AMO - Advanced Modeling and Optimization, Volume 15, Number 2, 2013

A New Fuzzy Similarity Measure Based on Cotangent Function For Medical Diagnosis

Tian Maoying[1]

Department of Physiology, Shandong Coal Mining Health School,
Taozhuang, Shandong, 277011, China.

Abstract. Similarity measure has important application in medical diagnosis. In this paper, a new fuzzy cotangent similarity(FCS) measure for intuitionistic fuzzy sets is presented. Moreover, considering the importance of each element, the weighted fuzzy cotangent similarity(WFCS) is proposed. To alleviate the influence of unduly large(or small) deviations on the aggregation results, we also present the ordered weighted fyzzy cotangent similarity(OWFCS). Some comparison are made between the CS and some existing similarity measures. Finally, a medical diagnosis is given to verify the proposed similarity.

Keywords. Intuitionistic fuzzy sets; Fuzzy similarity measure; Medical diagnosis.

AMS(2000) Subject Classification: 94A17

1 Introduction

Intuitionistic fuzzy set(IFS) was initially proposed by Atanassov in 1986[1], and it is a useful tool to express the fuzziness and uncertainty in real life. Gau and Buehere[2] researched vague sets. Bustince and Burillo[3] pointed out that the notion of vague sets is the same as intuitionistic fuzzy sets.

Similarity measure is an important topic in the fuzzy set theory, and has been intensive investigated by many researchers. Many similarity measures have been proposed. Chen and Tan[4] proposed two similarity measures for measuring the degree of similarity between vague sets. Szmidt and Kacprzyk[5] introduced the Hamming distance and the Euclidean distance between IFSs which incorporate the membership function, the non-membership function, and the hesitant function of the IFS, and proposed some efficient similarity measures between IFSs based on the above distance measures. Based on the ideas of the TOPSIS, Hwang and Yoon[9] gave a new similarity measure to avoid the strong similarity between two IFSs when their distance is quite little. Then, utilizing the geometric distance, Xu[10] generalized the two distance measures proposed by Szmidt and Kacprzyk[5], and proposed a generalized distance measure

[1] 1This work is supported by Shandong Outstanding Young Scientists Award Project Under Grant BS2010SF031, and the Foundation of Zhejiang Provincial Education Department Under Grant Y201225096.

***AMO-Advanced Modeling and optimization. ISSN: 1841-4311**

and a generalized similarity measure, and the latter includes many similarity measures as special cases. Liang and Shi[6] also proposed some similarity measures for IFSs and discussed their relationships. Xia and Xu[7] presented a series of similarity measures for intuitionistic fuzzy values based on the intuitionistic fuzzy operators. Recently, Xu and Xia[8] extended the above similarity measures to the hesitant fuzzy sets(HFSs), and proposed a variety of similarity measures for HFSs. Quite recently, based on the cotangent function, Wang et al.[12] presented a new entropy measure for IFSs, and applied it to multiple attribute decision making with incomplete weight.

In this paper, based on the entropy proposed by Wang et al.[12], a new fuzzy cotangent similarity measure for IFSs is proposed. Moreover, considering the importance of each element, the weighted fuzzy cotangent similarity(WFCS) is proposed. To alleviate the influence of unduly large(or small) deviations on the aggregation results, we also present the ordered weighted fyzzy cotangent similarity(OWFCS). Some comparison are made between the CS and some existing similarity measures. Finally, a medical diagnosis is given to verify the proposed similarity.

The remainder of this paper is organized as follows: In Section 2, some related definitions are presented. In Section 3, the new fuzzy cotangent similarity measure and its weighted form, the ordered weighted form are introduced, and some comparisons are also made between the new similarity measure and the existing ones. In Section 4, we apply the proposed similarity measure to deal with the problem related to medical diagnosis. Some concluding remarks are made in Section 5.

2 Preliminaries

In 1986, Atanassov[1] gave the definition of IFS as follows.

Definition 2.1. [1] Let X be an universe of discourse, then the concept of intuitionistic fuzzy set(IFS) A on X is defined as:

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\}, \tag{1}$$

where $\mu_A(x)$ and $v_A(x)$ are mappings from X to the closed interval [0,1] such that $0 \leq \mu_A(x) \leq 1, 0 \leq v_A(x) \leq 1$ and $0 \leq \mu_A(x) + v_A(x) \leq 1, \forall x \in X$, and they are called the degrees of membership and nonmembership of element $x \in X$ to the set A, respectively.

For convenience of notations, we denote by IFS(X) the set of all the IFSs in X.

Let $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$, then it is usually called the intuitionistic fuzzy index of the element $x \in X$ to set A. From the definition of $\pi_A(x)$, we can see that it represents the degree of hesitation or indeterminacy of x to A. Then it is obviously that $0 \leq \pi_A(x) \leq 1$ for all $x \in X$.

Similarity measure has important applications in many areas, which is defined as follows:

Definition 2.2.[11] A real-valued function S : IFS$(X) \times$IFS$(X) \to [0, 1]$ is called a similarity measure on IFS(X), if it satisfies the following axiomatic requirements:

(S1) $0 \leq S(A,B) \leq 1$; (S2) $S(A,B) = 1$ iff $A = B$;

(S3) $S(A,B) = S(B,A)$; (S4) If $A \subseteq B \subseteq C$, then $S(A,C) \leq S(A,B) \wedge S(B,C)$.

3 Fuzzy cotangent similarity between IFSs

In [12], Wang et al. proposed the following entropy measure for IFSs:

Theorem 3.1.[12] For any intuitionistic fuzzy set $A \in$IFS(X), the $E_W(A)$ which is defined as follows

$$E(A) = \frac{1}{n}\sum_{i=1}^{n} \cot\big(\frac{1}{4}\pi + \frac{|\mu_A(x) - v_A(x)|\pi}{4(1+\pi_A(x)}\big)$$

is an entropy of IFSs.

For two given IFSs

$$A = \{\langle x, \mu_A(x), v_A(x), \rangle | x \in X\}, B = \{\langle x, \mu_B(x), v_B(x)\rangle | x \in X\},$$

similar to [11], we first define a new IFS as follows:

$$M(A,B) = \{\langle x, \mu_{M(A,B)}(x), v_{M(A,B)}(x)\rangle | x \in X\},$$

where

$$\mu_{M(A,B)}(x) = \frac{1}{2}(1 + |\mu_A(x_i) - \mu_B(x_i)| \vee |v_A(x_i) - v_B(x_i)|);$$
$$v_{M(A,B)}(x) = \frac{1}{2}(1 - |\mu_A(x_i) - \mu_B(x_i)| \vee |v_A(x_i) - v_B(x_i)|)$$

and $\vee$ stands for max operator. Then $M(A,B)$ is an IFS on X. The following theorem is Theorem 4 in [11].

Theorem 3.2.[11] Let E be an entropy of IFSs, for $A, B \in$ IFS(X), then $E(M(A,B))$ is a similarity measure of IFSs A and B.

Therefore, based on Theorem 3.1 and Theorem 3.2, we can propose the following fuzzy cotangent similarity measure for IFSs:

$$FCS(A,B) = \frac{1}{n}\sum_{i=1}^{n} \cot\big(\frac{\pi + \pi|\mu_A(x_i) - \mu_B(x_i)| \vee |v_A(x_i) - v_B(x_i)|}{4}\big). \tag{2}$$

Usually, the weight of each element $x_i \in X$ should be taken into account. Assume that the weight of the element $x_i \in X$ is $w_i (i = 1, 2, \ldots, n)$ with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, and so, we present the following weighted fuzzy cotangent similarity(WFCS) measure for IFSs.

$$WFCS(A,B) = \sum_{i=1}^{n} w_i \cot\big(\frac{\pi + \pi|\mu_A(x_i) - \mu_B(x_i)| \vee |v_A(x_i) - v_B(x_i)|}{4}\big). \tag{3}$$

To alleviate the influence of unduly large(or small) deviations on the aggregation results, we present the following ordered weighted fyzzy cotangent similarity(OWFCS) for IFSs.

$$OWFCS(A,B)=\sum_{i=1}^{n} w_i \cot\big(\frac{\pi+\pi|\mu_A(\sigma(x_i))-\mu_B(\sigma'(x_i))|\vee|v_A(\sigma(x_i))-v_B(\sigma'(x_i))|}{4}\big), \quad (4)$$

where $\langle\mu_A(\sigma(x_i)),v_A(\sigma(x_i))\rangle$ and $\langle\mu_A(\sigma'(x_i)),v_A(\sigma'(x_i))\rangle$ are the ith largest element in the fuzzy set A and B, respectively.

We review the following similarity measures for IFSs A and B as follows.

(a) [5] defined the similarity measure of IFSs A and B as follows:

$$S_{\mathrm{SK}}(A,B)=\frac{d(A,B)}{d(A,\bar{B})}, \quad (5)$$

where

$$d(A,B)=\frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i)-\mu_B(x_i)|+|v_A(x_i)-v_B(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|),$$

and $\bar{B}=\{\langle x,v_B(x),\mu_B(x)\rangle|x\in X\}$ is the complementary set of B.

(b) [9] defined the similarity measure of IFSs A and B as follows:

$$S_{\mathrm{HY}}(A,B)=\frac{d(A,\bar{B})}{d(A,B)+d(A,\bar{B})}. \quad (6)$$

(c) [10] defined the similarity measure of IFSs A and B as follows:

$$S_{\mathrm{X}}(A,B)=1-[\frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i)-\mu_B(x_i)|^{\lambda}+|v_A(x_i)-v_B(x_i)|^{\lambda}+|\pi_A(x_i)-\pi_B(x_i)|^{\lambda})]^{1/\lambda}, \quad (7)$$

where $\lambda\geq 1$.

Example 3.1. We consider two IFSs A_1, A_2 and compare the proposed new similarity measure Equation (2) with the existing ones (5)-(7)(set $\lambda=1$ in (7)). Assume that $X=\{x\}$ and two IFSs $A_1=\{\langle x,0.1,0.2\rangle\}$, $A_2=\{\langle x,0.2,0.2\rangle\}$. Then $S_{\mathrm{SK}}(A_1,A_2)=1$, $S_{\mathrm{HY}}(A_1,A_2)=0.5$, $S_{\mathrm{X}}(A_1,A_2)=0.9$, and $FCS(A_1,A_2)=0.8546$. The above results show that S_{SK} and S_{HY} are not efficient, because the result of the former is 1(too large), and the result of the latter is 0.5(too little). The results of S_{X} and our proposed similarity measures are reasonable.

4 Application to medical diagnosis

Let us consider a set of diagnosis $\tilde{A}=\{\tilde{A}_1,\tilde{A}_2,\tilde{A}_3\}$, where $\tilde{A}_1$: Viral fever, $\tilde{A}_2$: Malaria, $\tilde{A}_3$: Typhoid, and a set of symptoms $S=\{x_1,x_2,x_3\}$, where x_1: Temperature, x_2: Headache, x_3: Cough. Suppose a patient, with respect to all the symptoms, can be represented by the following intuitionistic fuzzy numbers:

$$\tilde{P}=\{\langle x_1,0.6,0.3\rangle,\langle x_2,0.3,0.2\rangle,\langle x_3,0.6,0.3\rangle\}.$$

Each diagnosis $\tilde{A}_i(i=1,2,3)$ can also be represented by intuitionistic fuzzy numbers with respect to all the symptoms as follows:

$$\tilde{A}_1 = \{\langle x_1, 0.4, 0.3\rangle, \langle x_2, 0.4, 0.2\rangle, \langle x_3, 0.4, 0.1\rangle\}.$$

$$\tilde{A}_2 = \{\langle x_1, 0.3, 0.3\rangle, \langle x_2, 0.5, 0.3\rangle, \langle x_3, 0.4, 0.1\rangle\}.$$

$$\tilde{A}_3 = \{\langle x_1, 0.7, 0.1\rangle, \langle x_2, 0.6, 0.1\rangle, \langle x_3, 0.3, 0.1\rangle\}.$$

Now, we want to classify the patient $\tilde{P}$ belong to the class $\tilde{A}_1$, or $\tilde{A}_2$, or $\tilde{A}_3$. According to the recognition principle of maximum degree of similarity between intuitionistic fuzzy numbers, the process of diagnosis $\tilde{A}_i(i=1,2,3)$ to patient $\tilde{P}$ is derived according to

$$k = \operatorname{argmin}\{FCS(\tilde{A}_i, \tilde{P})\}.$$

From (2), we can compute the cotangent similarity measure between $\tilde{A}_i(i=1,2,3)$ and $\tilde{P}$ as follows:

$$FCS(\tilde{A}_1, \tilde{P}) = 0.7691, FCS(\tilde{A}_2, \tilde{P}) = 0.6886, FCS(\tilde{A}_3, \tilde{P}) = 0.6507.$$

Then, we can assign the patient to diagnosis $\tilde{A}_1$(Viral fever) according to the recognition of principal.

5 Conclusion

In this paper, we have proposed a new fuzzy similarity measure for intuitionistic fuzzy sets based on the cotangent function. We have given its weighted form and the ordered weighted form. Finally, an illustrative example has been given to show the efficiency of the developed similarity measure.

References

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986), 87-96.

[2] W.L. Gau, D.J. Buehere, Vague sets. IEIE T Syst Man Cyb, 23 (1993), 610-614.

[3] H. Bustince, P. Burillo, Vague sets are intuitionistic fuzzy sets. Fuzzy Sets and Systems, 79 (1996), 403-405.

[4] S.M. Chen, J.M. Tan, Handling multi-criteria fuzzy decison-making problems based on vague sets. Fuzzy Sets and Systems, 67 (1994), 163-172.

[5] Szmidt E., Kacprzyk J., Entropy for intuitionistic fuzzy sets. Fuzzy Sets and Systems, 2001, 118: 467-477.

[6] Z. Liang, P. Shi, Similarity measure on intuitionistic fuzzy sets. Pattern Recog. Lett., 24 (2003), 2687-2693.

[7] M.M. Xia, Z.S. Xu, Some new similarity measures for intuitionistic fuzzy values and their application in group decision making. J Syst Sci Syst Engin, 19 (2010), 430-452.

[8] Z.S. Xu, M.M. Xia, Distance and similarity measures for hesitant fuzzy sets. Inform Sciences, 181 (2011), 2128-2138.

[9] C.L. Hwang, K. Yoon, Multiple attribute decision making: methods and applications. Berlin: Springer, 1981.

[10] Z.S. Xu, Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making. Fuzzy Optimization and Decision Making, 6(2), (2007), 109-121.

[11] Q.S. Zhang, S.Y. Jiang, Relationships Between Entropy and Similarity Measure of Interval-Valued Intuitionistic Fuzzy Sets. International Journal of Intelligent Systems, 25, (2010), 1121-1140.

[12] J.Q. Wang, P.Wang, Intuitionistic linguistic fuzzy multi-criteria decision-making method based on intuitionistic fuzzy entropy. Control and Decision, 27(11), (2012), 1694-1698.