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$\{e_m, v_k\}$-Cyclic Path Covering Number of Digraphs

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Abstract

In this paper we state a definition of v_k - Cyclic path covering to find the Cyclic path covering number of a digraph in which some of the vertex have privilege of being interior vertex of more than one path and we find v_k - Cyclic path Covering Number of simple digraph and we defined $\{ v_{k1}, v_{k2}, v_{k3} \ldots v_{km}\}$- Cyclic path covering and have found the $\{ v_{k1}, v_{k2}, v_{k3} \ldots v_{km}\}$- Cyclic path covering number of such coverings. Also we defined e_m - Cyclic path Covering Number and finally found the $\{e_m, v_k\}$-Cyclic path covering number of any digraph.

Keywords: Cyclic path cover, Cyclic path covering number, v_k - Cyclic path cover, e_m- Cyclic path Covering Number.

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1. Introduction

If we consider the context of mobile traffic streams in a city road network, every vehicle driver would wish to encounter as many green signals at junctions as possible so as to minimize fuel consumption and travel time as also maximize unhindered distance so travelled. This necessitates relative optimization of signal times at various junctions in the road network in order to provide maximum possible average length mobile traffic streams-one may expect such an optimal traffic flow in the network to occur with maximum probability when the number mobile traffic streams in the network is minimum. Thus for any finite digraph G the parameter $\psi(G) = \min\{|\psi| : \psi \in \mathcal{G}_G\}$, where $\mathcal{G}_G$ denotes the set of all distinct Cyclic path covers of G, is an ideal limit for any reasonably realistic measure of "mobility" of traffic flow in the city road network with G as the underlying digraph. In our recent development of city in many places there are so many flyovers and over bridges at the road crossings to minimise the traffic signals. This motivates the development of v_k - Cyclic path cover which is being discussed elaborately in this paper. The concept of path decomposition and path covering number of a digraph was introduced by Harary[**1**]. The preliminary results on this paper were obtained by Harary and Schwenk[**2**], Peroche[**3**] and Stanton etal.[**4**], [**5**]. Here the paths may intersect any number times only at the specified vertex of G. To get non-intersecting paths we impose some extra conditions in the definition of Path Covering. We follow the notations and terminology of Harary [**6**],[**7**]. All digraph considered in this paper are assumed to be connected digraphs without isolated points. Let D = (V,E) be a digraph. We denote the number of vertices if D by *n* and the number of edges in D by *e*.

Definition 1.1. A Path covering of a digraph D is a collection ψ of (not necessarily open) paths in D such that every edge of D is in exactly one path in ψ and the minimum cardinality taken over all Path covers ψ of D with | ψ | = η is called the Path covering number of D.

Definition 1.2. [**8**],[**9**]

A Cyclic Path covering of a digraph D is a collection Γ of (not necessarily open) paths in D whose union is D satisfying the conditions for distinct paths P_i and P_j with terminal vertices u, v and w, z respectively,

$$P_i \text{È} P_j = \begin{cases} A, \text{ A is the subset of the set } \{u, v, w, z\} \\ \phi, \text{ if } P_i \text{ and } P_j \text{ are cyclic.} \end{cases}$$

Definition 1.3. [**8**],[**9**],[10].[11]
The Cyclic Path covering number of D is defined to be the minimum cardinality taken over all Cyclic Path covers of D. Any Cyclic Path cover Γ of G with |Γ | = γ is called a minimum Cyclic Path cover of D.

2. v_k - Cyclic Path Cover

Definition 2.1. [11]
A v_k-Cyclic Path covering of a digraph D is a collection Γ_k of paths (not necessarily open) in D whose union is D satisfying the conditions for distinct paths P_i and P_j with terminal vertices u,v and w,z respectively,

$$P_i \text{È} P_j = \begin{cases} A, \text{ A is the subset of the set } \{u, v, w, z\} \\ \phi, \text{ if } P_i \text{ and } P_j \text{ are cyclic.} \\ v_k, \text{ if } v_k \text{ is the internal vertex of both } P_i \text{ and } P_j \end{cases}$$

Definition 2.2 [11]
The v_k- Cyclic path covering number γ_k of D is defined to be the minimum cardinality taken over all v_k-Cyclic path covers of D.

Example 2.3.
Consider the following digraph (Figure.1). The v_k - Cyclic path covers are

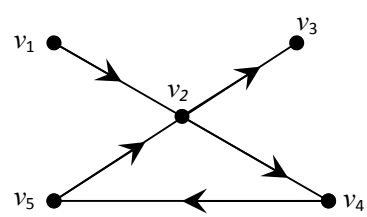

Figure. 1

Here the path covers are

$\psi_1 = \{v_1v_2v_4v_5,\ v_5v_2v_3\}$,
$\psi_2 = \{v_1v_2v_3,\ v_2v_4v_5\}$,
$\psi_3 = \{v_1v_2v_4,\ v_4v_5,\ v_5v_2v_3\}$
$\psi_4 = \{v_1v_2,\ v_2v_4,\ v_4v_5,\ v_5v_2,\ v_2v_3\}$
Then the path covering number = 2.

The v_2-Cyclic Path cover are $\{v_1v_2v_3, v_2v_4v_5v_2\}$, $\{v_1v_2v_4v_5v_2v_3\}$. Therefore the v_2- cyclic path covering number is $\gamma_2 = 1$.

Example 2.4. Consider the digraph D given in the following figure .2..

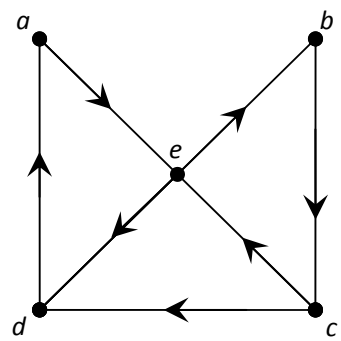

Figure. 2

Here the path covers are $\psi_1 = \{aebcda, ced\}$,

$\psi_2 = \{aeda, cebc, cd\}$,
$\psi_3 = \{aebcda, ced\}$
$\psi_4 = \{ae, eb, bc, ce, cd, ed, da\}$
Then the path covering number = 2.

The Cyclic path covers are $\{ae, eb, bc, ce, cd, ed, da\}$, $\{ebcdae, ce, ed\}$, $\{ebce, cdae, ed\}$, $\{edae, ebc, ce, cd\}$, $\{edae, ebcd, ce\}$, $\{dae, ebc, ced, cd\}$. Here the Cyclic path covering number $\gamma = 3$.

The v_3-cyclic path cover is $\{aebcda, ced\}$. Therefore the v_3- cyclic path covering number is $\gamma_3 = 2$.

Lemma 2.5. In a digraph D for a vertex v_k with degree 1 or 2 the v_k Cyclic path covering number is same as that of Cyclic path cover.

Proof: As there is almost one and only path is possible through the vertex v_k , there is no change in the number of directed paths to cover the digraph D. With reference to the definitions 1.2 and 1.3 we have the v_k -Cyclic path covering number is same as that of Cyclic path covering number.

Lemma 2.6. In a digraph D for a vertex of degree 3 the v_k - path covering number is γ.

Proof: Let v_k be a vertex of degree 3 on a digraph D. There is only one path through v_k . Further there is no chance for more than one path through v_k. Therefore $\gamma_k = \gamma$.

Lemma 2.7. In a simple digraph D with a vertex v_k with $d^-(v_k) = d^+(v_k) = 2$ the v_k -Cyclic path cover number is $\gamma_k = \gamma - 1$.

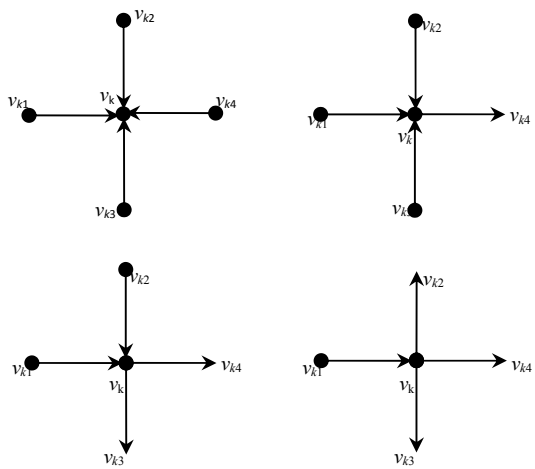

Figure. 3

Proof: Let v_k be a vertex with total degree 4. Then there must be 4 adjacent vertices say v_{k1}, v_{k2}, v_{k3}, v_{k4} and there are 5 different possible degree configuration at v_k. They are (i) $d^-(v_k) = 4$ and $d^+(v_k) = 0$ (ii) $d^-(v_k) = 3$ and $d^+(v_k) = 1$ (iii) $d^-(v_k) = 2$ and $d^+(v_k) = 2$, (iv) $d^-(v_k) = 1$ and $d^+(v_k) = 3$ and (v) $d^-(v_k) = 0$ and $d^+(v_k) = 4$. As more than one directed path is allowed through v_k, the v_k Cyclic path cover ψ_k, allows two different directed paths through the same vertex v_k as interior vertex. This is possible only in the case (iii). Actually, the Cyclic path cover ψ admits only one path P_1 through the vertex v_k as internal vertex. In the case (iii), there can be two directed paths in ψ_k in which v_k is the terminal vertex of two directed paths. These two paths create only one path P_2 in which v_k is internal vertex and $P_1 \neq P_2$. The remaining vertices in D follows the conditions that are same in ψ an ψ_k. Thus, we have ψ_k with one path less than the number of paths in ψ. Thus we have $\gamma_k = \gamma - 1$.

Lemma 2.8. In a simple digraph D with a vertex v_k with $d^-(v_k) = 2$ and $d^+(v_k) = 3$ or $d^-(v_k) = 3$ and $d^+(v_k) = 2$, the v_k-Cyclic path cover number is $\gamma_k = \gamma - 1$.

Proof: Let v_k is a vertex with degree 5. Then there will be 5 adjacent vertices say v_{k1}, v_{k2}, v_{k3}, v_{k4} and v_{k5} and there are 6 different possible degree configuration at v_k. They are (i) $d^-(v_k) = 5$ and $d^+(v_k) = 0$ (ii) $d^-(v_k) = 4$ and $d^+(v_k) = 1$ (iii) $d^-(v_k) = 3$ and $d^+(v_k) = 2$, (iv) $d^-(v_k) = 2$ and $d^+(v_k) = 3$, (v) $d^-(v_k) = 1$ and $d^+(v_k) = 4$ and (vi) $d^-(v_k) = 0$ and $d^+(v_k) = 5$. As more than one path allowed through v_k, the v_k-Cyclic path cover ψ_k, allows two

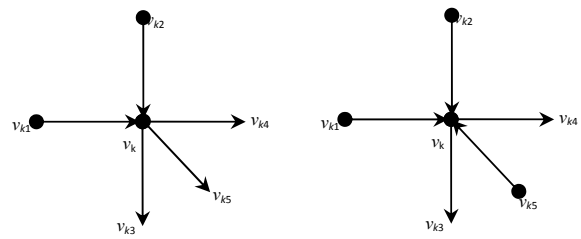

Figure. 4

different paths through the same vertex v_k as interior vertex. The Cyclic path cover ψ admits only one path P_1 through the vertex v_k as internal vertex. There are three paths in ψ in which v_k is one of terminal vertex in their respective paths. Any two of these paths can be joined at v_k and create only one path P_2 in which v_k is internal vertex and $P_1 \neq P_2$. The remaining vertices in D follows the conditions as they are in ψ and $\psi_{k.}$. Thus in $\psi_{k.}$ we have one path less than the number of paths in ψ .

Therefore we have $\gamma_k = \gamma - 1$.

Lemma 2.9. In a simple digraph D with a vertex v_k with $d^-(v_k) = 3$ and $d^+(v_k) = 3$, the v_k -Cyclic path cover number is $\gamma_k = \gamma - 2$.

Proof: Let v_k be a vertex with degree 6. Then there must be 6 adjacent vertices say v_{k1}, v_{k2}, v_{k3}, v_{k4}, v_{k5} and v_{k6} and there are 7 different possible degree configuration at v_k . They are (i) $d^-(v_k) = 6$ and $d^+(v_k) = 0$ (ii) $d^-(v_k) = 5$ and $d^+(v_k) = 1$ (iii) $d^-(v_k) = 4$ and $d^+(v_k) = 2$, (iv) $d^-(v_k) = 3$ and $d^+(v_k) = 3$, (v) $d^-(v_k) = 2$ and $d^+(v_k) = 4$, (vi) $d^-(v_k) = 1$ and $d^+(v_k) = 5$ and (vii) $d^-(v_k) = 0$ and $d^+(v_k) = 6$. *A*s more than one path are allowed through v_k , the v_k Cyclic path cover ψ_k , allows three different paths through the same vertex v_k as interior vertex. The Cyclic path cover ψ admits only one path P_1 through the vertex v_k as internal vertex. There are four directed paths in ψ_k in which v_k is one of terminal vertex in each directed paths. These four directed paths combined at v_k to create two different directed paths P_2 and P_3 different from P_1 in which v_k is internal vertex. The remaining vertices in D follows the conditions as they are in ψ and $\psi_{k..}$

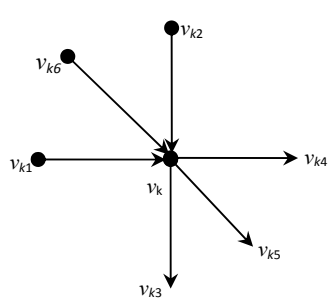

Figure. 5

Thus in $\psi_{k.}$ we have two paths less than the number of paths in ψ .(ie) $\gamma_k = \gamma - 2$.

Theorem 2.10. Let v_k be the vertex in D with $d^-(v_k) = m$ and $d^+(v_k) = n$. Then the v_k - Cyclic path covering number $\gamma_k = \gamma - \left[\min\left\{d^-(v_k), d^+(v_k)\right\} - 1\right]$, where γ is the Cyclic path covering number of D.

Proof: The vertex v_k may have the degree 1,2,3…. If v_k has degree 1 or 2 then the result is obvious by using the lemma 2.5 . If the degree of the vertex is 3 then we have $\gamma_k = \gamma$ (lemma 2.6). If the vertex v_k is such that $d^-(v_k) = d^+(v_k) = 2$ the v_k -Cyclic path cover number is $\gamma_k = \gamma - 1$(lemma 2.7). For the vertex v_k with total degree 5 such that with $d^-(v_k) = 2$ and $d^+(v_k) = 3$ or $d^-(v_k) = 3$ and $d^+(v_k) = 2$, the v_k -Cyclic path cover number is $\gamma_k = \gamma - 1$.(lemma 2.8) . Even if the vertex v_k with degree 6 such that $d^-(v_k) = 3$ and $d^+(v_k) = 3$, the v_k -Cyclic path cover number is $\gamma_k = \gamma - 2$.(lemma 2.9). Thus by considering all these results by induction we have the v_k - Cyclic path covering number $\gamma_k = \gamma - \left[\min\left\{d^-(v_k), d^+(v_k)\right\} - 1\right]$, where γ is the Cyclic path covering number of D.

Definition 2.11. Let $v_{k1}, v_{k2}, v_{k3} \ldots v_{km}$ are some of the vertices of D. Then $\{v_{k1}, v_{k2}, v_{k3} \ldots v_{km}\}$-Cyclic path covering of the digraph D is a collection $\Gamma_{vk_1, vk_2, vk_3 \ldots vk_m}$ of (not necessarily open) paths in D satisfying the conditions for distinct paths P_i and P_j with terminal vertices u, v and w, z respectively,

$$P_i \grave{E} P_j = \begin{cases} A, \text{ A is the subset of the set } \{u, v, w, z\} \\ \phi, \text{ if } P_i \text{ and } P_j \text{ are cyclic.} \\ v_{k1}, v_{k2}, \ldots v_{km} \text{ if } v_{ki} \text{ is the internal vertex} \\ \quad \text{of some } P_{ik} \text{ and } P_{jk}. \end{cases}$$

Definition 2.12. The $\left\{v_{k_1}, v_{k_2}, v_{k_3}, \ldots v_{k_m}\right\}$- Cyclic path covering number $\gamma_{v_{k1}, v_{k2}, v_{k3}, \ldots v_{km}}$ of D is defined to be the minimum cardinality taken over all $\{v_{k1}, v_{k2}, v_{k3} \ldots v_{km}\}$- Cyclic path covers of D.

Corollary 2.13. Let $v_{k1}, v_{k2}, v_{k3} \ldots v_{km}$ are some of the vertices of D. Then $\left\{v_{k_1}, v_{k_2}, v_{k_3}, \ldots v_{k_m}\right\}$- Cyclic path covering number $\gamma_{v_{k1}, v_{k2}, v_{k3}, \ldots v_{km}}$ of the digraph D is

$$\gamma_{v_{k_1}, v_{k_2}, v_{k_3} \ldots v_{k_m}} = \gamma - \sum_{i=1}^{m} \left[min\left\{d^-(v_{k_i}), d^+(v_{k_i})\right\} - 1\right]$$

3. e_m - Cyclic Path Cover

Definition 3.1. [11],[12],[13],[14]

A e_m - Cyclic directed path cover of a digraph D is a collection ψ_m of paths (not necessarily open) in D whose union is D satisfying the following conditions for distinct paths P_i and P_j with terminal vertices u, v and w, z respectively,

$$\text{Pi È Pj} = \begin{cases} \text{A, A is the subset of the set } \{u, v, w, z\} \\ \phi, \text{ if } P_i \text{ and } P_j \text{ are cyclic.} \\ e_m, \text{ if } e_m \text{ is one of the arc in both } P_i \text{ and } P_j \end{cases}$$

Definition 3.2. [11],[12],[13],[14]

The $\boldsymbol{e_m}$ - Cyclic path covering number γ_m of D is defined to be the minimum cardinality taken over all $\boldsymbol{e_m}$ -Cyclic directed path covers of D.

Theorem 3.3. For a digraph D with edge e_m the $\boldsymbol{e_m}$ - Cyclic path covering number is $\gamma_m = \gamma - \left[\min\left\{d^+(v_m^t),\ d^-(v_m^h)\right\} - 1\right]$ where γ is the Cyclic path covering number of D and v_m^t and v_m^h are the tail and head vertex of the arc e_m.

Proof: let ψ_m be the $\boldsymbol{e_m}$ - Cyclic path covering of the digraph D and γ_m be the $\boldsymbol{e_m}$ Cyclic path covering number of D. Here e_m may be in any one of the following cases:
1) e_m may be the arc in a directed path P of length more than 1.
2) e_m may be a directed path of length 1.
Case (1): As e_m is an arc in P and let v_m^t and v_m^h be the tail and head vertices of the arc e_m.
Now arc e_m may be in one of the following 2 cases.
i) e_m is the intermediate arc in P.
ii) e_m is the terminal arc in P.
Case (ii) Then e_m has the following cases.
a) Degree of one vertex is 1.
Then there is nothing to prove.
b) the indegree and outdegree of both v_m^t and v_m^h equal to 1.
Now also there is nothing to prove.
c) out degree of v_m^h one end vertex is 2.
Even in this case there is no change in the number of path coverings.
d) the in degree and out degree of both v_m^t and v_m^h equal to 2.
As e_m is the intermediate arc and v_m^t and v_m^h are the start and end vertices of e_m, there must be two paths P_1 and P_2 whose end and start vertices are v_m^t and v_m^h respectively.

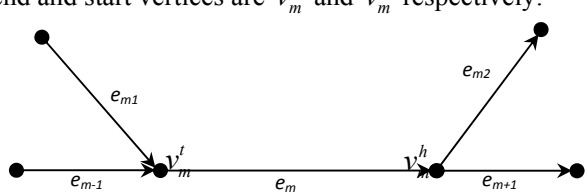

Figure. 6

As e_m allows more than one path, joining P_1 and P_2 as $P^* = P_1 » \{e_m\} » P_2$ we have a new path. Thus in the Cyclic directed path cover one path has been minimized.
$\gamma_m = \gamma - 1$.

e) $d^+(v_m^t) = 3$ and $d^-(v_m^h) = 2$.

Now in this case there must be two paths P_1^1 and P_1^2 are having end vertex v_m^t and the directed path P_2 has start vertex at v_m^h.

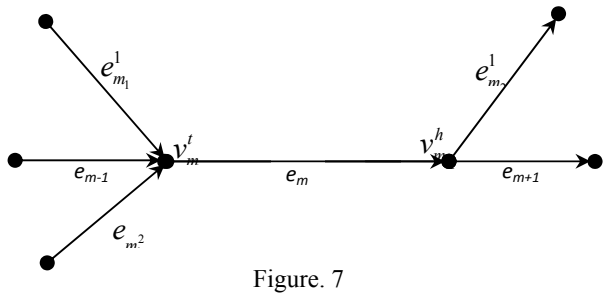

Figure. 7

Now we get the new path in one of the following manner

$P_1^1 » \{e_m\} » P_2$ or $P_1^2 » \{e_m\} » P_2$

Thus one path in the Cyclic path covering is reduced.

Therefore we have $\gamma_m = \gamma - 1$.

f) If $d^+(v_m^t) = d^-(v_m^h) = 3$.

Now in this case there must be two paths P_1^1 and P_1^2 having end terminal vertex as v_m^t and two paths P_2^1 and P_2^2 having start terminal vertex as v_m^h.

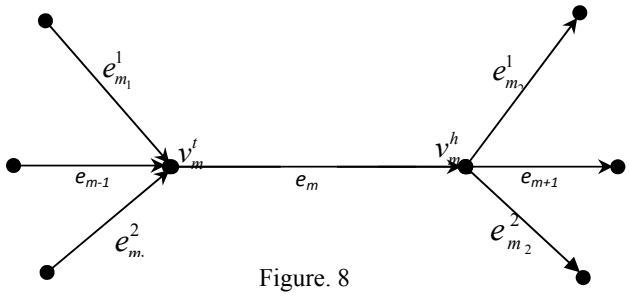

Figure. 8

Now we get two paths in the following manner.

(i) $P_1^1 » \{e_m\} » P_2^1$ or $P_1^1 » \{e_m\} » P_2^2$

(ii) $P_1^2 » \{e_m\} » P_2^1$ or $P_1^2 » \{e_m\} » P_2^2$

Thus two paths are being reduced from the total number of Cyclic path cover.

There fore $\gamma_m = \gamma - 2$.

In general, $\gamma_m = \gamma - \left[\min\left\{d(v_m^t),\ d(v_m^h)\right\} - 1\right]$

Case: (2) e_m is a directed path of length 1.

Let v_m^t and v_m^h be the start and end vertices of e_m.

a) If anyone of this vertex is of total degree ≤ 2 then there is nothing to prove.

(i.e there is no change in the minimal Cyclic path cover)

b) $d(v_m^t) = d(v_m^h) \geq 2$

Then v_m^t and v_m^h will be the intermediate vertex of some path P_{mt} and P_{mh}. Now the reduction in the number of paths is possible only if $d^+(v_m^t) \geq 2$ and $d^-(v_m^h) \geq 2$ (i). If $d^+(v_m^t) = 2$ and $d^-(v_m^h) = 2$, then there is a path P_{mt}^1 that is having v_m^t as the end vertex and another path P_{mh}^1 that is having v_m^h as the starting vertex apart from the paths P_{mt} and P_{mh} that are having v_m^t and v_m^h as internal vertices respectively. Now there is a possibility to make a new path as $P_{mh}^1 \gg \{e_m\} \gg P_{mt}^1$

Thus there is a reduction in the number of paths by 1.

$\gamma_m = \gamma - 1$.

(i.e. $\gamma_m = \gamma - 3 + 2 = \gamma - \min\{d(v_{m_1}), d(v_{m_2})\} + 2$

c) If $d^+(v_{m_1}) = 3$ and $d^-(v_{m_2}) = 2$

Then v_m^t and v_m^h will be the intermediate vertex of some path P_{mt} and P_{mh} respectively and as $d^+(v_{m_1}) = 3$, there must be a paths P_{mt}^1 and P_{mt}^2 that is incident at v_m^t and similarly as $d^-(v_{m_2}) = 2$ there is a path P_{mh}^1 that has the start vertex at v_m^h. Now there is possibility to get two new path as

(i) $P_{mt}^1 \gg \{e_m\} \gg P_{mh}^1$ or $P_{mt}^2 \gg \{e_m\} \gg P_{mh}^1$

Thus there is a reduction in the number of paths by one. $\gamma_m = \gamma - 1$.(i.e. $\gamma_m = \gamma - [2 - 1] = \gamma - [\min\{d(v_m^t), d(v_m^h)\} - 1]$.

This same is the case for $d^+(v_{m_1}) = 2$ and $d^-(v_{m_2}) = 3$

d) If $d^+(v_m^t) = 3$ and $d^-(v_m^h) = 4$

As usual v_m^t and v_m^h will be the intermediate edges in the path P_{mt} and P_{mh} and as $d^+(v_m^t) = 3$ and $d^-(v_m^h) = 4$ there must be paths P_{mt}^1 and P_{mt}^2 that are incident at v_m^t and the paths P_{mh}^1, P_{mh}^2 and P_{mh}^3 that are have the start vertex at v_m^h. Now there is a possibility to get 2 new paths as

(i) $P_{mt}^1 \gg \{e_m\} \gg P_{mh}^1$ or $P_{mt}^1 \gg \{e_m\} \gg P_{mh}^2$ or $P_{mt}^1 \gg \{e_m\} \gg P_{mh}^3$

(ii) $P_{mt}^2 \gg \{e_m\} \gg P_{mh}^1$ or $P_{mt}^2 \gg \{e_m\} \gg P_{mh}^2$ or $P_{mt}^2 \gg \{e_m\} \gg P_{mh}^3$

Thus there is a reduction in the number of path covers by 2 and so the following result $\gamma_m = \gamma - 2$. (i.e., $\gamma_m = \gamma - 3 + 1 = \gamma - [\min\{d^+(v_{m_1}), d^-(v_{m_2})\} - 1]$

e) If $d^+(v_m^t) = 4$ and $d^-(v_m^h) = 3$

As usual v_m^t and v_m^h will be the intermediate edges in the path P_{mt} and P_{mh} and as $d^+(v_m^t) = 4$ and $d^-(v_m^h) = 3$ there must be paths P_{mt}^1, P_{mt}^2 and P_{mt}^3 that are having end vertex at v_m^t and the paths P_{mh}^1 and P_{mh}^2 that have the start vertex at v_m^h. Now there is a possibility to get 2 new paths as

(i) $P_{mt}^1 \gg \{e_m\} \gg P_{mh}^1$ or $P_{mt}^2 \gg \{e_m\} \cup P_{mh}^1$ or $P_{mt}^3 \gg \{e_m\} \gg P_{mh}^1$

(ii) $P_{mt}^1 \gg \{e_m\} \gg P_{mh}^2$ or $P_{mt}^2 \gg \{e_m\} \gg P_{mh}^2$ or $P_{mt}^3 \gg \{e_m\} \gg P_{mh}^2$

Thus there is a reduction in the number of path covers by 2 and so the following result $\gamma_m = \gamma - 2$.

(i.e., $\gamma_m = \gamma - 3 + 1 = \gamma - [\min\{ d^+(v_{m_1}), d^-(v_{m_2})\} - 1]$

In general we have for any digraph the e_m-Cyclic path covering is

$$\gamma_m = \gamma - \left[\min\left\{d^+(v_m^t),\ d^-(v_m^h)\right\} - 1\right]$$

Definition 3.4. A $\{e_m, v_k\}$- Cyclic path cover of a digraph D is a collection ψ_{mk} of directed paths (may be closed) in D whose union is D satisfying the conditions for distinct paths P_i and P_j with terminal vertices u,v and w,z respectively,

$$P_i \grave{E} P_j = \begin{cases} A, \text{ A is the subset of the set } \{u, v, w, z\} \\ \phi, \text{ if } P_i \text{ and } P_j \text{ are cyclic.} \\ v_k, \text{if } v_k \text{ is internal vertex of both } P_i \text{ and } P_j, \\ e_m, \text{ if } e_m \text{ is one of the arc in both } P_i \text{ and } P_j \end{cases}$$

Definition 3.5. ***$\{e_m, v_k\}$- Cyclic path covering number*** γ_m

The $\{e_m, v_k\}$- Cyclic path covering number γ_{mk} of D is defined to be the minimum cardinality taken over all $\{e_m, v_k\}$-Cyclic path covers of D.

Theorem 3.6. For a digraph G with edge e_m and vertex v_k the $\{e_m, v_k\}$ - Cyclic path covering number is

$$\gamma_{mk} = \gamma - \left[\min\left\{d^-(v_k),\ d^+(v_k)\right\} - 1\right] - \left[\min\left\{d^+(v_m^t),\ d^-(v_m^h)\right\} - 1\right],$$

where γ is the Cyclic path covering number of D and v_m^t and v_m^h are tail and the head vertices of e_m.

Proof: Let e_m and v_k be any one edge and vertex of the digraph D. If v_k is one of the terminal vertex of e_m then $\gamma_{mk} = \gamma_m$. Otherwise if v_k is different from the terminal vertices of e_m, then we have

$$\gamma_{mk} = \gamma - (\textit{reduction of paths due to } v_k) - (\textit{reduction of path due to } e_m).$$

$$\text{(i.e), } \gamma_{mk} = \gamma - \left[\min\left\{d^-(v_k),\ d^+(v_k)\right\} - 1\right] - \left[\min\left\{d^+(v_m^t),\ d^-(v_m^h)\right\} - 1\right].$$

4. Conclusion

As the motivation behind the development of the cyclic path cover is mobile road traffic, some to avoid the congestion at the road junction, the junctions are made with flyovers. Even some of the roads are built with flyovers. Thus the kth junctions can be treated as v_k vertex and the mth roads can be treated as e_m edge. Thus if k^{th} junction and m^{th} road are made with flyovers these can be easily treated with $\{e_m, v_k\}$ cyclic path covers.

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