduced capacity (C_1) for different values of constant rate of allowed time to change mode of working from reduced to full capacity (γ_2)



Profit Vs Loss Per Unit Time due to low power generation for different values of γ_2

Loss Per Unit Time due to low power generation (C₄)

Fig. 3. Profit (P) vs Loss per unit time due to low power generation (C₄) for different values of constant rate of allowed time to change mode of working from reduced to full capacity (γ_2)

Fig. 4 shows the behavior of profit with respect to Revenue per unit time when system is working at Full Capacity (C_0) for different values of p. It can be concluded that profit increases with the increase in the value of C_0 and has higher values for higher p. If p = 0.8 then P > or = or < 0 accordingly as $C_0 > or = or < 30609.953125$. So, for the model to be beneficial for p = 0.8, the C_0 should be greater than 30609.953125. For p

= 0.9 and p =1, the values for C_0 should be greater than 30609.949219 and 30609.947266 respectively. The user of the system should fix the revenue per unit time when system is working at full capacity not less than that comes out to be at cut off point.



Fig. 4. Profit (P) vs. Revenue per unit time when system is working at full capacity (C₀) for different values of probability that parameters

get changed to make the system work at reduced capacity (p)



Profit Vs Rate of Allowed Time to get parameter changed to work at Reduced Capacity for different values of p

Constant Rate of Allowed time to get the parameters changed (γ_1)

Fig. 5. Profit (P) vs. Constant rate of allowed time to get the parameters changed (γ_1) for different values of probability that parameters get changed to make the system work at reduced capacity (p)

Cost Benefit Analysis of a 2-Out-of-3 Induced Draft Fans System with Priority for Operation to Cold Standby**

Fig. 5 shows the behavior of profit with respect to rate of allowed time to get parameters changed so that system can work at full capacity (γ_1) for different values of p. It can be concluded that profit increases with the increase in the value of γ_1 and has higher values for higher p. If p = 0.4 then P > or = or < 0 accordingly as γ_1 > or = or < 0.0177. So, for the model to be beneficial for p = 0.4, the γ_1 should be greater than 0.0177. If p = 0.5 and p = 0.6, the values for γ_1 should be greater than 0.0165 and 0.0152 respectively. Parameters should be changed with rate not less than that comes out at cut-off point so that system may work at reduced capacity instead of going to down state.

13. Conclusion

The analysis of this model plays a significant role in predicting reliability and availability of the system. From the graphical analysis made through various graphs we can conclude that different cut off points obtained will help the user of the system to decide the acceptable values of different costs/rates so that system becomes profitable. The user of such system may adopt the model discussed and implement it for deciding various costs like revenue generated at full capacity, revenue generated at reduced capacity and different rates used in the system.

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