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Some Aggregation Operators with Interval-valued Intuitionistic Trapezoidal Fuzzy Numbers and Their Application in Multiple Attribute Decision Making

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Abstract. This paper presents some aggregation operators, including interval-valued intuitionistic trapezoidal fuzzy ordered weighted averaging(IITFOWA) operator and interval-valued intuitionistic trapezoidal fuzzy hybrid averaging(IITFHA) operator. Some properties of these operators are also analyzed. Based on the IITFOWA and the IITFHA operators, an approach is developed for multiple attribute decision making problems with interval-valued intuitionistic trapezoidal fuzzy information. Finally, an illustrative example is given to verify the developed approach. The results shows the approach is simple, effective and easy to calculate.

Keywords. Multiple attribute decision making(MADM) problems; Interval-valued intuitionistic trapezoidal fuzzy numbers; Interval-valued intuitionistic trapezoidal fuzzy ordered weighted averaging(IITFOWA) operator; Interval-valued intuitionistic trapezoidal fuzzy hybrid averaging(IITFHA) operator

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1 Introduction

Since Atanassov(1986)[1] introduced the concept of intuitionistic fuzzy set(IFS), a lot of generalized forms have been proposed, among which there are interval-valued intuitionistic fuzzy sets(IVIFs)[2], triangular intuitionistic fuzzy numbers(TIFN)[3], intuitionistic trapezoidal fuzzy numbers(ITFN) and interval-valued intuitionistic trapezoidal fuzzy numbers(IITFN)[4]. From the references[5-7], we know that ITFN and IITFN are generalization of TIFN, and they extend TIFN theory from discrete sets to

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continuous sets. In this paper, we focus our attention on ITFN and IITFN theory.

Research on information aggregation methods with intuitionistic trapezoidal fuzzy numbers is active and some efficient aggregating operators are proposed. Wang[4] gave the definition of intuitionistic trapezoidal fuzzy number and interval intuitionistic trapezoidal fuzzy number. Wang and Zhang[5] gave the definition of expected values of ITFN and proposed a programming method of multi-criteria decision making based on intuitionistic trapezoidal fuzzy number with incomplete certain information. Wang and Zhang[6] gave the definition of the Hamming distance of ITFN and the intuitionistic trapezoidal fuzzy weighted arithmetic averaging(ITFWAA) operator, and proposed multi-criteria decision making method with incomplete certain information based on intuitionistic trapezoidal fuzzy numbers. Wan and Dong[7] presented new definitions of the expected values and the score function of ITFN from the geometric aspect, and proposed the intuitionistic trapezoidal fuzzy ordered weighted averaging(ITFOWA) operator and the intuitionistic trapezoidal fuzzy hybrid averaging(ITFHA) operator.

The above existing intuitionistic fuzzy aggregation techniques of ITFN have been applied widely in decision making. However, in these application, due to the increasing complexity of the social-economic environment and a lack of knowledge or data about the problem domains, the decision information may be provided with IITFN, which are characterized by membership functions and non-membership functions whose values are intervals, instead of real numbers. However, to our knowledge, not much research has been done in this respect. The followings are some of the research findings.

Wan[8] proposed some operational laws of IITFN and some related properties are researched, and the weighted arithmetic average operator and weighted geometric average operator for IITFN are given. He also gave the definition of the score function and accurate function of IITFN, then a multi-attribute decision making method is proposed based on interval-valued intuitionistic trapezoidal fuzzy numbers.

In this paper, we will further study the IITFN theory, and develop some new aggregating operators, including interval-valued intuitionistic trapezoidal fuzzy ordered weighted averaging(IITFOWA) operator and interval-valued intuitionistic trapezoidal fuzzy hybrid averaging(IITFHA) operator, and propose a new multi-attribute decision making method based on the new aggregating operators.

The rest of the paper is structured as follows. In the next section, we introduce some basic concepts related to IITFN and some operational laws of IITFN. Then we develop two new interval-valued intuitionistic trapezoidal fuzzy aggregating operators(IITFOWA and IITFHA) and study their desirable properties, such as commutativity, idempotency and monotonicity. A numerical example is presented in Section 3 to illustrate the effective of the new aggregating operators. Some conclusions are presented in the finial section.

2 Interval-valued intuitionistic trapezoidal fuzzy aggregating operators

In the following, we shall introduce some basic concepts related to interval-valued intuitionistic trapezoidal fuzzy numbers, and some aggregation operators with interval-valued intuitionistic trapezoidal fuzzy numbers are also developed.

Definition 1[4]. Let $\tilde{a}$ be an interval-valued intuitionistic trapezoidal fuzzy number, its membership is defined as:

$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{x-a}{b-a}\mu_{\tilde{a}}, a\le x<b;\\ \mu_{\tilde{a}}, b\le x\le c;\\ \dfrac{d-x}{d-c}\mu_{\tilde{a}}, c<x\le d;\\ 0, \text{otherwise}.\end{cases}\tag{1}$$

its non-membership function is

$$v_{\tilde{a}}(x)=\begin{cases}\dfrac{b-x+v_{\tilde{a}}(x-a_1)}{b-a_1}, a_1\le x<b;\\ v_{\tilde{a}}, b\le x\le c;\\ \dfrac{x-c+v_{\tilde{a}}(d_1-x)}{d_1-c}, c<x\le d_1;\\ 0, \text{otherwise}.\end{cases}\tag{2}$$

where $0\le\mu_{\tilde{a}}\le 1; 0\le v_{\tilde{a}}\le 1; a,b,c,d,a_1,d_1\in R$. Then $\tilde{a}=\langle([a,b,c,d];\mu_{\tilde{a}}),([a_1,b,c,d_1];v_{\tilde{a}})\rangle$ is called an intuitionistic trapezoidal fuzzy number(ITFN). If $b=c$, then ITFN reduces to TIFN. If $\mu_{\tilde{a}}, v_{\tilde{a}}\in \mathrm{int}(0,1)$, where $\mathrm{int}(0,1)$ denotes all closed subintervals of the interval $[0,1]$, then $\tilde{a}$ is called an interval-valued intuitionistic trapezoidal fuzzy number(IITFN).

In general, we have $[a,b,c,d]=[a_1,b,c,d_1]$ in IITFN $\tilde{a}=\langle([a,b,c,d];\mu_{\tilde{a}}),([a_1,b,c,d_1];v_{\tilde{a}})\rangle$. Thus for convenience, let $\tilde{a}=\langle[a,b,c,d];\mu_{\tilde{a}},v_{\tilde{a}}\rangle$. For each IITFN $\tilde{a}$, we call $\pi_{\tilde{a}}(x)=1-\mu_{\tilde{a}}(x)-v_{\tilde{a}}(x)$ the degree of indeterminacy of x to $\tilde{a}$. Let $\mu_{\tilde{a}}=[\underline{\mu},\overline{\mu}], v_{\tilde{a}}=[\underline{v},\overline{v}]$, then an IITFN $\tilde{a}$ can be denoted by $\tilde{a}=([a,b,c,d];[\underline{\mu},\overline{\mu}],[\underline{v},\overline{v}])$.

The following are some operational laws of IITFN.

Definition 2 [8] Let $\tilde{a}_i=([a_i,b_i,c_i,d_i];[\underline{\mu}_i,\overline{\mu}_i],[\underline{v}_i,\overline{v}_i])(i=1,2)$ be two IITFN, and $\lambda\ge 0$, then

(1) $\tilde{a}_1+\tilde{a}_2=([a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2];[\underline{\mu}_1+\underline{\mu}_2-\underline{\mu}_1\underline{\mu}_2,\overline{\mu}_1+\overline{\mu}_2-\overline{\mu}_1\overline{\mu}_2],[\underline{v}_1\underline{v}_2,\overline{v}_1\overline{v}_2])$.

(2) $\tilde{a}_1\tilde{a}_2=([a_1a_2,b_1b_2,c_1c_2,d_1d_2];[\underline{\mu}_1\underline{\mu}_2,\overline{\mu}_1\overline{\mu}_2],[\underline{v}_1+\underline{v}_2-\underline{v}_1\underline{v}_2,\overline{v}_1+\overline{v}_2-\overline{v}_1\overline{v}_2])$, where $a_1\ge 0, a_2\ge 0$.

(3) $\lambda\tilde{a}_1=([\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1];[1-(1-\underline{\mu}_1)^\lambda,1-(1-\overline{\mu}_1)^\lambda],[(\underline{v}_1)^\lambda,(\overline{v}_1)^\lambda])$, where $a_1\ge 0$.

(4) $(\tilde{a}_1)^\lambda=([(a_1)^\lambda,(b_1)^\lambda,(c_1)^\lambda,(d_1)^\lambda];[(\underline{\mu}_1)^\lambda,(\overline{\mu}_1)^\lambda],[1-(1-\underline{v}_1)^\lambda,1-(1-\overline{v}_1)^\lambda])$, where $a_1\ge 0$.

Definition 3 [8] The expected value E of a trapezoidal fuzzy number $[a, b, c, d]$ is defined as follows:

$$E = \frac{a+b+c+d}{4}. \tag{3}$$

For an IITFN $\tilde{a} = ([a, b, c, d]; [\underline{\mu}, \overline{\mu}], [\underline{v}, \overline{v}])$, its score function $S(\tilde{a})$ and accuracy function $H(\tilde{a})$ are defined as follows:

$$S(\tilde{a}) = \frac{1}{2} E(\underline{\mu} - \underline{v} + \overline{\mu} - \overline{v}), \tag{4}$$

and

$$H(\tilde{a}) = \frac{1}{2} E(\underline{\mu} + \underline{v} + \overline{\mu} + \overline{v}). \tag{5}$$

Obviously, the larger the value of $H(\tilde{a})$, the more the degree of accuracy of the IITFN $\tilde{a}$ is. Based on the score function S and the accuracy function H, in the following, we shall give an order relation between two IITFN, which is defined as follows:

Definition 4 Let $\tilde{a}_1$ and $\tilde{a}_2$ be two interval-valued intuitionistic trapezoidal fuzzy numbers; then

- If $S(\tilde{a}_1) < S(\tilde{a}_2)$, then $\tilde{a}_1$ is smaller then $\tilde{a}_2$, denoted by $\tilde{a}_1 < \tilde{a}_2$.
- If $S(\tilde{a}_1) = S(\tilde{a}_2)$, then

  (1) If $H(\tilde{a}_1) = H(\tilde{a}_2)$, then $\tilde{a}_2$ and $\tilde{a}_2$ represent the same information, denoted by $\tilde{a}_1 = \tilde{a}_2$.

  (2) If $H(\tilde{a}_1) < H(\tilde{a}_2)$, then $\tilde{a}_1$ is smaller than $\tilde{a}_2$, denoted by $\tilde{a}_1 < \tilde{a}_2$.

In [8], some aggregation operators with interval-valued intuitionistic trapezoidal fuzzy numbers are developed, and the following is one of these operators.

Definition 5 Let $\tilde{a}_j (j = 1, 2, \cdots, n)$ be a collection of interval-valued intuitionistic trapezoidal fuzzy numbers, and let IITFWAA: $Q^n \to Q$, if

$$\text{IITFWAA}_\omega(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) = \sum_{j=1}^n \omega_j \tilde{a}_j = ([\sum_{j=1}^n \omega_j a_j, \sum_{j=1}^n \omega_j b_j,$$

$$\sum_{j=1}^n \omega_j c_j, \sum_{j=1}^n \omega_j d_j], [1 - \prod_{j=1}^n (1 - \underline{\mu}_j)^{\omega_j}, 1 - \prod_{j=1}^n (1 - \overline{\mu}_j)^{\omega_j}], [\prod_{j=1}^n (\underline{v}_j)^{\omega_j}, \prod_{j=1}^n (\overline{v}_j)^{\omega_j}]), \tag{6}$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^\top$ be the weight vector of $\tilde{a}_j (j = 1, 2, \cdots, n)$, and $\omega_j \geq 0, \sum_{j=1}^n \omega_j = 1$, then IITFWAA is called the interval-valued intuitionistic trapezoidal fuzzy weighted arithmetic averaging(IITFWAA) operator. Especially, if $\omega = (1/n, 1/n, \cdots, 1/n)$, then IITFWAA operator is reduced to a interval-valued trapezoidal fuzzy arithmetic averaging(IITFAA) operator:

$$\text{IITFWA}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) = \sum_{j=1}^n \frac{\tilde{a}_j}{n}.$$

In the following, we will propose the interval-valued intuitionistic trapezoidal fuzzy order weighted averaging(IITFOWA) operator and the interval-valued intuitionistic trapezoidal fuzzy hybrid averaging(IITFHA) operator.

Definition 6 Let $\tilde{a}_j(j=1,2,\cdots,n)$ be a collection of interval-valued intuitionistic trapezoidal fuzzy numbers. An interval-valued intuitionistic trapezoidal fuzzy order weighted averaging(IITFOWA) operator of dimension n is a mapping IITFOWA: $Q^n \to Q$, that has an associated vector $w=(w_1,w_2,\cdots,w_n)^\top$ such that $\omega_j \geq 0, \sum_{j=1}^n \omega_j = 1$. Furthermore,

$$\text{IITFOWA}_w(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\sum_{j=1}^n w_j\tilde{a}_{\sigma(j)}=([\sum_{j=1}^n w_j a_{\sigma(j)},\sum_{j=1}^n w_j b_{\sigma(j)},$$

$$\sum_{j=1}^n w_j c_{\sigma(j)},\sum_{j=1}^n w_j d_{\sigma(j)}],[1-\prod_{j=1}^n(1-\underline{\mu}_{\sigma(j)})^{w_j},1-\prod_{j=1}^n(1-\overline{\mu}_{\sigma(j)})^{w_j}],[\prod_{j=1}^n(\underline{v}_{\sigma(j)})^{w_j},\prod_{j=1}^n(\overline{v}_{\sigma(j)})^{w_j}]), \quad (7)$$

where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $\tilde{a}_{\sigma(j-1)} \geq \tilde{a}_{\sigma(j)}$ for all $j=2,3,\cdots,n$.

The IITFOWA operator has the following properties:

Theorem 1. (Commutativity)

$$\text{IITFOWA}_w(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\text{IITFOWA}_w(\tilde{a}'_1,\tilde{a}'_2,\cdots,\tilde{a}'_n),$$

where $(\tilde{a}'_1,\tilde{a}'_2,\cdots,\tilde{a}'_n)$ is any permutation of $(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)$.

Theorem 2. (Idempotency) If $\tilde{a}_j=\tilde{a}(j=1,2,\cdots,n)$, then

$$\text{IITFOWA}_w(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\tilde{a}.$$

Theorem 3. (Monotonicity) If $\tilde{a}_j \leq \tilde{a}'_j(j=1,2,\cdots,n)$, then

$$\text{IITFOWA}_w(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n) \leq \text{IITFOWA}_w(\tilde{a}'_1,\tilde{a}'_2,\cdots,\tilde{a}'_n).$$

The proof of above three theorems is quite easy, thus is omitted.

From Definitions 5 and 6, we know that the IITFWAA operator weights only the interval-valued intuitionistic trapezoidal fuzzy numbers, while the IITFOWA operator weights only the ordered positions of the interval-valued intuitionistic trapezoidal fuzzy numbers instead of weighting the interval-valued intuitionistic trapezoidal fuzzy numbers themselves. Therefore, weights represent different aspects in both the IITFWAA and IITFOWA operators. However, both the operators consider only one of them. To solve this drawback, motivated by the idea of combining the weighted average and the OWA operator[9], in the following we shall propose an interval-valued intuitionistic trapezoidal fuzzy hybrid aggregation(IITFHA) operator which weights both the given interval-valued intuitionistic trapezoidal fuzzy numbers and their ordered position.

Definition 7 An interval-valued intuitionistic trapezoidal fuzzy hybrid aggregating(IITFHA) operator of dimension n is a mapping IITFHA: $Q^n \to Q$, that has an associated vector $w=(w_1,w_2,\cdots,w_n)^\top$

such that $\omega_j \geq 0, \sum_{j=1}^{n} \omega_j = 1$. Furthermore,

$$\mathrm{IITFHA}_{\omega,w}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) = \sum_{j=1}^{n} w_j \dot{\tilde{a}}_{\sigma(j)} = ([\sum_{j=1}^{n} w_j \dot{a}_{\sigma(j)}, \sum_{j=1}^{n} w_j \dot{b}_{\sigma(j)},$$

$$\sum_{j=1}^{n} w_j \dot{c}_{\sigma(j)}, \sum_{j=1}^{n} w_j \dot{d}_{\sigma(j)}], [1 - \prod_{j=1}^{n}(1 - \dot{\underline{\mu}}_{\sigma(j)})^{w_j}, 1 - \prod_{j=1}^{n}(1 - \dot{\bar{\mu}}_{\sigma(j)})^{w_j}], [\prod_{j=1}^{n}(\dot{\underline{v}}_{\sigma(j)})^{w_j}, \prod_{j=1}^{n}(\dot{\bar{v}}_{\sigma(j)})^{w_j}]), \quad (8)$$

where $\dot{\tilde{a}}_{\sigma(j)}$ is the jth largest of the weighted interval-valued intuitionistic trapezoidal fuzzy numbers $\dot{\tilde{a}}_j(\dot{\tilde{a}}_j = n\omega_j\tilde{a}_j, j = 1, 2, \cdots, n)$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^\top$ is the weight vector of $\tilde{a}_j(j = 1, 2, \cdots, n)$, $\omega_j \geq 0, \sum_{j=1}^{n} \omega_j = 1$, and n is the balancing coefficient.

Theorem 4. The IITFWAA operator is a special case of the IITFHA operator.

Proof. Let $w = (1/n, 1/n, \cdots, 1/n)^\top$; then

$$\begin{aligned} & \mathrm{IITFHA}_{\omega,w}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) \\ = & \frac{1}{n}(\dot{\tilde{a}}_1 + \dot{\tilde{a}}_2 + \cdots + \dot{\tilde{a}}_n) \\ = & \omega_1\tilde{a}_1 + \omega_2\tilde{a}_2 + \cdots + \omega_n\tilde{a}_n \\ = & \mathrm{IITFWAA}_{\omega}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n). \end{aligned}$$

This completes the proof of Theorem 4.

Theorem 5. The IITFOWA operator is a special case of the IITFHA operator.

Proof. Let $\omega = (1/n, 1/n, \cdots, 1/n)^\top$; then $\dot{\tilde{a}}_j = \tilde{a}_j, j = 1, 2, \cdots, n$, thus

$$\begin{aligned} & \mathrm{IITFHA}_{\omega,w}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) \\ = & w_1\dot{\tilde{a}}_{\sigma(1)} + w_2\dot{\tilde{a}}_{\sigma(2)} + \cdots + w_n\dot{\tilde{a}}_{\sigma(n)} \\ = & w_1\tilde{a}_1 + w_2\tilde{a}_2 + \cdots + w_n\tilde{a}_n \\ = & \mathrm{IITFOWA}_{w}(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n). \end{aligned}$$

This completes the proof of Theorem 5.

Clearly, from Theorem 4 and 5, we know that the IITFHA operator generalizes both the IITFWAA and IITFOWA operators and reflects the importance degrees of both the given interval-valued intuitionistic trapezoidal fuzzy arguments and their ordered positions.

3 Illustrative example

In this section, a multiple attribute decision making problem is concerned with a customer who intends to buy a air conditioner. Three type of air conditioners are available(A_1, A_2, A_3). The customer takes into account the following four attributes: 1. G_1 is the quality; 2. G_2 is the design; 3. G_3 is the price; 4. G_4

is the level of after-sale service. The three possible suppliers $A_i(i = 1, 2, 3)$ are to be evaluated using the interval-valued trapezoidal intuitionistic fuzzy numbers under the above four attributes (whose weighting vector $\omega = (0.2, 0.1, 0.3, 0.4)^\top$). The decision matrix is listed in the following matrices $\tilde{D} = (\tilde{d}_{ij})_{3\times 4}$ as follows:

$$D = \begin{bmatrix} ([0.1, 0.2, 0.3, 0.4]; [0.5, 0.6], [0.2, 0.3]) & ([0.2, 0.3, 0.4, 0.5]; [0.4, 0.7], [0.2, 0.3]) \\ ([0.4, 0.5, 0.6, 0.7]; [0.3, 0.5], [0.2, 0.4]) & ([0.1, 0.3, 0.5, 0.6]; [0.2, 0.4], [0.4, 0.5]) \\ ([0.2, 0.4, 0.5, 0.8]; [0.5, 0.7], [0.1, 0.2]) & ([0.2, 0.3, 0.4, 0.5]; [0.1, 0.4], [0.5, 0.6]) \end{bmatrix.$$

$$\begin{matrix} ([0.3, 0.4, 0.5, 0.6]; [0.3, 0.6], [0.3, 0.4]) & ([0.1, 0.3, 0.5, 0.6]; [0.4, 0.5], [0.2, 0.4]) \\ ([0.2, 0.4, 0.6, 0.7]; [0.4, 0.7], [0.2, 0.3]) & ([0.3, 0.5, 0.6, 0.8]; [0.0, 0.3], [0.5, 0.7]) \\ ([0.1, 0.2, 0.4, 0.5]; [0.6, 0.8], [0.1, 0.2]) & ([0.1, 0.2, 0.4, 0.6]; [0.2, 0.4], [0.3, 0.6]) \end{matrix}\right].$$

Step 1. Firstly, utilizing the decision information d_{ij} given in the matrix D, the attribute weighting vector $\omega = (0.2, 0.1, 0.3, 0.4)^\top$, and multiple the balance coefficient $n = 4$, we can get the weighted attribute values $\dot{d}_{ij} = 4\omega_j d_{ij}(i = 1, 2, 3; j = 1, 2, 3, 4)$, which compose the following weighted decision matrix $\dot{D}$.

$$\dot{D} = \begin{bmatrix} ([0.08, 0.16, 0.24, 0.32]; [0.43, 0.52], [0.28, 0.38]) & ([0.08, 0.12, 0.16, 0.20]; [0.18, 0.38], [0.53, 0.62]) \\ ([0.32, 0.40, 0.48, 0.56]; [0.25, 0.43], [0.28, 0.48]) & ([0.04, 0.12, 0.20, 0.24]; [0.09, 0.19], [0.69, 0.76]) \\ ([0.16, 0.32, 0.40, 0.64]; [0.43, 0.62], [0.16, 0.28]) & ([0.08, 0.12, 0.16, 0.20], [0.04, 0.18], [0.76, 0.82]) \end{bmatrix.$$

$$\begin{matrix} ([0.36, 0.48, 0.60, 0.72]; [0.35, 0.67], [0.24, 0.33]) & ([0.16, 0.48, 0.80, 0.96]; [0.56, 0.67], [0.08, 0.23]) \\ ([0.24, 0.48, 0.72, 0.84]; [0.46, 0.76], [0.15, 0.24]) & ([0.48, 0.80, 0.96, 1.28]; [0.00, 0.43], [0.33, 0.57]) \\ ([0.12, 0.24, 0.48, 0.60]; [0.67, 0.86], [0.06, 0.15]) & ([0.16, 0.32, 0.64, 0.96]; [0.30, 0.56], [0.15, 0.44]) \end{matrix}\right].$$

Step 2. Let the weighted vector $w = (0.2, 0.4, 0.3, 0.1)^\top$. Utilize the IITFHA operator to derive the collective overall preference interval-valued trapezoidal intuitionistic fuzzy values $\tilde{r}_i(i = 1, 2, 3)$ of the alternative $A_i(i = 1, 2, 3)$:

$$\tilde{r}_1 = ([0.208, 0.348, 0.488, 0.596]; [0.408, 0.607], [0.310, 0.447]).$$

$$\tilde{r}_2 = ([0.208, 0.344, 0.464, 0.560]; [0.219, 0.448], [0.360, 0.511]).$$

$$\tilde{r}_3 = ([0.144, 0.284, 0.464, 0.684]; [0.427, 0.648], [0.256, 0.433]).$$

Step 3. Calculate the scores $S(\tilde{r}_i)(i = 1, 2, 3)$ of the collective overall preference values $\tilde{r}_i(i = 1, 2, 3)$:

$$S(\tilde{r}_1) = 0.0529, S(\tilde{r}_2) = -0.0402, S(\tilde{r}_3) = 0.0760.$$

Step 4. Rank all the alternatives $A_i(i = 1, 2, 3)$ in accordance with the scores $S(\tilde{r}_i)(i = 1, 2, 3)$ of the overall preference values $\tilde{r}_i(i = 1, 2, 3)$: $A_3 \succ A_1 \succ A_2$, and thus the most desirable alternative is A_3.

4 Conclusion

In this paper, we have proposed some interval-valued intuitionistic trapezoidal fuzzy averaging operators. We have studied some desirable properties of the proposed operators, such as commutativity, idempotency and monotonicity, and applied the IITFHA operator to decision making with interval-valued intuitionistic trapezoidal fuzzy information. Finally, an illustrative example has been given to show the developed operators.

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