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Sdm labeling of $C_{2k+1}\otimes K_{1,m}$ and $C_{2k}\otimes K_{1,m}$

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Abstract

A graph $G=(V,E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices x ϵ V with distinct elements $f(x)$ from 1,2…p+q in such a way that the edge $e=uv$ is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting labels of the edges are distinct and are from 1,2…q. A graph that admits skolem difference mean labeling is called skolem difference mean graph. In this paper we study the skolem difference mean labeling of $C_{2k+1}\otimes K_{1,m}$ and $C_{2k}\otimes K_{1,m}$.

Keywords: Cycle, bipartite graphs, star

2010 Mathematics Subject Classification: 05C78

1. Introduction

Throughout this paper we consider only finite, undirected, simple graphs. Let *G* be a graph with *p* vertices and *q* edges. For all terminologies and notations we follow *[2]*. The symbols $V(G)$and $E(G)$ denote respectively the vertex set and edge set of a graph. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or edge labeling). There are several types of labeling and a detailed survey can be found in *[3]*. The concept of mean labeling was introduced in *[5],* skolem mean labeling in *[1]* and skolem difference mean labeling in *[4]*. Following definitions are necessary for the present study.

Definition 1.1. A cycle in a graph *G* is a sequence of distinct vertices $\{v_0, v_1, v_2 \dots v_{n-1}, v_0\}$ where v_i and v_{i+1} are adjacent for all $\mathrm{i}=0,1,2\dots n-2$ and v_{n-1} and $v_0$are adjacent in *G*.A cycle with $n\geq 3$ vertices is denoted by $C_{n.}$

Definition 1.2. The complete bipartite graph $K_{1,n}$ or $K_{n,1}$ is called a star.

Definition 1.3. Let *G* be any graph and $K_{1,m}$ be a star with m spokes. We denote by $G\otimes K_{1,m}$, the graph obtained from *G* by identifying one vertex of *G* with any vertex of $K_{1,m}$ other than the centre of $K_{1,m}$.

Definition 1.4. A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling $(sdml)$ if it is possible to label the vertices $x \epsilon V$ with distinct elements $f(x)$ from 1,2…p+q in such a way that the edge $e = uv$ is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u) - f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u) - f(v)|$ is odd and the resulting labels of the edges are distinct and are from 1,2…q. A graph that admits skolem difference mean labeling is called skolem difference mean graph. The skolem difference mean labeling of the path $P_7$is given in Figure *1*

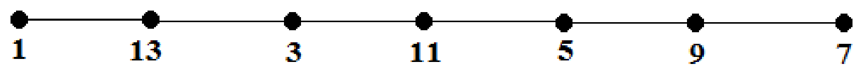

Figure 1

2. Results

Theorem 2.1. $C_{2k+1} \otimes K_{1,m}$ is skolem difference mean for all k, m ≥ 1.

Proof: Let *G* be the graph $C_{2k+1} \otimes K_{1,m}$

Let *V* (G) = $\{u_{i,} v_{j,}$w,w_t / $1 \le i \le k+1, 1 \le j \le k, 1 \le t \le m\}$

Identify w_1 with the vertex u_1 of $C_{2k+1.}$ Then

E (G) = $\{u_1 v_{1,} u_{k+1} v_{k,} u_i u_{i+1,} v_j v_{j+1,} u_1$w,$w w_t$ /1≤ $i \le k, 1 \le j \le k-1, 2 \le t \le m\}$

$|V(G)|$ = 2k+m+1 and $|E(G)|$ = 2k+m+1.

Let *f*: *V* (G)→{1,2…4k+2m+2} be defined as follows.

Case (i) when *k* and *m* are odd.

$f(u_{2s+1}) = 4s+1;\ 0 \le s < \frac{k+1}{2}$

$f(u_{2s}) = 4k+2m+5-4s;\ 1 \le s \le \frac{k+1}{2}$

$f(v_{2s+1}) = 4k+2m+2-4s;\ 0 \le s < \frac{k+1}{2}$

$f(v_{2s}) = 4s;\ 1 \le s < \frac{k+1}{2}$

$f(w) = 2m+3$

$f(w_{2i}) = 4i-1;\ 1 \le i < \frac{m+1}{2}$

$f(w_{2i+1}) = 4i+2;\ 1 \le i < \frac{m+1}{2}$

Case (ii) when *k* is odd and *m* is even

$f(u_{2s+1}) = 4s+1;\ 0 \le s < \frac{k+1}{2}$

$f(u_{2s}) = 4k+2m+5-4s;\ 1 \le s \le \frac{k+1}{2}$

$f(v_{2s+1}) = 4k+2m+2-4s;\ 0\leq s < \frac{k+1}{2}$

$f(v_{2s}) = 4s;\ 1\leq s < \frac{k+1}{2}$

$f(w) = 2m+3$

$f(w_{2i}) = 4i-1;\ 1\leq i \leq \frac{m}{2}$

$f(w_{2i+1}) = 4i+2;\ 1\leq i < \frac{m}{2}$

Case (iii) when *k* is even and *m* is odd

$f(u_{2s+1}) = 4s+1;\ 0\leq s \leq \frac{k}{2}$

$f(u_{2s}) = 4k+2m+5-4s;\ 1\leq s \leq \frac{k}{2}$

$f(v_{2s+1}) = 4k+2m+2-4s;\ 0\leq s < \frac{k}{2}$

$f(v_{2s}) = 4s;\ 1\leq s \leq \frac{k}{2}$

$f(w) = 2m+3$

$f(w_{2i}) = 4i-1;\ 1\leq i < \frac{m+1}{2}$

$f(w_{2i+1}) = 4i+2;\ 1\leq i < \frac{m+1}{2}$

Case (iv) when *k* and *m* are even.

$f(u_{2s+1}) = 4s+1;\ 0 \leq s \leq \frac{k}{2}$

$f(u_{2s}) = 4k+2m+5-4s;\ 1\leq s < \frac{k}{2}$

$f(v_{2s+1}) = 4k+2m+2-4s;\ 0\leq s < \frac{k}{2}$

$f(v_{2s}) = 4s;\ 1\leq s \leq \frac{k}{2}$

$f(w) = 2m+3$

$f(w_{2i}) = 4i-1;\ 1\leq i \leq \frac{m}{2}$

$f(w_{2i+1}) = 4i+2;\ 1\leq i < \frac{m}{2}$

In all the cases let *f** be the induced edge labeling of *f*. Then

$f^*(u_iu_{i+1}) = 2k+m+2-2i;\ 1 \leq i \leq k$

$f^*(v_iv_{i+1}) = 2k+m+1-2i;\ 1 \leq i \leq k-1$

$f^*(u_1v_1) = 2k+m+1$

$f^*(u_{k+1}v_k) = 1$

$f^*(u_1w) = m+1$

$f^*(ww_i) = m+1-i;\ 1 \le i \le m-1$

The induced edge labels distinct and are *1, 2…2k*+m+1.Hence the theorem. ■

Example 2.2. The skolem difference mean labeling of the graphs$C_{11} \otimes K_{1,7}$, $C_{11} \otimes K_{1,6}$, $C_9 \otimes K_{1,5}$ *and* $C_9 \otimes K_{1,6}$ are given in figures 2, 3, 4 and 5 respectively.

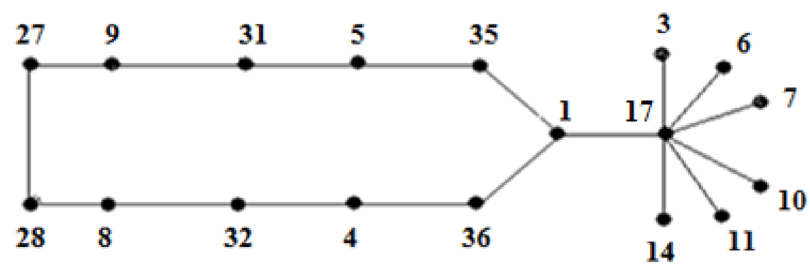

Figure 2

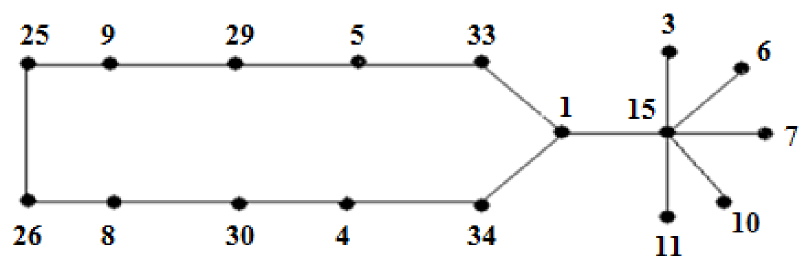

Figure 3

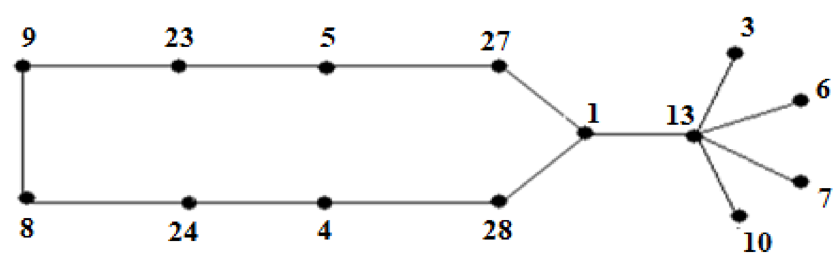

Figure 4

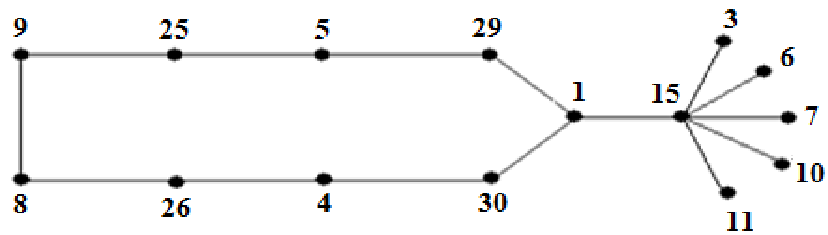

Figure 5

Theorem 2.3. $C_{2k} \otimes K_{1,m}$ is skolem difference mean for all k ≥ 2 and m ≥ 1.

Proof: Let *G* be the graph $C_{2k} \otimes K_{1,m}$

Let *V* (G) = $\{u_{i,} v_{j,} \text{w}, w_t \ / \ 1 \leq i \leq k, 1 \leq j \leq k, 1 \leq t \leq m\}$

Identify w_1 with the vertex u_1 of $C_{2k.}$ Then

E (G) = $\{u_1 v_{1,} u_k v_{k,} u_i u_{i+1,} v_j v_{j+1,} u_1 \text{w}, w w_t \ /1 \leq i \leq k-1, 1 \leq j \leq k-1, 2 \leq t \leq m\}$

$|V(G)|$ = 2k+m and $|E(G)|$ = 2k+m.

Let *f*: V (G)→{1,2…4k+2m} be defined as follows.

Case (i) when *k* and *m* are odd

$f(u_1) = 1$

$f(u_{2s+1}) = 4s; \ 1 \leq s < \frac{k+1}{2}$

$f(u_{2s}) = 4k+2m-4(s-1); \ 1 \leq s < \frac{k+1}{2}$

$f(v_{2s+1}) = 4k+2m-1-4s; \ 0 \leq s < \frac{k+1}{2}$

$f(v_{2s}) = 4s+1 \ ; \ 1 \leq s < \frac{k+1}{2}$

$f(w) = 2m+1$

$f(w_{2i}) = 4i-1; \ 1 \leq i < \frac{m+1}{2}$

$f(w_{2i+1}) = 4i+2; \ 1 \leq i < \frac{m+1}{2}$

Case (ii) when *k* is odd and *m* is even

$f(u_1) = 1$

$f(u_{2s+1}) = 4s; \ 1 \leq s < \frac{k+1}{2}$

$f(u_{2s}) = 4k+2m-4(s-1);\ 1 \leq s < \frac{k+1}{2}$

$f(v_{2s+1}) = 4k+2m-1-4s;\ 0 \leq s < \frac{k+1}{2}$

$f(v_{2s}) = 4s+1;\ 1 \leq s < \frac{k+1}{2}$

$f(w) = 2m+1$

$f(w_{2i}) = 4i-1;\ 1 \leq i \leq \frac{m}{2}$

$f(w_{2i+1}) = 4i+2;\ 1 \leq i < \frac{m}{2}$

Case (iii) when *k* is even and *m* is odd

$f(u_1) = 1$

$f(u_{2s+1}) = 4s;\ 1 \leq s < \frac{k}{2}$

$f(u_{2s}) = 4k+2m-4(s-1);\ 1 \leq s \leq \frac{k}{2}$

$f(v_{2s+1}) = 4k+2m-1-4s;\ 0 \leq s < \frac{k}{2}$

$f(v_{2s}) = 4s+1;\ 1 \leq s \leq \frac{k}{2}$

$f(w) = 2m+1$

$f(w_{2i}) = 4i-1;\ 1 \leq i < \frac{m+1}{2}$

$f(w_{2i+1}) = 4i+2;\ 1 \leq i < \frac{m+1}{2}$

Case (iv) when *k* and *m* are even

$f(u_1) = 1$

$f(u_{2s+1}) = 4s;\ 1 \leq s < \frac{k}{2}$

$f(u_{2s}) = 4k+2m-4(s-1);\ 1 \leq s \leq \frac{k}{2}$

$f(v_{2s+1}) = 4k+2m-1-4s;\ 0 \leq s < \frac{k}{2}$

$f(v_{2s}) = 4s+1;\ 1 \leq s \leq \frac{k}{2}$

$f(w) = 2m+1$

$f(w_{2i}) = 4i-1;\ 1 \leq i \leq \frac{m}{2}$

$f(w_{2i+1}) = 4i+2;\ 1 \leq i < \frac{m}{2}$

In all the cases let *f** be the induced edge labeling of *f*. Then

$f^*(u_i u_{i+1}) = 2k+m+2-2i;\ 1 \le i \le k-1$

$f^*(v_i v_{i+1}) = 2k+m-1-2i;\ 1 \le i \le k-1$

$f^*(u_1 v_1) = 2k+m-1$

$f^*(u_k v_k) = m+2,\ f^*(u_1 w) = m$

$f^*(ww_i) = m-i;\ 1 \le i \le m-1$

The induced edge labels are distinct and are $1,2\ldots 2k+m$.Hence the theorem. ■

Example 2.4. The skolem difference mean labeling of the graphs$C_{10} \otimes K_{1,5}$, $C_{10} \otimes K_{1,6}$, $C_{12} \otimes K_{1,5}$ and $C_{12} \otimes K_{1,6}$ are given in figures 6, 7, 8 and 9 respectively.

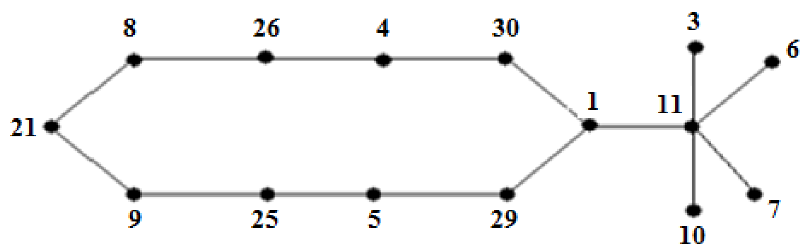

Figure 6

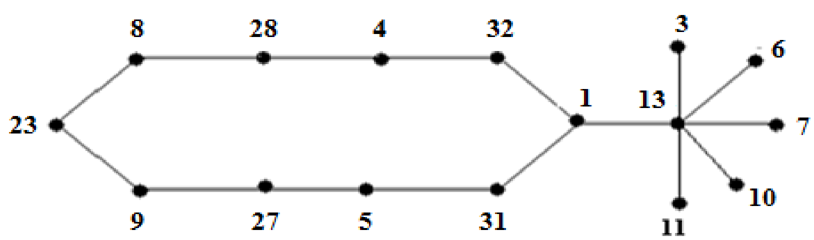

Figure 7

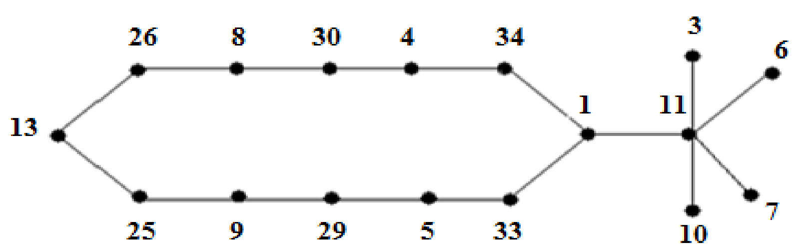

Figure 8

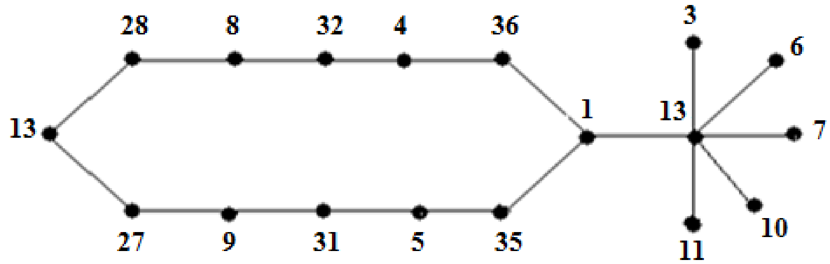

Figure 9

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