Analysis of a Two-Unit Automatic Power Factor Controller System with Inspection and Four Types of Failure

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Abstract

The electrical energy provided by power systems is generated, transmitted and distributed in the form of alternating current. Therefore, the importance of power factor increases and high power factor is desirable in such situations. Most of the A.C. motors, arc lamps and heating furnaces in industries operate at low lagging power factor, so some special power factor improvement equipments are installed in such industrial companies. One such equipment is automatic power factor controller (APFC). In the present study, system comprising two identical automatic power factor controller (APFC) cold standby units is analyzed using semi-Markov processes and regenerative point technique. Initially the system is operative with controlled power factor then it may undergo either to a state with uncontrolled power factor or to a failure. Inspection is carried out to detect the type of failure which may be due to fuse blown off, transformer burnt, programming problem, output relay faulty. The different measures of system effectiveness like MTSF, availability, busy period of repairman, number of visits etc. are obtained and cost benefit analysis is also done to suggest limits of some costs for significant profit.

Keywords: *Automatic Power Factor Controller (APFC) system, controlled/uncontrolled power factor, four types of failure, inspection, measures of system effectiveness*

2000 Mathematics Subject Classification: *90B25 and 60K10.*

1. Introduction

In reliability analysis, various measures of system effectiveness are obtained which help the consumers and service providers to increase the system effectiveness by minimizing failures and its maintenance costs. The primary objective of reliability is to make a system reliable as much as possible and to keep it in working condition as long as possible. A large number of researchers including Taneja (2005) [8], Rizwan (2007) [5] have probabilistically analyzed 2-unit standby system under various assumptions. However, some researchers including Taneja et al (2007) [9], and Parashar and Taneja (2007) [4] studied some reliability models collecting real data on failure and repair rates of the units used in such systems. Thereafter, Goyal et al (2009) [2] studied reliability and profit analysis of a two-unit standby system working in a sugar mill with operating and rest periods. Rizwan et al (2010) [6] analyzed different measures of system effectiveness of desalination unit with nine different kinds of failure. Sharma et al (2011) [7] investigated stochastically two standby units wherein on the failure of both the units the other system is switched on to avoid down time.

Bhatia et al (2012) [1] developed a model for single unit automatic power factor controller (APFC) unit by taking four types of failure—fuse blown off, transformer burnt, programming problem and output relay faulty. They considered single unit system. However, there exist many practical situations where two APFC units are also used and hence there arises need to develop and analyse a reliability model on two-unit APFC system considering the state of controlled / uncontrolled power factor.

Thus, a reliability model for the system comprising of two identical automatic power factor controller units is developed. Initially, one unit is operative and other is cold standby with controlled power factor. Then it may transit to a failed state or to an uncontrolled power factor state. On failure, an inspection is carried out to detect the one out of four types of failure—namely, fuse blown off, transformer burnt, programming problem, output relay faulty. In case of failure due to "fuse blown off" or "transformer burnt", the components are replaced whereas in the last two types of failure, the repairs are done and hence there is no need of replacement in the latter case. The repairman gives priority to control the power factor over inspection. Also, the uncontrolled power factor is taken care of by the repairman before declaring the unit undertaken by him operable so that failed state with uncontrolled power factor changes to operative state with controlled power factor after repair. The system is analyzed by making use of semi-Markov processes and regenerative point technique. Various measures of system effectiveness are obtained such as

- Mean time to system failure (MTSF)
- Availability when power factor is controlled
- Availability when power factor is not controlled
- Busy period of the repairman when fuse is blown off (Type I)
- Busy period of the repairman when transformer is burnt (Type II)
- Busy period of the repairman when there is programming problem (Type III)
- Busy period of the repairman when output relay is faulty (Type IV)
- Expected number of visits of the repairman
- Expected number of fuse replacement
- Expected number of transformer replacement
- Profit incurred to the system

2. Notations and Nomenclature

λ	constant rate of failure
β_1	rate with which power factor changes from controlled mode to uncontrolled mode
β_2	rate with which power factor changes from uncontrolled mode to controlled mode
β	rate of inspection
p_1	probability of failure of type I (Fuse blown off)
p_2	probability of failure of type II (Transform burnt)
p_3	probability of failure of type III (Programming Problem)
p_4	probability of failure of type IV (output relay faulty)
$g_1(t),G_1(t)$	p.d.f. and c.d.f. of failure of type I with controlled power factor
$g_2(t), G_2(t)$	p.d.f. and c.d.f. of failure of type II with controlled power factor
$g_3(t), G_3(t)$	p.d.f. and c.d.f. of failure of type III with controlled power factor
$g_4(t), G_4(t)$	p.d.f. and c.d.f. of failure of type IV with controlled power factor
$i(t),I(t)$	p.d.f. and c.d.f. of inspection time
© / Ⓢ	Laplace/ Stieltjes convolution
$h(t),H(t)$	p.d.f. and c.d.f. of the time of conversion of power factor from uncontrolled to controlled mode
O	the unit is operative
CS	cold standby unit
C	power factor controlled
$\overline{C}$	power factor not controlled
F_i	unit is under inspection on failure
F_{r1}	the main unit is under repair in case of failure of type I (fuse blown off)
F_{r2}	the main unit is under repair in case of failure of type II (transformer burnt)
F_{r3}	the main unit is under repair in case of failure of type III (programming problem)
F_{r4}	the main unit is under repair in case of failure of type IV (output relay faulty)
F_{wi}	unit waiting for inspection by the repairman
F_{R_1}	the continuation of repair of main unit from previous state in case of failure of type I
F_{R_2}	the continuation of repair of main unit from previous state in case of failure of type II
F_{R_3}	the continuation of repair of main unit from previous state in case of failure of type III
F_{R_4}	the continuation of repair of main unit from previous state in case of failure of type IV

3. Assumptions of Model

For the probabilistic analysis of the model, following assumptions are made :

- Initially, the system is operative with controlled power factor.
- Both units are identical. One unit is operative while other unit is in cold standby mode.
- On failure, an inspection is carried out to detect the type of failure.
- During inspection, other events can also take place i.e. cold standby may also fail.
- The system can fail due to different types of failures like fuse blown off, transformer burnt, programming problem and output relay faulty.
- Failure times and inspection time are assumed to have exponential distribution whereas repair/replacement times have general distribution.
- The repairman comes immediately as soon as the unit fails.
- On the arrival of the repairman, power factor is controlled first, if it is not controlled already.
- All random variables are independent.

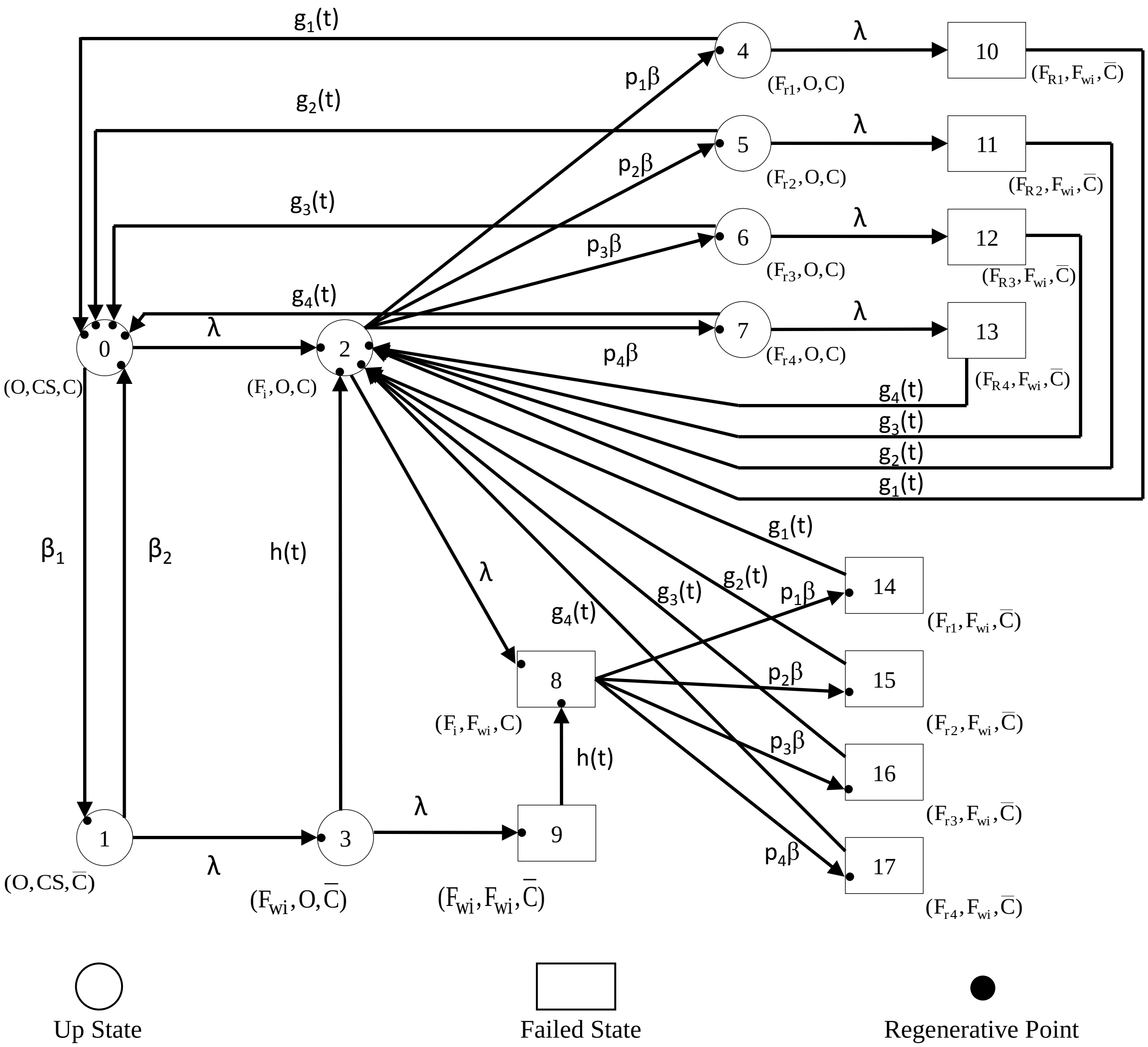

Fig.1 State Transition Diagram

4. Essential Pre-requisites for the calculation of various measures of system effectiveness

4.1 Transition Probabilities

A transition diagram showing the various states of system is shown in Fig. 1. The epochs of entry into states 0, 1,2,3,4,5,6,7,8,9,14,15,16,17 are regenerative points while 10,11,12,13 are failed states. The transition probabilities from regenerative state to regenerative state are given below:

$q_{01}(t)=\beta_1 e^{-(\lambda+\beta_1)t}$, $q_{02}(t) = \lambda e^{-(\lambda+\beta_1)t}$, $q_{10}(t)= \beta_2 e^{-(\lambda+\beta_2)t}$, $q_{14}(t)= \lambda e^{-(\lambda+\beta_2)t}$, $q_{24}(t) = p_1 \beta e^{-(\lambda+\beta)t}$

$q_{25}(t)= p_2 \beta e^{-(\lambda+\beta)t}$, $q_{26}(t)= p_3 \beta e^{-(\lambda+\beta)t}$, $q_{27}(t)= p_4 \beta e^{-(\lambda+\beta)t}$, $q_{28}(t)= \lambda e^{-(\lambda+\beta)t}$

$q_{32}(t) = e^{-\lambda t} h(t)$, $q_{39}(t)= \lambda e^{-\lambda t}\overline{H}(t)$, $q_{40}(t)= e^{-\lambda t} g_1(t)$, $q_{4,10}(t)=\lambda e^{-\lambda t}\overline{G_1}(t)$, $q_{42}^{(10)}(t)= g_1(t)(\lambda e^{-\lambda t} ©1)$

$q_{50}(t)=e^{-\lambda t} g_2(t)$, $q_{5,11}(t)=\lambda e^{-\lambda t} \overline{G_2}(t)$, $q_{52}^{(11)}(t)=g_2(t)(\lambda e^{-\lambda t} ©1)$, $q_{60}(t) =e^{-\lambda t} g_3(t)$, $q_{6,12}(t)=\lambda e^{-\lambda t} \overline{G_3}(t)$,

$q_{62}^{(12)}(t)=g_3(t)(\lambda e^{-\lambda t} ©1)$, $q_{70}(t)=e^{-\lambda t} g_4(t)$, $q_{7,13}(t) = \lambda e^{-\lambda t} \overline{G_4}(t)$, $q_{72}^{(13)}(t)=g_4(t)(\lambda e^{-\lambda t} ©1)$,

$q_{8,14}(t)=p_1 \beta e^{-\beta t}$, $q_{8,15}(t) =p_2 \beta e^{-\beta t}$, $q_{8,16}(t) =p_3 \beta e^{-\beta t}$, $q_{8,17}(t) =p_4 \beta e^{-\beta t}$, $q_{98}(t)= h(t)$, $q_{14,2}(t)= g_1(t)$

$q_{15,2}(t)= g_2(t)$, $q_{16,2}(t)= g_3(t)$, $q_{17,2}(t)= g_4(t)$

The corresponding non-zero element $p_{ij} = \lim_{s\to 0} q_{ij}^*(s)$ are given by

$p_{01} = \frac{\beta_1}{\lambda+\beta_1}$, $p_{02} = \frac{\lambda}{\lambda+\beta_1}$, $p_{10} = \frac{\beta_2}{\lambda+\beta_2}$, $p_{14} = \frac{\lambda}{\lambda+\beta_2}$, $p_{24} = \frac{p_1\beta}{\lambda+\beta}$, $p_{25} = \frac{p_2\beta}{\lambda+\beta}$, $p_{26} = \frac{p_3\beta}{\lambda+\beta}$, $p_{27} = \frac{p_4\beta}{\lambda+\beta}$

$p_{28} = \frac{\lambda}{\lambda+\beta}$, $p_{32} = h^*(\lambda)$, $p_{39} = 1-h^*(\lambda)$, $p_{40} = g_1^*(\lambda)$, $p_{4,10} = p_{42}^{(10)} = 1-g_1^*(\lambda)$, $p_{50} = g_2^*(\lambda)$

$p_{5,11} = p_{52}^{(11)} = 1-g_2^*(\lambda)$, $p_{60} = g_3^*(\lambda)$, $p_{6,12} = p_{62}^{(12)} = 1-g_3^*(\lambda)$, $p_{70} = g_4^*(\lambda)$, $p_{7,13} = p_{72}^{(13)} = 1-g_4^*(\lambda)$

$p_{8,14} = p_1$, $p_{8,15} = p_2$, $p_{8,16} = p_3$, $p_{8,17} = p_4$, $p_{98} = p_{14,2} = p_{15,2} = p_{16,2} = p_{17,2} = 1$

From above mentioned transition probabilities it can be verified that

$p_{01} + p_{02} = 1$, $p_{10} + p_{13} = 1$, $p_{24} + p_{25} + p_{26} + p_{27} + p_{28} = 1$, $p_{32} + p_{39} = 1$

$p_{40} + p_{4,10} = p_{40} + p_{42}^{(10)} = 1$, $p_{50} + p_{5,11} = p_{50} + p_{52}^{(11)} = 1$, $p_{60} + p_{6,12} = p_{60} + p_{62}^{(12)} = 1$

$p_{70} + p_{7,13} = p_{70} + p_{72}^{(13)} = 1$, $p_{8,14} + p_{8,15} + p_{8,16} + p_{8,17} = 1$, $p_{98} = p_{14,2} = p_{15,2} = p_{16,2} = p_{17,2} = 1$

4.2 Mean Sojourn Times

The mean sojourn times (μ_i) in the regenerative state i is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in the regenerative state i, then

$$\mu_i = E(T) = Pr(T>y)$$

$\mu_0 = \frac{1}{\lambda+\beta_1}$, $\mu_1 = \frac{1}{\lambda+\beta_2}$, $\mu_2 = \frac{1}{\lambda+\beta}$, $\mu_3 = \frac{1-h^*(\lambda)}{\lambda}$, $\mu_4 = \frac{1-g_1^*(\lambda)}{\lambda}$, $\mu_5 = \frac{1-g_2^*(\lambda)}{\lambda}$, $\mu_6 = \frac{1-g_3^*(\lambda)}{\lambda}$

$\mu_7 = \frac{1-g_4^*(\lambda)}{\lambda}$, $\mu_8 = \frac{1}{\beta}$, $\mu_9 = \int_0^\infty t h(t)dt$, $\mu_{14} = \int_0^\infty t g_1(t)dt$, $\mu_{15} = \int_0^\infty t g_2(t)dt$, $\mu_{16} = \int_0^\infty t g_3(t)dt$, $\mu_{17} = \int_0^\infty t g_4(t)dt$

The unconditional mean time taken by the system to transit to any regenerative state j when time is counted from the epoch of entrance into state i is mathematically stated as

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$$

Also,

$m_{01} + m_{02} = \mu_0$, $m_{10} + m_{13} = \mu_1$, $m_{24} + m_{25} + m_{26} + m_{27} + m_{28} = \mu_2$, $m_{32} + m_{39} = \mu_3$

$$m_{40}+m_{4,10}=\mu_4\,,\ m_{40}+m_{42}^{(10)}=\int_0^\infty \overline{G_1}(t)dt=\mu_{14}\,,\ m_{50}+m_{5,11}=\mu_5\,,\ m_{50}+m_{52}^{(11)}=\int_0^\infty \overline{G_2}(t)dt=\mu_{15}$$

$$m_{60}+m_{6,12}=\mu_6\,,\ m_{60}+m_{62}^{(12)}=\int_0^\infty \overline{G_3}(t)dt=\mu_{16}\,,\ m_{70}+m_{7,13}=\mu_7\,,\ m_{70}+m_{72}^{(13)}=\int_0^\infty \overline{G_4}(t)dt=\mu_{17}$$

$$m_{8,14}+m_{8,15}+m_{8,16}+m_{8,17}=\mu_8\,,\ m_{98}=\mu_9\,,\ m_{14,2}=\mu_{14}\,,\ m_{15,2}=\mu_{15}\,,\ m_{16,2}=\mu_{16}\,,\ m_{17,2}=\mu_{17}$$

3. Mathematical Analysis of various Measures of System Effectiveness

For all the calculations given below failed states are considered as absorbing states and using the probabilistic arguments used for regenerative processes, the recursive relations are obtained for probabilistic analysis of measures of system effectiveness.

5.1 Mean Time to System Failure (MTSF)

This measure is defined as the expected time for which the system is in operation before it completely fails. $\phi_i(t)$ is defined as the cumulative distribution function of first passage time from ith state to a failed state.

$\phi_0(t) = Q_{01}(t)$ Ⓢ $\phi_1(t) + Q_{02}(t)$ Ⓢ $\phi_2(t)$

$\phi_1(t) = Q_{10}(t)$ Ⓢ $\phi_0(t) + Q_{13}(t)$ Ⓢ $\phi_3(t)$

$\phi_2(t) = Q_{24}(t)$ Ⓢ $\phi_4(t) + Q_{25}(t)$ Ⓢ $\phi_5(t) + Q_{26}(t)$ Ⓢ $\phi_6(t) + Q_{27}(t)$ Ⓢ $\phi_7(t) + Q_{28}(t)$

$\phi_3(t) = Q_{32}(t)$ Ⓢ $\phi_2(t) + Q_{39}(t)$

$\phi_4(t) = Q_{40}(t)$ Ⓢ $\phi_0(t) + Q_{4,10}(t)$

$\phi_5(t) = Q_{50}(t)$ Ⓢ $\phi_0(t) + Q_{5,11}(t)$

$\phi_6(t) = Q_{60}(t)$ Ⓢ $\phi_0(t) + Q_{6,12}(t)$

$\phi_7(t) = Q_{70}(t)$ Ⓢ $\phi_0(t) + Q_{7,13}(t)$

Taking Laplace Stieltjes Transform on both sides and solving equations for $\phi_0^{**}(s)$;

$$\text{MTSF} = \lim_{s\to 0}\frac{1-\phi_0^{**}(s)}{s} = \frac{D_0'(0)-N_0'(0)}{D_0(0)} = \frac{N_1}{D_1}$$

where

$$N_1=\mu_0+p_{01}\mu_1+p_{01}p_{13}\mu_3+(\mu_2+p_{27}\mu_7+p_{26}\mu_6+p_{25}\mu_5+p_{24}\mu_4)(p_{02}+p_{01}p_{13}p_{32})$$
$$D_1=1-p_{01}p_{10}-(p_{27}p_{70}+p_{26}p_{60}+p_{25}p_{50}+p_{24}p_{40})(p_{02}+p_{01}p_{13}p_{32})$$

5.2 Availability when Power Factor is controlled

Using the probabilistic arguments and defining $AC_i(t)$ as the probability that the system is in up state when power factor is controlled at instant t, given that the system entered regenerative state i at t=0, we have the following recursive relations :

$AC_0(t) = M_0(t) + q_{01}(t)$ © $AC_1(t) + q_{02}(t)$ © $AC_2(t)$

$AC_1(t) = q_{10}(t)$ © $AC_0(t) + q_{13}(t)$ © $AC_3(t)$

$AC_2(t) = M_2(t) + q_{24}(t)$ © $AC_4(t) + q_{25}(t)$ © $AC_5(t) + q_{26}(t)$ © $AC_6(t) + q_{27}(t)$ © $AC_7(t) + q_{28}(t)$ © $AC_8(t)$

$AC_3(t) = q_{32}(t)$ © $AC_2(t) + q_{39}(t)$ © $AC_9(t)$

$AC_4(t) = M_4(t) + q_{40}(t)$ © $AC_0(t) + q_{42}^{(10)}(t)$ © $AC_2(t)$

$AC_5(t) = M_5(t) + q_{50}(t)$ © $AC_0(t) + q_{52}^{(11)}$ © $AC_2(t)$

$AC_6(t) = M_6(t) + q_{60}(t)$ © $AC_0(t) + q_{62}^{(12)}$ © $AC_2(t)$

$AC_7(t) = M_7(t) + q_{70}(t)$ © $AC_0(t) + q_{72}^{(13)}$ © $AC_2(t)$

$AC_8(t) = M_8(t) + q_{8,14}(t)$ © $AC_{14}(t) + q_{8,15}(t)$ © $AC_{15}(t) + q_{8,16}(t)$ © $AC_{16}(t) + q_{8,17}(t)$ © $AC_{17}(t)$

$AC_9(t) = q_{98}(t) © AC_8(t)$
$AC_{14}(t) = q_{14,2}(t) © AC_2(t)$
$AC_{15}(t) = q_{15,2}(t) © AC_2(t)$
$AC_{16}(t) = q_{16,2}(t) © AC_2(t)$
$AC_{17}(t) = q_{17,2}(t) © AC_2(t)$

where $M_0(t) = e^{-(\lambda+\beta_1)t}$, $M_2(t) = e^{-(\lambda+\beta)t}$, $M_4(t)= e^{-\lambda t}\overline{G_1}(t)$, $M_5(t) = e^{-\lambda t}\overline{G_2}(t)$, $M_6(t) = e^{-\lambda t}\overline{G_3}(t)$, $M_7(t) = e^{-\lambda t}\overline{G_4}(t)$, $M_8(t) = e^{-\beta t}$

and $M_0^*(s) = \dfrac{1}{\lambda+\beta_1+s}$, $M_2^*(s) = \dfrac{1}{\lambda+\beta+s}$, $M_4^*(s) = \dfrac{1-g_1^*(\lambda+s)}{\lambda+s}$, $M_5^*(s) = \dfrac{1-g_2^*(\lambda+s)}{\lambda+s}$, $M_6^*(s) = \dfrac{1-g_3^*(\lambda+s)}{\lambda+s}$,

$M_7^*(s) = \dfrac{1-g_4^*(\lambda+s)}{\lambda+s}$, $M_8^*(s) = \dfrac{1}{\beta+s}$

Taking Laplace transforms on both sides of above equations and solving for $AC_0^*(s)$;

$$AC_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

For steady state, availability of the system is given by

$$AC_0 = \lim_{s\to 0} sAC_0^*(s) = \frac{N_2}{D_2}$$

where

$$N_2 = (\mu_0 + p_{01}p_{13}p_{39}\mu_8)(p_{27}p_{70} + p_{26}p_{60} + p_{25}p_{50} + p_{24}p_{40}) + (1-p_{01}p_{10})(\mu_2 + p_{28}\mu_8 + p_{27}\mu_7 + p_{26}\mu_6 + p_{25}\mu_5 + p_{24}\mu_4)$$

$$D_2 = (\mu_0 + \mu_1 p_{01} + \mu_3 p_{01}p_{13} + \mu_8 p_{01}p_{13}p_{39} + \mu_9 p_{01}p_{13}p_{39})(p_{27}p_{70} + p_{26}p_{60} + p_{25}p_{50} + p_{24}p_{40}) + (1-p_{01}p_{10})(\mu_2 + p_{28}\mu_8 + p_{27}\mu_{17} + p_{26}\mu_{16} + p_{25}\mu_{15} + p_{24}\mu_{14}) + (1-p_{01}p_{10})p_{28}(p_{8,17}m_{17,2} + p_{8,16}m_{16,2} + p_{8,15}m_{15,2} + p_{8,14}m_{14,2}) + p_{01}p_{13}p_{39}(p_{27}p_{70} + p_{26}p_{60} + p_{25}p_{50} + p_{24}p_{40})(p_{8,17}m_{17,2} + p_{8,16}m_{16,2} + p_{8,15}m_{15,2} + p_{8,14}m_{14,2})$$

5.3 Availability when Power Factor is not controlled

Using the probabilistic arguments and defining $\overline{AC_i}(t)$ as the probability that the system is in up state when power factor is not controlled at instant t, given that the system entered regenerative state i at t=0, we have the following recursive relations :

$\overline{AC_0}(t) = q_{01}(t) © \overline{AC_1}(t) + q_{02}(t) © \overline{AC_2}(t)$
$\overline{AC_1}(t) = M_1(t) + q_{10}(t) © \overline{AC_0}(t) + q_{13}(t) © \overline{AC_3}(t)$
$\overline{AC_2}(t) = q_{24}(t) © \overline{AC_4}(t) + q_{25}(t) © \overline{AC_5}(t) + q_{26}(t) © \overline{AC_6}(t) + q_{27}(t) © \overline{AC_7}(t) + q_{28}(t) © \overline{AC_8}(t)$
$\overline{AC_3}(t) = M_3(t) + q_{32}(t) © \overline{AC_2}(t) + q_{39}(t) © \overline{AC_9}(t)$
$\overline{AC_4}(t) = q_{40}(t) © \overline{AC_0}(t) + q_{42}^{(10)}(t) © \overline{AC_2}(t)$
$\overline{AC_5}(t) = q_{50}(t) © \overline{AC_0}(t) + q_{52}^{(11)} © \overline{AC_2}(t)$
$\overline{AC_6}(t) = q_{60}(t) © \overline{AC_0}(t) + q_{62}^{(12)} © \overline{AC_2}(t)$
$\overline{AC_7}(t) = q_{70}(t) © \overline{AC_0}(t) + q_{72}^{(13)} © \overline{AC_2}(t)$
$\overline{AC_8}(t) = q_{8,14}(t) © \overline{AC_{14}}(t) + q_{8,15}(t) © \overline{AC_{15}}(t) + q_{8,16}(t) © \overline{AC_{16}}(t) + q_{8,17}(t) © \overline{AC_{17}}(t)$
$\overline{AC_9}(t) = M_9(t) + q_{98}(t) © \overline{AC_8}(t)$
$\overline{AC_{14}}(t) = M_{14}(t) + q_{14,2}(t) © \overline{AC_2}(t)$

$\overline{AC_{15}}(t) = M_{15}(t) + q_{15,2}(t) © \overline{AC_2}(t)$

$\overline{AC_{16}}(t) = M_{16}(t) + q_{16,2}(t) © \overline{AC_2}(t)$

$\overline{AC_{17}}(t) = M_{17}(t) + q_{17,2}(t) © \overline{AC_2}(t)$

where $M_1(t) = e^{-(\lambda+\beta_2)t}$, $M_3(t) = e^{-\lambda t}\overline{H}(t)$, $M_9(t)=\overline{H}(t)$, $M_{14}(t)=\overline{G_1}(t)$, $M_{15}(t)=\overline{G_2}(t)$, $M_{16}(t)=\overline{G_3}(t)$, $M_{17}(t)=\overline{G_4}(t)$

and $M_1^*(s) = \dfrac{1}{\lambda+\beta_2+s}$, $M_3^*(s) = \dfrac{1-h^*(\lambda+s)}{\lambda+s}$, $M_9^*(s) = \int_0^\infty e^{-st}\overline{H}(t)dt$, $M_{14}^*(s) = \int_0^\infty e^{-st}\overline{G_1}(t)dt$, $M_{15}^*(s) = \int_0^\infty e^{-st}\overline{G_2}(t)dt$,

$M_{16}^*(s) = \int_0^\infty e^{-st}\overline{G_3}(t)dt$, $M_{17}^*(s) = \int_0^\infty e^{-st}\overline{G_4}(t)dt$

Taking laplace transforms on both sides of above equations and solving for $\overline{AC}_0^*(s)$;

$$\overline{AC}_0^*(s) = \frac{N_3(s)}{D_2(s)}$$

For steady state availability of the system is given by

$$\overline{AC_0} = \lim_{s\to 0} s\overline{AC}_0^*(s) = \frac{N_3}{D_2}$$

where

$$N_3 = p_{01}(p_{27}p_{70} + p_{26}p_{60} + p_{25}p_{50} + p_{24}p_{40})(\mu_1 + \mu_3 p_{13} + \mu_9 p_{13}p_{39} + p_{13}p_{39}p_{8,14}\mu_{14}$$
$$+ p_{13}p_{39}p_{8,15}\mu_{15} + p_{13}p_{39}p_{8,16}\mu_{16} + p_{13}p_{39}p_{8,17}\mu_{17}) + p_{28}(1 - p_{01}p_{10})(p_{8,14}\mu_{14} + p_{8,15}\mu_{15}$$
$$+ p_{8,16}\mu_{16} + p_{8,17}\mu_{17})$$

and D_2 is already specified.

5.4 Busy Period Analysis of Type I Repair

Using the probabilistic arguments and defining $BF_i(t)$ as the probability that the repairman is busy in the repair of Type I failure at instant t, given that the system entered regenerative state i at t=0, we have the following recursive relations :

$BF_0(t) = q_{01}(t) © BF_1(t) + q_{02}(t) © BF_2(t)$

$BF_1(t) = q_{10}(t) © BF_0(t) + q_{13}(t) © BF_3(t)$

$BF_2(t) = q_{24}(t) © BF_4(t) + q_{25}(t) © BF_5(t) + q_{26}(t) © BF_6(t) + q_{27}(t) © BF_7(t) + q_{28}(t) © BF_8(t)$

$BF_3(t) = q_{38}(t) © BF_8(t) + q_{39}(t) © BF_9(t)$

$BF_4(t) = W_4(t) + q_{40}(t) © BF_0(t) + q_{42}^{(10)}(t) © BF_2(t)$

$BF_5(t) = q_{50}(t) © BF_0(t) + q_{52}^{(11)} © BF_2(t)$

$BF_6(t) = q_{60}(t) © BF_0(t) + q_{62}^{(12)} © BF_2(t)$

$BF_7(t) = q_{70}(t) © BF_0(t) + q_{72}^{(13)} © BF_2(t)$

$BF_8(t) = q_{8,14}(t) © BF_{14}(t) + q_{8,15}(t) © BF_{15}(t) + q_{8,16}(t) © BF_{16}(t) + q_{8,17}(t) © BF_{17}(t)$

$BF_9(t) = q_{98}(t) © BF_8(t)$

$BF_{14}(t) = W_{14}(t) + q_{14,2}(t) © BF_2(t)$

$BF_{15}(t) = q_{15,2}(t) © BF_2(t)$

$BF_{16}(t) = q_{16,2}(t) © BF_2(t)$

$BF_{17}(t) = q_{17,2}(t) © BF_2(t)$

where $W_4(t) = W_{14}(t) = \overline{G_1}(t)$

Taking Laplace transforms on both sides of the above equations and solving for $BF_0^*(s)$, we get

$$BF_0^*(s) = \frac{N_4(s)}{D_2(s)}$$

For steady state, the total fraction of time for which the system is under repair of failure of type I is given by

$$BF_0 = \lim_{s\to 0} sBF_0^*(s) = \frac{N_4}{D_2}$$

where

$$N_4 = (p_{27}p_{70} + p_{26}p_{60} + p_{25}p_{50} + p_{24}p_{40})p_{01}p_{13}p_{39}\mu_{14} + p_{24}\mu_{14}(1 - p_{01}p_{10})$$

and D_2 is already specified.

Similarly, busy period analysis of the repairman when transformer is burnt (Type II failure) (BT_0), busy period analysis of the repairman when there is programming problem (Type III failure) (BP_0), busy period analysis of the repairman when output relay is faulty (Type IV failure) (BO_0) can be obtained for steady state.

5.5 Expected Number of Visits of Repairman

Using the probabilistic arguments and defining $V_i(t)$ as the expected number of visits in (0,t], given that the system entered regenerative state i at t=0, we have the following recursive relations :

$V_0(t) = Q_{01}(t) \circledS V_1(t) + Q_{02}(t) \circledS (1+V_2(t))$

$V_1(t) = Q_{10}(t) \circledS V_0(t) + Q_{13}(t) \circledS (1+V_3(t))$

$V_2(t) = Q_{24}(t) \circledS V_4(t) + Q_{25}(t) \circledS V_5(t) + Q_{26}(t) \circledS V_6(t) + Q_{27}(t) \circledS V_7(t)$

$V_3(t) = Q_{32}(t) \circledS V_2(t) + Q_{39}(t) \circledS V_9(t)$

$V_4(t) = Q_{40}(t) \circledS V_0(t) + Q_{42}^{(10)}(t) \circledS V_2(t)$

$V_5(t) = Q_{50}(t) \circledS V_0(t) + Q_{52}^{(11)}(t) \circledS V_2(t)$

$V_6(t) = Q_{60}(t) \circledS V_0(t) + Q_{62}^{(12)}(t) \circledS V_2(t)$

$V_7(t) = Q_{70}(t) \circledS V_0(t) + Q_{72}^{(13)}(t) \circledS V_2(t)$

$V_8(t) = Q_{8,14}(t) \circledS V_{14}(t) + Q_{8,15}(t) \circledS V_{15}(t) + Q_{8,16}(t) \circledS V_{16}(t) + Q_{8,17}(t) \circledS V_{17}(t)$

$V_9(t) = Q_{98}(t) \circledS V_8(t)$

$V_{14}(t) = Q_{14,2}(t) \circledS V_2(t)$

$V_{15}(t) = Q_{15,2}(t) \circledS V_2(t)$

$V_{16}(t) = Q_{16,2}(t) \circledS V_2(t)$

$V_{17}(t) = Q_{17,2}(t) \circledS V_2(t)$

Taking Laplace Stietjes Transform on both sides of above equations and solving for $V_0^{**}(s)$;

$$V_0^{**}(s) = \frac{N_5(s)}{D_2(s)}$$

For steady state, the expected number of visits per unit time is given by

$$V_0 = \lim_{s\to 0} sV_0^{**}(s) = \frac{N_5}{D_2}$$

where

$$N_5 = (p_{27}p_{70} + p_{26}p_{60} + p_{25}p_{50} + p_{24}p_{40})(1 - p_{01}p_{10})$$

and D_2 is already specified.

5.6 Expected Number of Fuse Replacements

Using the probabilistic arguments and defining $FR_i(t)$ as the expected number of replacements in (0,t], given that the system entered regenerative state i at t=0, we have the following recursive relations :

$FR_0(t) = Q_{01}(t) \circledS FR_1(t) + Q_{02}(t) \circledS FR_2(t)$

$FR_1(t) = Q_{10}(t) \circledS FR_0(t) + Q_{13}(t) \circledS FR_3(t)$

$FR_2(t) = Q_{24}(t) \circledS FR_4(t) + Q_{25}(t) \circledS FR_5(t) + Q_{26}(t) \circledS FR_6(t) + Q_{27}(t) \circledS FR_7(t) + Q_{28}(t) \circledS FR_8(t)$

$FR_3(t) = Q_{32}(t) \circledS FR_8(t) + Q_{39}(t) \circledS FR_9(t)$

$FR_4(t) = Q_{40}(t) \circledS (1+FR_0(t)) + Q_{42}^{(10)}(t) \circledS (1+FR_2(t))$

$FR_5(t) = Q_{50}(t) \circledS FR_0(t) + Q_{52}^{(11)}(t) \circledS FR_2(t)$

$FR_6(t) = Q_{60}(t) \circledS FR_0(t) + Q_{62}^{(12)}(t) \circledS FR_2(t)$

$FR_7(t) = Q_{70}(t) \circledS FR_0(t) + Q_{72}^{(13)}(t) \circledS FR_2(t)$

$FR_8(t) = Q_{8,14}(t) \circledS FR_{14}(t) + Q_{8,15}(t) \circledS FR_{15}(t) + Q_{8,16}(t) \circledS FR_{16}(t) + Q_{8,17}(t) \circledS FR_{17}(t)$

$FR_9(t) = Q_{98}(t) \circledS FR_8(t)$

$FR_{14}(t) = Q_{14,2}(t) \circledS (1 + FR_2(t))$

$FR_{15}(t) = Q_{15,2}(t) \circledS FR_2(t)$

$FR_{16}(t) = Q_{16,2}(t) \circledS FR_2(t)$

$FR_{17}(t) = Q_{17,2}(t) \circledS FR_2(t)$

Taking Laplace Stietjes Transform on both sides of above equations and solving for $FR_0^{**}(s)$;

$$FR_0^{**}(s) = \frac{N_6(s)}{D_2(s)}$$

For steady state, the expected number of fuse replacements per unit time is given by

$$FR_0 = \lim_{s \to 0} sFR_0^{**}(s) = \frac{N_6}{D_2}$$

where $N_6 = (p_{27}p_{70} + p_{26}p_{60} + p_{25}p_{50} + p_{24}p_{40})p_{01}p_{13}p_{39}p_{8,17} + (p_{8,17}p_{28} + p_{24})(1 - p_{01}p_{10})$

and D_2 is already specified.

Also calculations are done to find expected number of transformer replacements (TR_0) and following result is obtained for steady state,

$$TR_0 = \frac{N_7}{D_2} \quad \text{where} \quad N_7 = (p_{27}p_{70} + p_{26}p_{60} + p_{25}p_{50} + p_{24}p_{40})p_{01}p_{13}p_{39}p_{8,15} + (p_{8,15}p_{28} + p_{25})(1 - p_{01}p_{10})$$

6. Cost-Benefit Analysis

At steady state, the expected total profit (P) per unit time incurred to the system is given by:

P (Profit) $= C_0(AC_0 + A\overline{C_0}) - C_{21}BF_0 - C_{22}BT_0 - C_{23}BP_0 - C_{24}BO_0 - C_1FR_0 - C_2TR_0 - C_3V_0 - (A\overline{C_0})(L\overline{C})$

where C_0 revenue per unit up time

C_{21} cost per unit up time for which the repairman is busy for repairing the unit having failure of type I

C_{22} cost per unit up time for which the repairman is busy for repairing the unit having failure of type II

C_{23} cost per unit up time for which the repairman is busy for repairing the unit having failure of type III

C_{24} cost per unit up time for which the repairman is busy for repairing the unit having failure of type IV

C_1 cost per fuse replacement

C_2 cost per transformer replacement

C_3 cost per visit of the repairman

$L\overline{C}$ loss per unit time when power factor is not controlled

7. Discussion and Results

A particular case is discussed by assuming the repair/replacement is exponentially distributed as under:

$g_1(t) = \alpha_1 e^{-\alpha_1 t}$, $g_2(t) = \alpha_2 e^{-\alpha_2 t}$, $g_3(t) = \alpha_3 e^{-\alpha_3 t}$, $g_4(t) = \alpha_4 e^{-\alpha_4 t}$, $h(t) = \gamma e^{-\gamma t}$

Using the values estimated from the data/information collected i.e. ($p_1 = 0.3$, $p_2 = 0.2$, $p_3 = 0.4$, $p_4 = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 2$, $\alpha_3 = 6$, $\alpha_4 = 10$, $\gamma = 2$, $\beta = 6$, $\beta_1 = 0.02$, $\beta_2 = 0.2$, $\lambda = 0.001$, $C_0 = 1000$, $C_1 = 50$, $C_2 = 250$, $C_3 = 1000$, $C_{21} = 100$, $C_{22} = 150$, $C_{23} = 50$, $C_{24} = 75$, $L\overline{C} = 500$) the following values of various measures of system effectiveness can be obtained.

1) Mean time to system failure (MTSF) = 2158448.25hrs
2) Availability when power factor is controlled (AC_0) = 0.909499
3) Availability when power factor is not controlled ($\overline{AC_0}$) = 0.090501
4) Busy period of Type I repair (BF_0) = 0.000075
5) Busy period of Type II repair (BT_0) =0.0001
6) Busy period of Type III repair (BP_0) = 0.000067
7) Busy period of Type IV repair (BO_0)= 0.00001
8) Expected number of visits of the repairman (V_0) =0.001
9) Expected number of fuse replacement (FR_0) = 0.0003
10) Expected number of transformer replacement (TR_0) = 0.0002
11) Profit incurred to the system (P) = 953.658 INR

8. Graphical analysis and Conclusions

Graphs have been plotted for the above particular case and it was observed from the graph that the MTSF decreases with the increase in the value of λ and has higher values for higher α_1. Also, availability when power factor is controlled increases with the increase in the value of λ and availability when power factor is not controlled decreases with the increase in the value of λ.

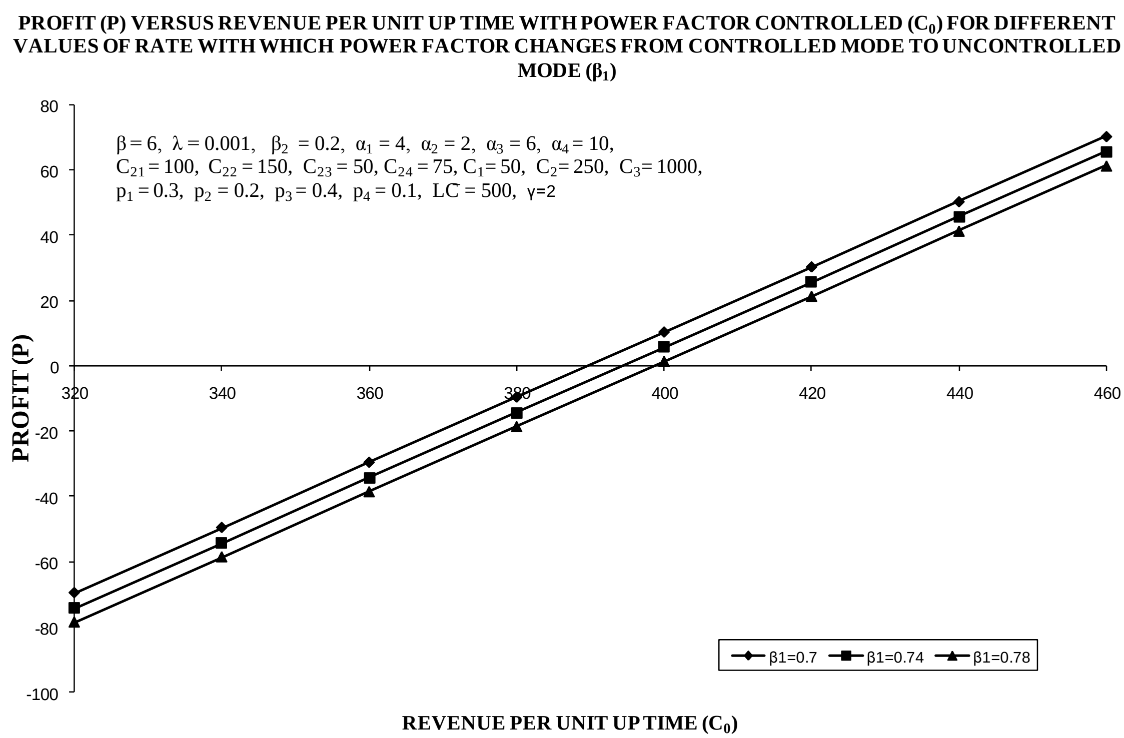

Fig. 2. Profit (P) versus revenue per unit up time with (C_0) for different values of the rate with which power factor changes from controlled mode to uncontrolled mode (β_1)

Fig. 2 shows the behavior of profit (P) with respect to revenue (C_0) per unit up time for different values of the rate with which power factor changes from controlled mode to uncontrolled mode (β_1). It can be concluded that the profit (P) increases with the increase in the value of C_0 and has higher values for lower rates of β_1. It can also be noticed that

(i) For β_1= 0.7 then P > or = or < 0 accordingly as C_0 > or = or < 390. So, for the model to be beneficial for β_1= 0.7, the C_0 should be > 390.

(ii) Similarly, for β_1 = 0.74 and β_1 = 0.78, the values for C_0 should be greater than 395 and 398 respectively.

(iii)It can be suggested to the user of the system to fix the prices in such a way so as to get the revenue per unit up time not less than that comes out to be at cut off point.

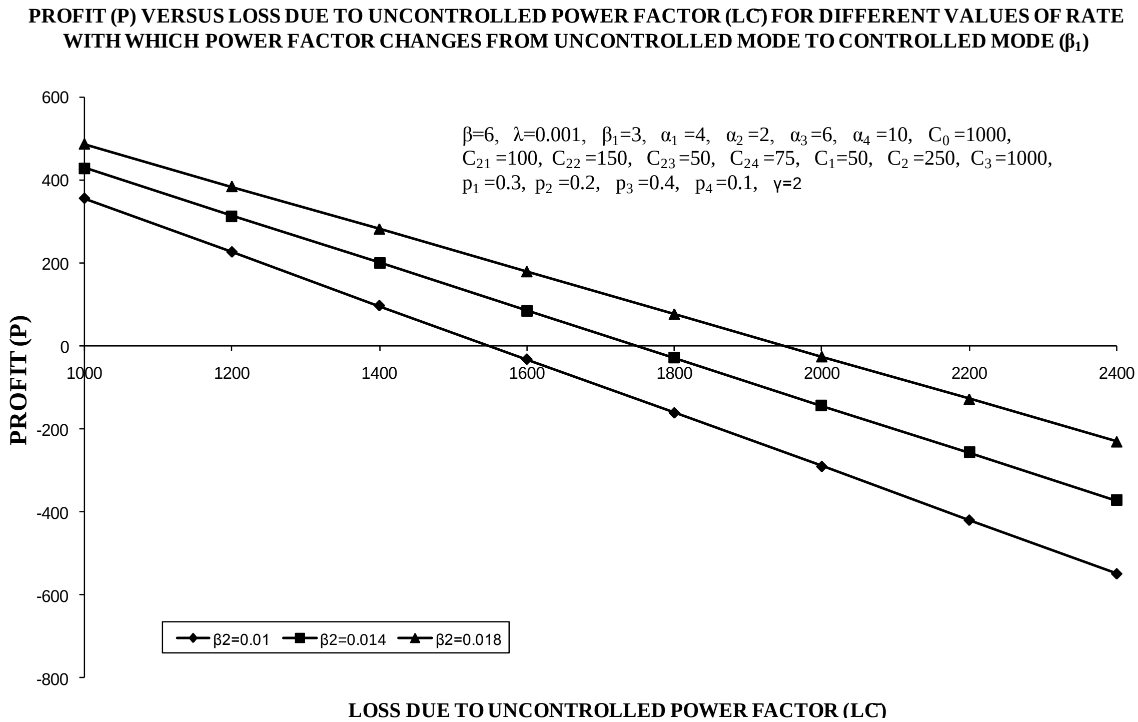

Fig. 3. Profit (P) versus loss due to uncontrolled power factor ($L\overline{C}$) for different values of the rate with which power factor changes from uncontrolled mode to controlled mode (β_2)

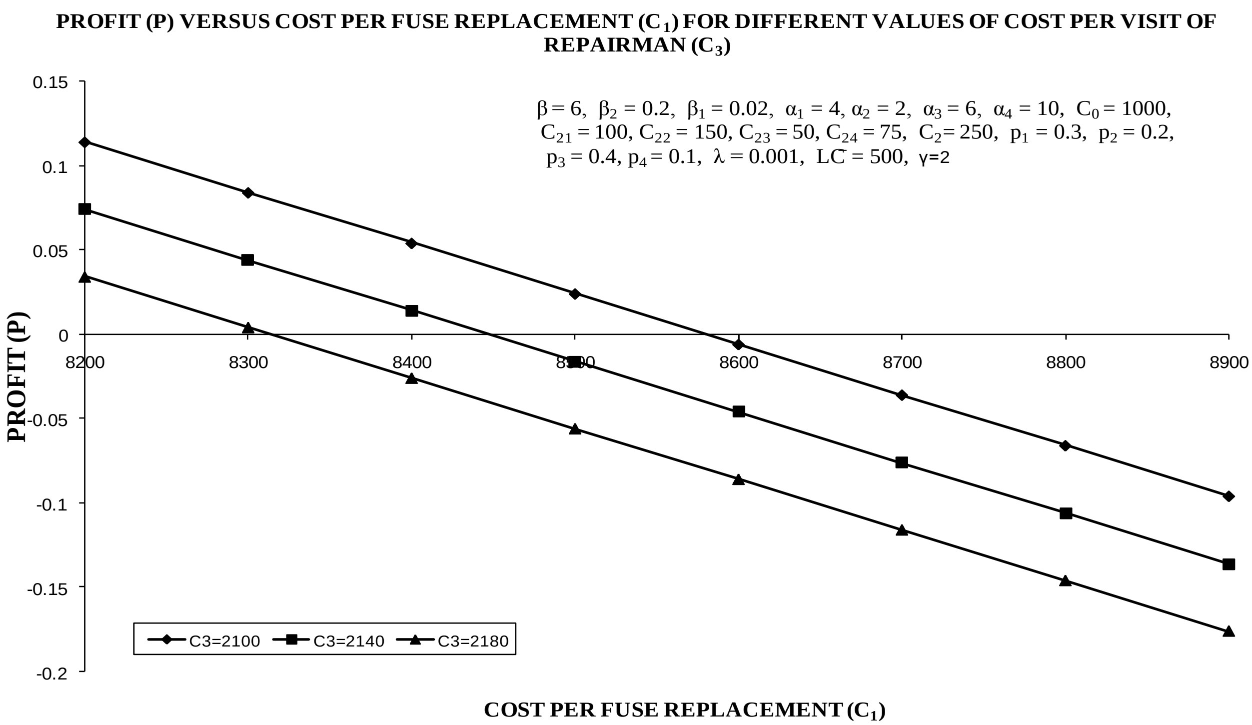

Fig. 4. Profit (P) versus cost per fuse replacement (C_1) for different values of the cost per visit of repairman (C_3)

Fig. 3 reveals the behavior of profit (P) with respect to loss ($L\overline{C}$) due to uncontrolled power factor for different values of the rate with which power factor changes from uncontrolled mode to controlled mode (β_2). It can be concluded that the profit decreases with the increase in the value of $L\overline{C}$ and has higher values for higher rates of β_2.

It can also be noticed that

(i) For $\beta_2 = 0.01$ then P < or = or > 0 accordingly as $L\bar{C}$ > or = or < 1550. So, for the model to be beneficial for $\beta_2 = 0.01$, $L\bar{C}$ should be < 1550.

(ii) Similarly, for $\beta_2 = 0.014$ and $\beta_2 = 0.018$, the values of $L\bar{C}$ should be less than 1750 and 1950 respectively.

(iii) It can be suggested to the user of the system to fix the prices in such a way so as to get the losses not more than that comes out to be at cut off point.

Fig. 4 depicts the behavior of profit (P) with respect to cost per fuse replacement (C_1) for different values of cost per visit of repairman (C_3). It is obvious from the graph that the profit decreases with the increase in the value of cost per fuse replacement (C_1). It can also be noticed that

(i) For C_3 =2100 then P < or = or > 0 accordingly as C_1 > or = or < 8590. So, for the model to be beneficial for $C_3 = 2100$, the C_1 should be < 8590.

(ii)Similarly, for $C_3 = 2140$ and $C_3 = 2180$, the value of cost per fuse replacement (C_1) should be less than 8450 and 8310

Many other graphs can be plotted by the users of such systems from the data/information given above to get the cut-off points to know the desired rates/costs for making the system more profitable. Also, from the cut-off points profit can be increased to fix the cost of the product by the manufacturers.

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