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# Neighbourhood Resolving sets in Graphs – II

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**Abstract**

P.J.Slater **[21]** in 1975 introduced the concepts of locating sets and locating number in graphs. Subsequently with minor changes in terminology, this concept was elaborately studied by Harary and Melter **[14],** Chartrand et al **[5]**, Robert C. Brigham et al **[19],** Chartrand et al **[10]**and Varaporn Saenpholphat and Ping Zhang **[29][30]**. Given an $k$-tuple of vectors, $S=(v_1,v_2,\dots,v_k)$, the neighbourhood adjacency code of a vertex $v$ with respect to $S$, denoted by $nc_S(v)$ and defined by $(a_1,a_2,\dots,a_k)$ where $a_i$ is 1 if $v$ and $v_i$ are adjacent and 0 otherwise. $S$ is called a neighbourhood resolving set or a neighbourhood $r$-set if $nc_S(u)\neq nc_S(v)$ for any $u,v\in V(G)$. The least(maximum) cardinality of a minimal neighbourhood resloving set of $G$ is called the neighbourhood(upper neighbourhood) resolving number of $G$ and is denoted by $nr(G)$ ($NR(G)$). In this paper, Bounds for $nr(G)$, Neighbourhood resolving number for sum and composition of two graphs are obtained. Neighbourhood resolving number for Mycielski Graphs are discussed. Also nice results involving neighbourhood resolving numbers of $G$ and $\overline{G}$ are obtained.

***Keywords:*** locating sets, locating number, neighbourhood resolving sets, neighbourhood resolving number, Mycielski graph

***2010 Mathematics Subject Classification*** : 05C69

## 1. Introduction

In the case of finite dimensional vector spaces, every ordered basis induces a scalar coding of the vectors where the scalars are from the base field. While finite dimensional vector spaces have rich structures, graphs have only one structure namely adjacency. If a graph is connected, the adjacency gives rise to a metric. This metric can be used to define a code for the verices. P. J. Slater [**21**] defined the code of a vertex $v$ with respect to a $k$-tuple of vertices $S=(v_1,v_2,\dots,v_k)$ as $(d(v,v_1),d(v,v_2),\dots,d(v,v_k))$ where $d(v,v_j)$ denotes the distance of the vertex $v$ from the vertex $v_j$. Thus, entries in the code of a vertex may vary from 0 to diameter of $G$. If the codes of the vertices are to be distinct, then the number of vertices in $G$ is less than or equal to $(diam(G)+1)^k$. If it is required to extend this concept to disconnected graphs, it is not possible to use the distance property. One can use adjacenty to define binary codes, the motivation for this having come from finite dimensional vector spaces over $Z_2$. There is an advantage as well as demerit in this type of codes. The advantage is that the codes of the vertices can be defined even in disconnected graphs. The drawback is that not all graphs will allow resolution using this type of codes.

Given an $k$-tuple of vectors, $S=(v_1,v_2,\ldots,v_k)$, the neighbourhood adjacency code of a vertex $v$ with respect to $S$ is defined as $(a_1,a_2,\ldots,a_k)$ where $a_i$ is 1 if $v$ and $v_i$ are adjacent and 0 otherwise. Whereas in a connected graph $G=(V,E)$, $V$ is always a resolving set, the same is not true if we consider neighbourhood resolvability. If $u$ and $v$ are two vertices which are non-adjacent and $N(u)=N(v)$, $u$ and $v$ will have the same binary code with respect to any subset of $V$, including $V$. The least(maximum) cardinality of a minimal neighbourhood resloving set of $G$ is called the neighbourhood(upper neighbourhood) resolving number of $G$ and is denoted by $nr(G)$ ($NR(G)$).

In section 1, Bounds for Neighbourhood Resolving number of a graph is discussed. Neighbourhood resolving number for sum and composition of two graphs are obtained. Neighbourhood resolving number for Mycielski Graphs are also discussed. The second section deals with results involving neighbourhood resolving numbers of $G$ and $\overline{G}$.

**2.Bounds for Neighbourhood Resolving number of a Graph**

Definition 2.1 Let $G$ be any graph. Let $S\subset V(G)$. Consider the $k$-tuple $(u_1,u_2,\ldots,u_k)$ where $S$ = $\{u_1,u_2,\ldots,u_k\}$, $k\geq 1$. Let $v\in V(G)$. Define a binary neighbourhood code of $v$ with respect to the $k$-tuple $(u_1,u_2,\ldots,u_k)$, denoted by $nc_S(v)$ as a $k$-tuple $(r_1,r_2,\ldots,r_k)$ where

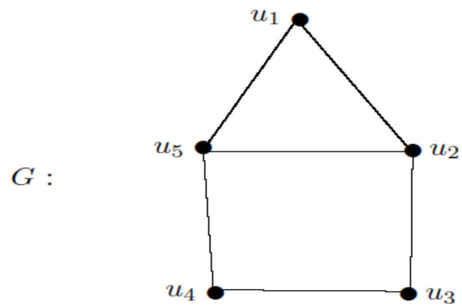
$$r_i=\begin{cases}1,\ if\ \ v\in N(u_i), 1\leq i\leq k\\ \quad 0,\ otherwise\end{cases}$$

$S$ is called a neighbourhood resolving set or a neighbourhood $r$-set if $nc_S(u)\neq nc_S(v)$ for any $u,v\in V(G)$.

The least cardinality of a minimal neighbourhood resloving set of $G$ is called the neighbourhood resolving number of $G$ and is denoted by $nr(G)$. The maximum cardinality of a minimal neighbourhood resolving set of $G$ is called the upper neighbourhood resolving number of $G$ and is denoted by $NR(G)$.

Clearly $nr(G)\leq NR(G)$. A neighbourhood resolving set $S$ of $G$ is called a minimum neighbourhood resolving set or nr-set if $S$ is a neighbourhood resolving set with cardinality $nr(G)$.

**Example 2.2.**



Now $S_1 = \{u_1, u_2, u_5\}$ is a neighbourhood resolving set of $G$, since $nc_S(u_1) = (0,1,1)$; $nc_S(u_2) = (1,0,1)$;
$nc_S(u_3) = (0,1,0)$; $nc_S(u_4) = (0,0,1)$ and
$nc_S(u_5) = (1,1,0)$. Also $S_2 = \{u_1, u_3, u_4\}$, $S_3 = \{u_1, u_2, u_4\}$, $S_4 = \{u_1, u_3, u_5\}$ are neighbourhood resolving sets of $G$. For this graph, $nr(G) = NR(G) = 3$.

**Observation 2.3.**The above definition holds good even if $G$ is disconnected.

**Theorem 2.4.[25]** Let $G$ be a connected graph of order $n \geq 3$. Then G does not have any neighbourhood resolving set if and only if there exist two non adjacent vertices $u$ and $v$ in $V(G)$ such that $N(u) = N(v)$.

**2.1.Bounds for $nr(G)$**

**Theorem 2.5.**For any graph $G$, $nr(G) \leq n-1$.

**Proof :** Suppose there exist a graph $G$ such that $nr(G) = n$.
Then $V(G)$ is the minimal neighbourhood resolving set of $G$. For any $u \in V(G)$, there exists two vertices $x, y$ different from $u$ such that $x$ and $y$ are privately resolved by $u$. That is one of $x$ and $y$ is adjacent to $u$ while the other is not and $N_{G-\{u\}}(x) = N_{G-\{u\}}(y)$.
Case (i) : $N_{G-\{u\}}(x) = N_{G-\{u\}}(y) = \varnothing$.
Then $x$ is an isolate of $G$. Therefore $V - \{x\}$ is a neighbourhood resolving set of $G$, a contradiction, since $V(G)$ is a minimal neighbourhood resolving set of $G$.
Case (ii) : $N_{G-\{u\}}(x) = N_{G-\{u\}}(y) \neq \varnothing$.
Subcase (i) : $x$ and $y$ are non-adjacent.
Subsubcase (i) : $T = V - \{u, x, y\}$.
Subsubcase (iA) : $u$ is adjacent with $v$, for all $v \in T$.
Then $x$ does not resolve privately any pair of vertices. Therefore $V - \{x\}$ is a neighbourhood resolving set of $G$, a contradiction, since $V(G)$ is a minimal neighbourhood resolving set of $G$.
Subsubcase (iB) : There exists a vertex $v \in T$ such that $u$ is not adjacent to $v$.
Then $y$ does not resolve partially any pair of vertices. Therefore $V - \{y\}$ is a neighbourhood

resolving set of $G$, a contradiction, since $V(G)$ is a minimal neighbourhood resolving set of $G$.
Subsubcase (ii) : $T \subset V - \{u, x, y\}$.
Let $T = \{u_1, u_2, \ldots, u_k\}$. Then $V - T = \{u_{k+1}, \ldots, u_n\}$.
Subsubcase (iiA) : $u$ is adjacent with $v$ for all $v \in V - \{u, x, y\}$.
Then $y$ does not resolve privately any pair of vertices. Therefore $V - \{y\}$ is a neighbourhood resolving set of $G$, a contradiction, since $V(G)$ is a minimal neighbourhood resolving set of $G$.
Subsubcase (iiB) : There exists $v \in T$ such that $u$ is not adjacent to $v$.
Then either $x$ or $y$ does not resolve privately any pair of vertices.
Therefore $V - \{x\}$ or $V - \{y\}$ is a neighbourhood resolving set of $G$, a contradiction, since $V(G)$ is a minimal neighbourhood resolving set of $G$.
Subcase (ii) : $x$ and $y$ are not adjacent.
Arguing as in Subcase(i) we get a contradiction.
Hence $nr(G) \leq n - 1$.
Observation 2.6 If $|V(G)| \geq 3$ then $2 \leq nr(G) \leq n - 1$ and bounds are sharp. (The lower bound is attained in $K_3$ and upper bound is attained in $K_n$). The upper bound is attained in $(\frac{n}{2})$ $K_2$ if $n$ is even and $(\frac{n-1}{2})K_2 \cup K_1$ if $n$ is odd.

**Theorem 2.7.[26]**Let $G$ be a connected graph of order $n$ such that $nr(G) = k$. Then $log_2 n \leq k$

**Observation 2.8.[26]** There exists a graph $G$ in which $n = 2^k$ and there exists a neighbourhood resolving set of cardinality $k$ such that $nr(G) = k$. Hence all the distinct binary $k$-vectors appear as codes for the n vertices.

**Theorem 2.9**Let $G$ be a connected graph of order $n$ admitting neighbourhood resolving sets of $G$ and let $nr(G) = k$. Then $k = 1$ if and only if $G$ is either $K_2$ or $K_1$.

**Proof :** If $G$ is $K_1$ or $K_2$ then $nr(G) = 1$.
Suppose $nr(G) = 1$. Let $S = \{u\}$ be an $nr$-set of $G$. Then $nc_S(u)$ = (0). Therefore there exists no vertex in $G$ which is not adjacent to $u$.
If there exist two vertices $x, y \in V(G)$ which are adjacent to $u$ then they have the same neighbourhood code as (1) with respect to $S$, a contradiction.
Therefore there exist atmost one vertex other than $u$ in $G$. Therefore $G = K_1$ or $K_2$.
**Theorem 2.10.**Let $G$ be a connected graph of order $n$ admitting neighbourhood resolving sets of $G$.Then $nr(G)$ = 2 if and only if $G$ is either $K_3$ or $K_3 +$ a pendant edge or $K_3 \cup K_1$ or $K_2 \cup K_1$.

**Proof :** If $G$ is either $K_3$ or $K_3 +$ a pendant edge or $K_3 \cup K_1$ or $K_2 \cup K_1$, then $nr(G) = 2$.
Suppose $nr(G) = 2$. Let $S = \{x, y\}$ be a minimum neighbourhood resolving set of $G$. Then there

are only two possibilities, either the two vertices say $x$ and $y$ in $S$ adjacent or non adjacent.

If they are non adjacent then there the neighbourhood code of the two vertices with respect to $S$ is $(0,0)$, which is a contradiction.

If they are adjacent, then since $n \le 2^k$ there can be at most two more vertices in $G$. $nc_S(x)$ = $(0,1)$ and $nc_S(y)$ = $(1,0)$. Therefore the neighbourhood code of other two vertices are $(1,1)$ and $(0,0)$ which means that one vertex is adjacent to both the vertices of $S$ and other vertex is not adjacent to both the vertices of $S$.

Hence $G$ is either $K_3$ or $K_3 +$ a pendant edge or $K_3 \cup K_1$ or $K_2 \cup K_1$.

3 Neighbourhood Resolving Number for binary operations on Graphs

**Theorem 3.1.**Let $G$, $H$ be two connected vertex disjoint graphs of order $\ge 2$. Then we have $nr(G+H) \le nr(G) + nr(H) + 1$. Also $nr(G+H)$ = $nr(G) + nr(H)$ if and only if at least one of $G$, $H$ has an $nr$-set $S$ for which no vertex receives 1-code with respect to the set $S$.

**Proof :** Let $S_1$ and $S_2$ be $nr$-sets of $G$ and $H$.

Suppose both $S_1$ and $S_2$ allow 1-codes. Then $S_1 \cup S_2$ is not a neighbourhood resolving set for $G+H$.

Suppose $u \in V(G)$ receives 1-code with respect to $S_1$ and $v \in V(H)$ receives 1-code with respect to $S_2$.

Then $S_1 \cup \{u\}$ is a neighbourhood resolving set for $G$ not allowing 1-code for any vertex in $V(G)$ and $S_2 \cup \{v\}$ is a neighbourhood resolving set for $H$ not allowing 1-code for any vertex in $V(H)$.

Therefore $S_1 \cup \{u\} \cup S_2$ and $S_1 \cup S_2 \cup \{v\}$ are neighbourhood resolving sets of $G+H$.

Therefore $nr(G+H) \le nr(G) + nr(H) + 1$.

Suppose there exist $nr$-sets $S_1$, $S_2$ for $G$, $H$ respectively such that at least one of them does not allow 1-code.

Then $S_1 \cup S_2$ is a neighbourhood resolving set of $G+H$.

Therefore $nr(G+H) \le nr(G) + nr(H)$.

Let $S$ be an $nr$-set of $G+H$. Let $T_1 = V(G) \cap S$ and $T_2 = V(H) \cap S$.

Clearly $T_1$ cannot resolve any two vertices of $V(G_2)$ and $T_2$ cannot resolve any two vertices of $V(G_1)$. Therefore $T_1$ and $T_2$ are neighbourhood resolving sets of $G$ and $H$ respectively.

Therefore $nr(G) \le |T_1|$ and $nr(H) \le |T_2|$.

Therefore $nr(G) + nr(H) \le |T_1| + |T_2| = |S|$.

Therefore $nr(G) + nr(H) \le nr(G+H)$ and hence $nr(G+H)$ = $nr(G) + nr(H)$.

Suppose $nr(G+H)$ = $nr(G) + nr(H)$.

Let $S$ be an $nr$-set of $G+H$. Let $T_1 = V(G) \cap S$ and $T_2 = V(H) \cap S$.

Proceeding as before we get that

$nr(G) + nr(H) \le |T_1| + |T_2| = |S| = nr(G+H)$.

Suppose $nr(G) < |T_1|$. Then $nr(G) + nr(H) < nr(G+H)$, a contradiction.

Therefore $nr(G) = |T_1|$. similarly $nr(H) = T_2$.
Therefore $T_1$ and $T_2$ are $nr$-sets of $G$ and $H$ respectively such that $T_1 \cup T_2$ is an $nr$-set of $G+H$.Therefore at least one of $T_1$, $T_2$ does not allow 1-code.

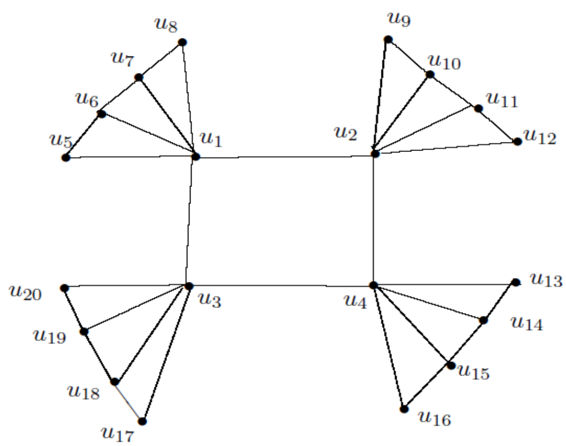
**Theorem 3.2.**Suppose $G$ has a neighbourhood resolving set and $H$ has no neighbourhood resolving set. Then $GoH$ has no neighbourhood resolving set.

**Proof :** Since $H$ has no neighbourhood resolving sets, there exists non-adjacent vertices $u$ and $v$ in $V(H)$ such that $N(u) = N(v)$.
Let $w \in V(G)$. Let $H_1$ be the copy of $H$ attached with $w$ in $GoH$. Then $u$, $v$ in $V(H_1)$ are adjacent to $x$ in addition to their neighbours in $H_1$ which are the same for both $u$ and $v$.
Therefore $N_{GoH}(u) = N_{GoH}(v)$. Hence $GoH$ has no neighbourhood resolving set.

**Theorem 3.3.**There exist examples of graphs $G$ for which neighbourhood resolving set does not exist but $GoH$ has a neighbourhood resolving set where $H$ admits neighbourhood resolving sets exist.

**Illustration 3.4.**

1. $G$:




The above graph $G$ is $C_4 \, o \, P_4$ in which $C_4$ has no neighbourhood resolving set but in $P_4$ neighbourhood resolving sets exist.
For $G$, $nr(G) = 12$ and an $nr$-set is
$\{u_5, u_6, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{20}\}$.
2. $K_{1,n} o P_m$ has neighbourhood resolving sets even though $K_{1,n}$ has no such set.

**Theorem 3.5.**Let $G$ be a simple graph of order $n$. Let $H$ be a graph allowing neighbourhood resolving sets. Then
(i) If there exists an $nr$-set of $H$ which neither allows 0-code nor the 1-code, then $nr(GoH) = n.nr(H)$.
(ii) Suppose every $nr$-set of $H$ allows 0-code but not 1-code, then $nr(GoH) = n.nr(H) + n - 1$.
(iii) Suppose every $nr$-set of $H$ allows 1-code but not 0-code, then $nr(GoH) = n.nr(H) + \gamma(G)$.

(iv) Suppose every $nr$-set of $H$ allows both 0-code and 1-code, then
$$nr(GoH) = \begin{cases} n.nr(H) + n, & \text{if, G is totally disconnected} \\ n.nr(H) + n - 1, & \text{otherwise} \end{cases}.$$

**Proof:** Let $G$ be any graph of order $n$. Let $V(G) = \{u_1, u_2, \ldots, u_n\}$. Let $H_{u_i}$, $1 \le i \le n$ be $n$-copies of $H$ and join $u_i$ of $G$ with every vertex of $H_{u_i}$, $1 \le i \le n$.

(i) Let $u \in V(G)$. Let $S_u$ be an $nr$-set of $H_u$ which neither allows 0-code nor the 1-code.

Let $T = \bigcup_{u \in V(G)} S_u$. Further $u \notin T$ for any $u \in V(G)$ and $nc_T(u)$ receives 1 at every place corresponding to the vertices of $S_u$ and 0 at every other place. Since $S_u$ does not allow 0-code or 1-code, $nc_T(u) \ne nc_T(x)$ for any $x \in V(H_u)$. Also for any $x \in V(H_u)$ and any $y \in V(H_v)$, $u \ne v$, $nc_T(x)$ will receive a non-zero entry in at least one place corredponding to the vertices of $S_u$ and 0 in all other places corresponding to the vertices of $S_v$, $v \ne u$, since $S_u$ does not admit 0-code. Therefore $nc_T(x) \ne nc_T(y)$.

Therefore $T$ is a neighbourhood resolving set of $GoH$ and also $|T| = n.nr(H)$.

Suppose $T^1$ is an $nr$-set of $GoH$ and let $T_u^1 = T^1 \cap V(H_u)$.

$u$ can not resolve any two vertices of $H_u$, since $u$ is adjacent with every vertex of $H_u$. Any vertex of $G$ other than $u$ as well as any vertex of $H_v$, $v \ne u$ is not adjacent with any vertex of $H_u$ and hence it can not resolve any two vertices of $H_u$.

Therefore $T_u^1$ is a neighbourhood resolving set of $H_u$.

If $|T_u^1| > nr(H_u)$, then $T_u$ can be replaced by an $nr$-set of $H_u$ giving rise to a neighbourhood resolving set of $GoH$ whose cardinality is less than $|T^1|$, a contradiction.

Therefore $T_u^1$ is an $nr$-set of $H_u$ and hence $|T^1| = n.nr(H)$.

Therefore $nr(GoH) = n.nr(H)$ and $T$ is an $nr$-set of $GoH$.

(ii) Let $u \in V(G)$. Let $S_u$ be an $nr$-set of $H_u$. Since every $nr$-set of $H$ allows 0-code but not 1-code, $S_u$ allows 0-code but not 1-code.

Therefore there exists an unique $x_u \in V(H_u)$ such that code of $x_u$ with respect to $S_u$ is (0).

Let $T = \bigcup_{u_i \in V(G)} S_{u_i} \cup \{u_1, u_2, \ldots, u_{n-1}\}$.

Since any $nr$-set of $H$ allows 0-code for exactly one vertex, there exists unique $x_{u_i}$ receiving 0-code with respect to $S_{u_i}$. For any $x \in V(H_{u_i})$ and $y \in V(H_{u_j})$, $i \ne j$, $1 \le i, j \le n-1$, $x$ and $y$ will be resolved at the places $u_i$ and $u_j$ even when $x$ and $y$ receives 0-code with respect to $S_{u_i}$ and $S_{u_j}$ respectively.

The vertex $x_{u_n}$ receiving 0-code with respect to $S_{u_n}$ is the only vertex receiving 0-code with respect to $T$.

For any $x \in V(H_{u_i})$, $1 \le i \le n-1$, and $y \in V(H_{u_n})$, $(j \ne n)$, $y$ receives a non-zero entry in a place corresponding to a vertex of $S_{u_n}$ and $x$ receives 0-code in that place. Any $u_i$, $1 \le i \le n$ receives 1-code in the places corresponding to the vertices of $S_{u_i}$ and 0 at places corresponding to vertices of $S_{u_j}$, $j \ne i$, $1 \le j \le n$, and code $a_r$ in the place corresponding to $u_r$, $1 \le r \le n-1$.

Therefore $T$ resolves $u_i$ and $u_j$. Since $S_{u_i}$ does not resolve 1-code, $T$ resolves $x$ and $u_i$ for any $x \in V(H_{u_i})$, $1 \le i \le n$ and hence $T$ is a neighbourhood resolving set of $GoH$.

Therefore $|T| = n.nr(H) + n - 1$.

Therefore $nr(GoH) \le |T| = n.nr(H) + n - 1$.

Let $T^1$ be an $nr$-set of $GoH$. Let $T^1_{u_i}$ = $T^1 \cap V(H_{u_i})$ and $T^1_G$ = $T^1 \cap V(G)$.

Clearly $T^1_{u_i}$ is a neighbourhood resolving set of $H_{u_i}$.

If $|T^1_{u_i}| > nr(H_{u_i})$, then $T_{u_i}$ can be replaced by an $nr$-set of $H_{u_i}$ giving rise to the neighbourhood resolving set of $GoH$ whose cardinality is less than $|T^1|$, a contradiction.

Therefore $T^1_{u_i}$ is an $nr$-set of $H_{u_i}$.

Suppose $u_{i_1}$ and $u_{i_2} \notin T^1_u$.

Suppose $|T^1_G| \le n-2$. Then the unique vertices $x_{u_{i_1}}$ and $x_{u_{i_2}}$ in $S_{u_{i_1}}$ and $S_{u_{i_2}}$ respectively receiving 0-code have 0-code with respect to $T^1$, a contradiction (since $T^1$ is an $nr$-set of $GoH$).

Therefore $|T^1_G| \ge n-1$ and hence $|T^1| \ge n.nr(H) + n - 1$.

Therefore $nr(GoH) \ge n.nr(H) + n - 1$.

Therefore $nr(GoH) = n.nr(H) + n - 1$.

(iii) Let $S_{u_i}$, $1 \le i \le n$, be an $nr$-set of $H_{u_i}$. Since every $nr$-set of $H$ allows 1-code but not 0-code, $S_{u_i}$ does not allow 0-code but allows 1-code.

Therefore there exists a unique $x_{u_i}$ in $V(H_{u_i})$ receiving 1-code with respect to $S_{u_i}$, $1 \le i \le n$.

Let $T = \bigcup_{i=1}^{n} S_{u_i} \cup D$ where $D$ is a $\gamma$ set of $G$.

Let $D = \{u_1, u_2, \ldots, u_\gamma\}$. For any $x \in S_{u_i}$ and $y \in S_{u_j}$, $i \ne j$, $1 \le i, j \le n$, $x$ and $y$ are resolved by $T$ even if $x = x_{u_i}$ and $y = x_{u_j}$. Since any $nr$-set of $H$ does not allow 0-code, $x$ and $y$ are resolved by $T$ even when $u_i$ and $u_j$ do not belong to $D$.

$x_{u_i}$ and $u_i$ receive same code in all places corresponding to vertices of $\bigcup_{i=1}^{n} S_{u_i}$.

If $u_i \in D$, then $x_{u_i}$ and $u_i$ receive distinct codes 1 and 0 at the place corresponding to $u_i$.

If $u_i \notin D$, then $u_i$ is adjacent to some $u_t \in D$.

At the place corresponding to $u_t$ in $T$, $u_i$ receives 1 and $x$ receives 0.

Therefore $u_i$ and $x_{u_i}$ are resolved by $T$.

Clearly $u_i$ and $u_j$ are resolved by $T$, since they receive different codes corresponding to the vertices of $S_{u_i}$ as well as $S_{u_j}$.

Therefore $T$ is a neighbourhood resolving set of $G$ and hence $nr(GoH) \leq n.nr(H) + \gamma(G)$.

Let $T^1$ be an $nr$-set of $GoH$.

Let $T^1_{u_i} = T^1 \cap V(H_{u_i})$ and $T^1_G = T^1 \cap V(G)$.

Clearly $T^1_{u_i}$ is a neighbourhood resolving set of $H_{u_i}$.

If $|T^1_{u_i}| > nr(H_{u_i})$, then $T_{u_i}$ can be replaced by a nr-set of $H_{u_i}$ giving rise to the neighbourhood resolving set of $GoH$ whose cardinality is less than $|T^1|$, a contradiction.

Therefore $T^1_{u_i}$ is an $nr$-set of $H_{u_i}$.

Suppose there exists $u_j \in T^1_G$ such that $u_j$ is not adjacent with any vertex of $T^1_G$. Then $x_{u_j}$ and $u_j$ receive the same code with respect to $T^1$, a contradiction.

Therefore $u_j$ is adjacent with some vertex of $T^1_G$ and hence $T^1_G$ is a a dominating set of $G$, which means $|T^1_G| \geq \gamma(G)$.

Therefore $|T| \geq n.nr(H) + \gamma(G)$.

That is $nr(GoH) \geq n.nr(H) + \gamma(G)$.

Therefore $nr(GoH) = n.nr(H) + \gamma(G)$.

(iv) Let $u \in V(G)$. Let $S_u$ be an $nr$-set of $H_u$. Since every $nr$-set of $H$ allows 0-code and 1-code, $S_u$ allows 0-code and 1-code.

Therefore there exists a unique $x_u \in V(H_u)$ such that code of $x_u$ with respect to $S_u$ is $(0,0,\ldots,0)$ and a unique $x^1_u \in V(H_u)$ such that code of $x^1_u$ with respect to $S_u$ is $(1,\ldots,1)$.

Case (i) : Suppose $G$ is not totally disconnected.

Then there exists a $(n-1)$-subset (say) $\{u_1, u_2, \ldots, u_{n-1}\}$ of $V(G)$ which is a dominating set of $G$.

Let $T = \bigcup_{u_i \in V(G)} S_{u_i} \cup \{u_1, u_2, \ldots, u_{n-1}\}$.

Arguing as in proof of $(ii)$, it can easily seen that that $T$ resolves any two vertices $w_1, w_2$ in $V(GoH)$ where $\{w_1, w_2\} \neq \{x^1_{u_i}, u_i\}$ for some $i$, $1 \leq i \leq n$.

Since $\{u_1, u_2, \ldots, u_{n-1}\}$ is a dominating st of $G$, $T$ resolves $x^1_{u_i}$ and $u_i$, for some $i$, $1 \leq i \leq n$.

Therefore $T$ is a neighbourhood resolving set of $GoH$.

Therefore $nr(GoH) \leq n.nr(H) + n - 1$.

Similar to the proof of $(ii)$ and $(iii)$, it is obvious that $nr(GoH) \geq n.nr(H) + n - 1$.

Therefore $nr(GoH) = n.nr(H) + n - 1$.

Case (ii) : Suppose $G$ is totally disconnected.

Since $H$ admits 1-code, $nr(GoH) \geq n.nr(H) + \gamma(G) = n.nr(H) + n$.

But $nr(GoH) \le n.nr(H)+n$, since $T = \bigcup_{u_i \in V(G)} S_{u_i} \cup V(G)$ is a neighbourhood resolving set of $GoH$.

Therefore $nr(GoH) = n.nr(H)+n$.

**Theorem 3.6.** Let $G$ be a $(n,m)$ graph admitting neighbourhood resolving sets. Then (i) $nr(G)$ = $n-m$ if and only if $G = K_1$ or $K_2$ or $K_1 \cup K_2$.

(ii) Let $G$ have $t$ components and let $G$ have no isolates. Then

$nr(G) = n-m+t-1$ if and only if $G = tK_2$.

**Proof :** (i) If $G = K_1$ or $K_2$ or $K_1 \cup K_2$ then $nr(G) = n-m$.

Conversely, let $nr(G) = n-m$.

For any connected graph $G$, $n-m \le 1$.

Since $nr(G) \ge 1$, $n-m = 1$. Therefore $nr(G) = 1$ and hence $G$ is $K_1$ or $K_2$.

Suppose $G$ is disconnected .

Let $G_1, G_2, \ldots, G_k$ be the components of $G$.

Then atmost one $G_i$ is $K_1$.

Since $nr(G_i) \ge 1 \ge n(G_i) - m(G_i)$, $nr(G_i) \ge n(G_i) - m(G_i)$.

Now $n-m = \sum_{i=1}^{k}(n(G_i) - m(G_i)) \le \sum_{i=1}^{k} nr(G_i)$.

Let $S$ be an $nr$-set of $G$.

Let $T_i = S \cap V(G_i)$, $1 \le i \le k$.

Then $T_i$, $1 \le i \le k$, is a neighbourhood resolving set of $G_i$.

Therefore $nr(G_i) \le |T_i|$.

$n-m = \sum_{i=1}^{k}(n(G_i) - m(G_i)) \le \sum_{i=1}^{k} nr(G_i) \le \sum_{i=1}^{k} |T_i| = |S| = n-m$.

Therefore $\sum_{i=1}^{k}(n(G_i) - m(G_i)) = \sum_{i=1}^{k} nr(G_i)$.

Since $(n(G_i) - m(G_i)) \le nr(G_i)$, we get that

$(n(G_i) - m(G_i)) = nr(G_i)$, for every $i$, $1 \le i \le k$.

Therefore $(n(G_i) - m(G_i)) = nr(G_i) = 1$ (Since $(n(G_i) - m(G_i)) \le 1$ and $nr(G_i) \ge 1$).

Therefore $G_i$ is $K_1$ or $K_2$. Also $n-m = \sum_{i=1}^{k} nr(G_i) = \sum_{i=1}^{k} |T_i| = |S|$.

If $k \ge 3$, then there exist at least two components in $G$ which are $K_2$.

Therefore $\sum_{i=1}^{k} nr(G_i) < \sum_{i=1}^{k} |T_i|$, a contradiction.

Therefore $k \le 2$.

Therefore $G$ = $K_1$ or $K_2$ or $K_1 \cup K_2$.

(ii)If $G = tK_2$ then $nr(G) = n - m + t - 1$.
Conversely, let $nr(G) = n - m + t - 1$.
Case (i) : Let $t = 1$.
Then $G$ is connected and $nr(G) = n - m$.
Therefore $G = K_1$ or $K_2$.
Case (ii) : Let $t \geq 2$. Let $G_1, G_2, \ldots, G_t$ be the components of $G$.
Since $G$ has no isolates, $|G_i| \geq 2$, for every $i$.
Since $nr(G_i) \geq 1 \geq n(G_i) - m(G_i)$, we get $nr(G_i) \geq n(G_i) - m(G_i)$.

$n - m = \sum_{i=1}^{t}(n(G_i) - m(G_i)) \leq \sum_{i=1}^{t} nr(G_i)$.

Let $S$ be an $nr$-set of $G$.
Let $T_i = S \cap V(G_i)$, $1 \leq i \leq t$.
Then $T_i$, $1 \leq i \leq t$ is a neighbourhood resolving set of $G_i$.
Therefore $nr(G_i) \leq |T_i|$.

$n - m = \sum_{i=1}^{t}(n(G_i) - m(G_i)) \leq \sum_{i=1}^{t} nr(G_i) \leq \sum_{i=1}^{t} |T_i| = |S| = n - m + t - 1$.

Suppose $n(G_i) - m(G_i) \leq 0$, for some $i$, $1 \leq i \leq t$.
Then $G_i$ is not a tree. Therefore $G_i$ contains a cycle.

$n - m = \sum_{i=1}^{t}(n(G_i) - m(G_i)) \leq t - 1$.

$|S| = \sum_{i=1}^{t} |T_i| \geq 2(t-1) + 1 = 2t + 1$. (since $G$ has no isolates).

$n - m + t - 1 \leq t - 1 + t - 1 = 2t - 2 < 2t_1 \leq |S|$, a contradiction.
Therefore $n(G_i) - m(G_i) \geq 1$, for every $i$.
But $n(G_i) - m(G_i) \leq 1$, for every $i$.
Therefore $n(G_i) - m(G_i) = 1$, for every $i$.
Therefore $G_i$ is a tree for every $i$ and $G_i \neq K_1$, for every $i$.
Therefore $n - m = t$.
Therefore $n - m + t - 1 = t + t - 1 = 2t - 1$.
Suppose $G_i \neq K_2$. Then $|T_i| \geq 3$.

$|S| = \sum_{i=1}^{t} |T_i| \geq 3 + 2(t-2) + 1 = 2t$.

But $|S| = 2t - 1$, a contradiction.
Therefore $G = tK_2$.

**Theorem 3.7.**Let $G$ be a disconnected graph with $t+1$ components and one of the components is a singleton (say) $u$. Then $G = tK_2 \cup K_1$ if and only if $nr(G_1) = n - m + t - 2$ where $G_1 = G - \{u\}$.

**Proof :** Suppose $G = tK_2 \cup K_1$. Then $nr(G_1) = 2t - 1$.
$n - m + t - 2 = 2t + 1 - t + t - 2 = 2t - 1$.
Therefore $nr(G_1) = n - m + t - 2$.
Conversely, let $nr(G_1) = n - m + t - 2 = n(G_1) - m(G_1) + t - 1$, (since $n(G_1) = n - 1$).
Now $G_1$ has $t$ components and no isolates.
Then by the theorem 3.6, $G_1 = tK_2$.
Therefore $G = tK_2 \cup K_1$.

**Corollary 3.8.**Let $G$ be a disconnected graph with $t+1$ components and one of the components is a singleton (say) $u$. Then $G = tK_2 \cup K_1$ if and only if $n - m + t - 1 = nr(G)$.

**Definition 3.9.**Mycielski Graphs Let $G = (V, E)$ be a simple graph. The Mycielskian of $G$ is the graph $\mu(G)$ with vertex set equal to the disjoint union $V \cup V' \cup \{u\}$ where $V' = \{x' : x \in V\}$ and the edge set $E \cup \{xy', x'y : xy \in E\} \cup \{y'u : y' \in V'\}$. The vertex $x'$ is called the twin of the vertex and the vertex $u$ is called the root of $\mu(G)$.

**Theorem 3.10.**For any graph $G$, $nr(G) + 1 \le nr(\mu(G)) \le nr(G) + 2$.

**Proof:** Let $V(G) = \{u_1, u_2, \ldots, u_n\}$ and let $V(\mu(G)) = \{u_1, u_2, \ldots, u_n, u_1^1, u_2^1, \ldots, u_n^1, u\}$ where $N(u_i) = N(u_i^1) \cup \{u_i\}$ and $N(u) = \{u_1^1, u_2^1, \ldots, u_n^1\}$.
Let $S = \{u_1, u_2, \ldots, u_{k-1}, u_k\}$ be an $nr$ -set of $G$. Since $N(u_i) \cap S = N(u_i^1) \cap S$, $nc_S(u_i) = nc_S(u_i^1)$. Therefore $S$ cannot be a neighbourhood resolving set of $\mu(G)$.
Let $D$ be an $nr$-set of $\mu(G)$. Suppose $|D| \le |S|$.
Case (i) : Suppose $D \subseteq V(G)$.
Then $nc_D(u_i) = nc_D(u_i^1)$, a contradiction, since $D$ is an $nr$-set of $\mu(G)$.
Case (ii) : Suppose $D \subseteq \{u_1^1, u_2^1, \ldots, u_n^1\}$.
Then $nc_D(u_i^1) = nc_D(u_j^1)$, $1 \le i, j \le n$, $i \ne j$, a contradiction, since $D$ is an $nr$-set of $\mu(G)$.
Case (iii) : Suppose $D$ contains atmost $k-1$ elements (say) $u_1, u_2, \ldots, u_t$ where $t \le k-1$ from $\{u_1, u_2, \ldots, u_n\}$ and $u$.
Since $u_1, u_2, \ldots, u_t \in V(G)$ and $t \le k-1$, the set $S^1 = \{u_1, u_2, \ldots, u_t\}$ is not a neighbourhood resolving set of $G$. Therefore there exist $u_a, u_b \in V(G)$, $1 \le a, b \le n$, $a \ne b$ such that $nc_{S^1}(u_a) = nc_{S^1}(u_b)$.
Since $u_a$ and $u_b$ are not adjacent to $u$, we have $nc_D(u_a) = nc_D(u_b)$, a contradiction, since $D$ is an $nr$-set of $\mu(G)$.
Case (iv) : Suppose $D$ contains atmost $k-1$ elements from $\{u_1^1, u_2^1, \ldots, u_n^1\}$ and $u$.
Then $nc_D(u_i^1) = nc_D(u_j^1) = (0, 0, \ldots, 0, 1)$, $1 \le i, j \le n$, $i \ne j$, a contradiction, since $D$ is an $nr$-set

of $\mu(G)$.

Case (v) : Suppose $D$ contains $l$ elements (say) $u_1, u_2, \ldots, u_l$ from $\{u_1, u_2, \ldots, u_n\}$ and $(t-l)$ elements where $l < t \leq k$ from $\{u_1^1, u_2^1, \ldots, u_n^1\}$.

Since $l < k$ and $u_1, u_2, \ldots, u_l \in V(G)$, $S^{11} = \{u_1, u_2, \ldots, u_l\}$ is not a neighbourhood resolving set of $G$. Therefore there exist $u_s, u_t \in V(G)$, $1 \leq s, t \leq n$, $s \neq t$ such that $nc_{S^{11}}(u_s) = nc_{S^{11}}(u_t)$. Since $< u_1^1, u_2^1, \ldots, u_n^1 >$ is independent and $N(u_i) \cap S^{11} = N(u_i^1) \cap S^{11}$, $nc_D(u_s^1) = nc_D(u_t^1)$, a contradiction, since $D$ is an $nr$-set of $\mu(G)$.

Case (vi) : Suppose $D$ contains $l$ elements from $\{u_1, u_2, \ldots, u_n\}$ and $(t-l)$ elements where $l < t \leq k-1$ from $\{u_1^1, u_2^1, \ldots, u_n^1\}$ and $u$.

Proceeding as in Case (v), this case leads to a contradiction.

Therefore $|D| > |S|$. That is $nr(\mu(G)) \geq nr(G)+1$.

Let $S_1 = S \cup \{u\}$. Then $nc_{S_1}(u) = (0,0,\ldots,0)$.

Case (A) : Suppose there is no element $v \in V(\mu(G))$ with $v \neq u$ such that $v$ receives 0-code with respect to $S_1$.

Then for $1 \leq i \leq n$, $nc_{S_1}(u_i) = (a_{i1}, a_{i2}, \ldots, a_{ik}, 0)$ and $nc_{S_1}(u_i^1) = (a_{i1}, a_{i2}, \ldots, a_{ik}, 1)$. Therefore $S_1$ is a neighbourhood resolving set of $\mu(G)$.

Therefore $nr(\mu(G)) \leq |S_1| = |S| + 1 = nr(G)+1$. Therefore $nr(\mu(G)) = nr(G)+1$.

Case (B) : Suppose there exists an element $v \in V(\mu(G))$ with $v \neq u$ such that $v$ receives 0-code with respect to $S_1$.

Let $v = u_a$, $1 \leq a \leq n$.

Subcase (i) : Suppose $u_a \in S$.

Without loss of generality, let $v_a = u_1$.

Since $S$ is an $nr$-set of $G$, there exist two vertices $u_s, u_t \in V(G)$, $1 \leq s, t \leq n$, $s \neq t$, such that $u_1$ privately resolves $u_s$ and $u_t$ with respect to $S$. Since $u$ is not adjacent with $u_s$ and $u_t$, $u_s$ and $u_t$ are privately resolved by $u_1$ with respect to $S_1$.

Subcase (ia) : Suppose $u_s$ and $u_t$ are different from $u_1$.

Consider $S_2 = S \cup \{u_1^1, u\}$.

Then $nc_{S_2}(u_s) = (a_1, a_2, \ldots, a_k, a_1, 0)$; $nc_{S_2}(u_t) = (b_1, a_2, \ldots, a_k, b_1, 0)$. Since $u_1$ privately resolves $u_s$ and $u_t$ with respect to $S_1$, $a_1 \neq b_1$.

$nc_{S_2}(u_1) = (0,0,\ldots,0,0,0)$ ; $nc_{S_2}(u_1^1) = (0,0,\ldots,0,0,1)$ ; $nc_{S_2}(u) = (0,0,\ldots,0,1,0)$ .

$nc_{S_2}(u_l) = (c_1, c_2, \ldots, c_k, c_1, 0)$ and $nc_{S_3}(u_l^1) = (c_1^1, c_2^1, \ldots, c_k^1, 0, 1)$, $2 \leq l \leq n$, $l \neq s, t$.

Clearly $S_2$ resolves any two vertices of $V(\mu(G))$. Therefore $S_2$ is a neighbourhood resolving set of $\mu(G)$.

Therefore $nr(\mu(G)) \leq |S_2| = nr(G)+2$.

Therefore $nr(G)+1 \le nr(\mu(G)) \le nr(G)+2$.

Subcase (ib) : Suppose either $u_s$ or $u_t$ is the same as $u_1$.

Let $u_s = u_1$. Then $nc_{S_2}(u_t) = (1, a_2, \ldots, a_k, 1, 0)$; $nc_{S_2}(u_1) = (0,0,\ldots,0,0,0)$;

$nc_{S_2}(u_1^1) = (0,0,\ldots,0,0,1)$ ; $nc_{S_2}(u) = (0,0,\ldots,0,1,0)$ . $nc_{S_2}(u_l) = (c_1, c_2, \ldots, c_k, c_1, 0)$ and $nc_{S_3}(u_l^1) = (c_1^1, c_2^1, \ldots, c_k^1, 0, 1)$, $2 \le l \le n$, $l \ne t$.

Therefore $S_2$ is a neighbourhood resolving set of $\mu(G)$.

Therefore $nr(\mu(G)) \le |S_2| = nr(G)+2$.

Therefore $nr(G)+1 \le nr(\mu(G)) \le nr(G)+2$.

Subcase (ii) : Suppose $u_a \notin S$.

Let $S_3 = S \cup \{u_a^1, u\}$.

Then $nc_{S_3}(u_a) = (0,0,\ldots,0,0,0)$ ; $nc_{S_3}(u_a^1) = (0,0,\ldots,0,0,1)$ ; $nc_{S_3}(u) = (0,0,\ldots,0,1,0)$ . $nc_{S_2}(u_l) = (c_1, c_2, \ldots, c_k, d, 0)$ and $nc_{S_3}(u_l^1) = (c_1^1, c_2^1, \ldots, c_k^1, 0, 0)$, $2 \le l \le n$, $l \ne t$.

Clearly $S_3$ is a neighbourhood resolving set of $\mu(G)$ and $nr(\mu(G)) \le |S_3| = nr(G)+2$.

Hence $nr(G)+1 \le nr(\mu(G)) \le nr(G)+2$.

## 4 Neighbourhood resolving numbers of $G$ and $\overline{G}$

We recall the

**Theorem 4.1.**Let $G$ be a simple graph and let $S$ be a subset of $V(G)$. Then $S$ does not resolve $x, y \in V(G)$ if and only if $N(x) \cap S = N(y) \cap S$.

**Observation 4.2.** $G$ and $\overline{G}$ will have neighbourhood resolving sets if and only if $N(x) \ne N(y)$ for every non-adjacent pair of vertices $x, y$ in $G$ and $V - N(x) \ne V - N(y)$, for every adjacent pair $x, y$ in $G$.

**Theorem 4.3.**Let $S$ be an $nr$-set of $G$. Then $S$ is not a neighbourhood resolving set of $\overline{G}$ if and only if there exist two adjacent vertices $x, y$ in $G$ such that $x \in S$ or $y \in S$ and $N[x] \cap S = N[y] \cap S$ in $G$.

**Proof :** Let $S$ be an $nr$-set of $G$.

Suppose there exist two adjacent vertices $x, y$ satisfying the hypothesis, then $x, y \notin S$, $x$ and $y$ are non-adjacent in $\overline{G}$ and $N_{\overline{G}}(x) \cap S = N_{\overline{G}}(y) \cap S$.

Therefore $S$ does not resolve $x$ and $y$ in $\overline{G}$. Hence the result.

Conversely, Suppose $S$ is an $nr$-set of $G$ which is not a neighbourhood resolving set of $\overline{G}$. Then there exist $x, y \in V(G)$ such that $S$ does not resolve $x$ and $y$ in $\overline{G}$ . Therefore $N_{\overline{G}}(x) \cap S = N_{\overline{G}}(y) \cap S$.

If $x, y \notin S$, then $S$ does not resolves $x$ and $y$ in $G$, a contradiction. Therefore $x \in S$ or $y \in S$ .

Suppose $x$ and $y$ are non-adjacent.

Then $N_G(x) \cap S = N_G(y) \cap S$ implies that $x$ and $y$ are not resolved by $S$, a contradiction. Therefore $x$ and $y$ are adjacent.

**Theorem 4.4.** Let $S$ be an $nr$-set of $G$. Then $S$ is a neighbourhood resolving set of $\overline{G}$ if and only if for every non-adjacent vertices $x, y \in V(G)$, $N(x) \cap S \neq N(y) \cap S$ in $G$.

**Proof :** Let $S$ be an $nr$-set of $G$.

Let $N_G(x) \cap S \neq N_G(y) \cap S$, for every non-adjacent vertices $x, y \in V(G)$.

Since $N_G(x) \cap S \neq N_G(y) \cap S$, $N_{\overline{G}}(x) \cap S \neq N_{\overline{G}}(y) \cap S$.

Therefore $x$ and $y$ are resolved by $S$ in $\overline{G}$.

Suppose $x$ and $y$ are adjacent vertices such that $N(x) \cap S = N(y) \cap S$.

Subcase (i) : $x \in S$, $y \notin S$.

Then $N(x) \cap S \neq N(y) \cap S$, a contradiction.

Similarly if $x \notin S$ and $y \in S$, the same result follows.

Subcase (ii) : $x, y \in S$.

$N(x) \cap S \neq N(y) \cap S$, a contradiction.

Subcase (iii) : $x, y \notin S$.

Then $S$ does not resolve $x$ and $y$, a contradiction.

Hence when $x$ and $y$ are adjacent, then $N(x) \cap S \neq N(y) \cap S$.

Therefore $N_{\overline{G}}(x) \cap S \neq N_{\overline{G}}(y) \cap S$ and hence $S$ resolves $x, y$ in $\overline{G}$. Therefore $S$ is a neighbourhood resolving set of $\overline{G}$.

Conversely, let $S$ be an $nr$-set of $G$ which is also a neighbourhood resolving set of $\overline{G}$. Let $x, y \in V(G)$ and let $x$ and $y$ be non-adjacent in $G$.

Suppose $N(x) \cap S = N(y) \cap S$. Then $S$ does not resolve $x$ and $y$ in $G$, a contradiction. Therefore $N(x) \cap S \neq N(y) \cap S$.

Theorem 4.4 can be restated as follows.

**Theorem 4.5.**Let $S$ be an $nr$-set of $G$. Then $S$ is a neighbourhood resolving set of $\overline{G}$ if and only if for every pair of vertices $x, y \in V(G)$, $N(x) \cap S \neq N(y) \cap S$.

**Corollary 4.6.**Suppose $S$ is an $nr$-set of $G$ such that for every non-adjacent vertices $x, y \in V(G)$ , $N(x) \cap S \neq N(y) \cap S$. Then $nr(\overline{G}) \leq nr(G)$.

**Corollary 4.7.**Suppose $S$ and $S^1$ are $nr$-sets of $G$ and $(\bar{G})$ respectively such that for every pair of vertices $x, y \in V(G)$, $N_G(x) \cap S \neq N_G(y) \cap S$ and $N_{\overline{G}}(x) \cap S^1 \neq N_{\overline{G}}(y) \cap S^1$. Then $nr(G) = nr(\overline{G})$.

**Observation 4.8.**If $G$ and $\overline{G}$ admit neighbourhood resolving sets, then $nr(G) + nr(\overline{G}) \leq 2n - 2$ and the upper bound is sharp as seen in $P_4$.

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