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A generalized bio-economic model for competing multiple-fish populations where prices depend on harvest

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Abstract

In the past ten years an increasing number of articles, books and conferences have raised the subject of the bio-economic models of fishery. Most of these models deal with the case of a one fish population. Recently, everyone has been trying to see what happens in the case of two or three competing fish populations. This paper presents a bio-economic model for several fish populations taking into consideration the fact that the prices of fish populations vary according to the quantity harvested. These fish populations compete with each other for space or food. The natural growth of each one is modeled using a logistic law. The objective of this work is multiple, it consists in defining the mathematical model; studying the existence and stability of the equilibrium point; calculating the fishing effort that maximizes the income of the fishing fleet exploiting all fish populations.

Keywords. Bio-economic model of fishery; Multi-fish populations; Price depends on quantity, Fish populations in competition; Sustainable management of the resources; Preservation of the biodiversity.

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	2004	2005	2006	2007	2008	2009
	(Million tonnes)					
PRODUCTION						
INLAND						
Capture	8.6	9.4	9.8	10.0	10.2	10.1
Aquaculture	25.2	26.8	28.7	30.7	32.9	35.0
Total inland	**33.8**	**36.2**	**38.5**	**40.6**	**43.1**	**45.1**
MARINE						
Capture	83.8	82.7	80.0	79.9	79.5	79.9
Aquaculture	16.7	17.5	18.6	19.2	19.7	20.1
Total marine	**100.5**	**100.1**	**98.6**	**99.2**	**99.2**	**100.0**
TOTAL CAPTURE	**92.4**	**92.1**	**89.7**	**89.9**	**89.7**	**90.0**
TOTAL AQUACULTURE	**41.9**	**44.3**	**47.4**	**49.9**	**52.5**	**55.1**
TOTAL WORLD FISHERIES	**134.3**	**136.4**	**137.1**	**139.8**	**142.3**	**145.1**

Figure 1: World fisheries and aquaculture production (FAO 2010).

1 Introduction

Fish has a substantial social and economic importance. The FAO estimates the value of fish traded internationally to be US$ 72 billion per annum (FAO, 2010). Over 56 million people are employed directly through fishing and aquaculture and as many as 260 million people derive direct and indirect income from fish (FAO, 2010). Consumption of food fish is increasing, having risen from 136.255.159 tonnes in 2000 to 162.821.400 tonnes in 2009 and is expected to reach 165 million tonnes by 2010 (FAO, 2012). Global capture fisheries production in 2009 was about 90 million tonnes, with an estimated first-sale value of US$ 93.9 billion, comprising about 80 million tonnes from marine waters and a record 10 million tonnes from inland waters as shown in Table 1 and Figure 1 (FAO: The State of World Fisheries and Aquaculture 2010). As we see, the fishery has a very interesting area for humans. Moreover, in the past ten years, an increasing number of articles (Botsford and al., 1997; Pauly and al., 1998; Pitcher, 2001; Watson and al., 2001; Pauly and al., 2002; Pikitch and al., 2004), Books (Hall, 1999) and conferences (Hollingworth, 2000; Sinclair and Valdimarsson, 2003; Daan and al., 2005) have raised the subject of the bio-economic models. It is, therefore, necessary to develop bio-economic models of multi-species fisheries. There are, however, many difficulties in the modeling of this model. Firstly, it is difficult to construct a realistic multi-species model which is amenable to analytical treatment. Secondly, one of the major difficulties lies in the fact that it is difficult to determine the existence and stability of an equilibrium point. Thirdly, the difficulty lies in calculating the income function of the fishing fleet where prices depend on quantity harvested. Finally, it is difficult to ensure the existence and uniqueness of

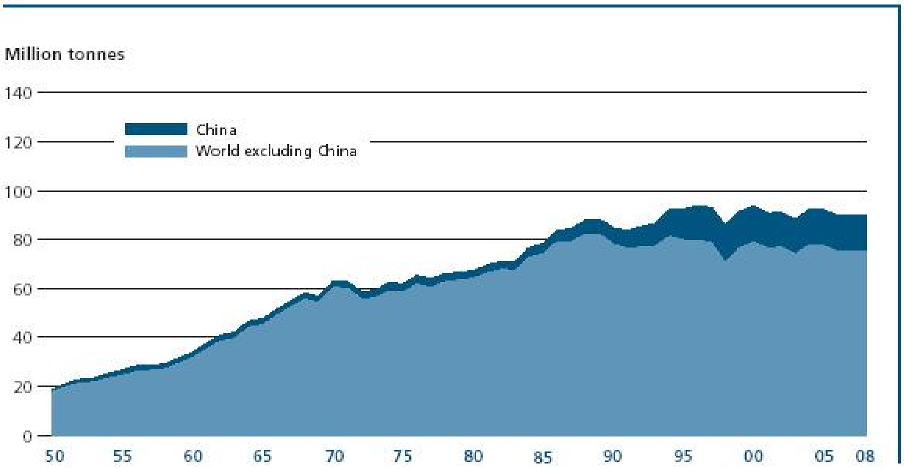

Figure 2: World capture fisheries production (FAO 2010).

the fishing effort that maximizes the income of the fishing fleet at biological equilibrium.

Otherwise, there are several works that deal with bio-economic model mono-specific (Gordon, 1954; Anderson and al., 1984; Clark, 1985; Meuriot, 1987; Auger and al., 2010); bio-economic model for two fish populations (Mchich and al., 2002; Kar and al., 2003; Purohit and al., 2004; Charouki and al., 2010; Kar and al., 2010; Elfoutayeni and al., 2011a) or bio-economic model for three fish populations (Kar and al., 2004; Mchich and al., 2006; Elfoutayeni and al., 2011b; Elfoutayeni and al., 2012b).

This work is intended to resolve these difficulties; precisely, this paper presents a bio-economic model for 'n' fish populations which compete with each other for space or food. The natural growth of each fish population is modeled using a logistic law. Moreover, most of the previous works on bio-economic theory applied to fisheries assumed that the price of the fish population is constant. Usually, the existing models consider that the prices of the fish populations are constant. In this work, we will take that the price of fish population depends on the quantity harvested; specifically we assumed that the price of the fish population increases with decreasing harvest and the price of the fish population decreases with the increase of the harvest, but the minimum price is equal to a fixed positive constant. More precisely we take that $p_i(H_i) = \frac{a_i}{H_i} + p_{0i}$ where a_i and p_{0i} are given positive parameters for all $i = 1, ..., n$. The model proposed here aims to determine the fishing effort strategy adopted by fishing fleet to maximize its income under two assumptions: i) the sustainable management of the resources, and ii) the preservation of the biodiversity.

The paper is structured as follows. Section 2 is devoted to the description of the biological model of fish populations; we will give the mathematical model and study the stability of the equilibrium of our system. Section 3 is intended to give the bio-economic model of the fish populations taking into consideration the fact that the prices of fish populations vary according to the quantity harvested; in this section we will show that the income of the fishing fleet has a unique solution; finally in section 4 we give a conclusion and some potential perspectives.

2 The biological model of fish populations

2.1 The mathematical model and hypotheses

Let b_i be the density of fish population i for all $i = 1, ..., n$; In this work we assume that the fish population i grows according to a logistic equation, this fish population competes with the other ones.

The evolution of the bio-mass of fish populations is modelled by the following equations

$$\begin{cases} db_1/dt = r_1 b_1(1 - \frac{b_1}{K_1}) - c_{12}b_1b_2... - c_{1n}b_1b_n \\ db_2/dt = r_2 b_2(1 - \frac{b_2}{K_2}) - c_{21}b_1b_2... - c_{2n}b_2b_n \\ .. \qquad\qquad\qquad\qquad .. \\ .. \qquad\qquad\qquad\qquad .. \\ db_n/dt = r_n b_n(1 - \frac{b_n}{K_n}) - c_{n1}b_1b_n... - c_{nn-1}b_{n-1}b_n \end{cases} \tag{1}$$

where r_i represents the intrinsic growth rate of the fish population i; K_i is the carrying capacity of the fish population i and c_{ij} is the coefficient of competition of the fish population j on the fish population i. We note that in order to ensure the existence and stability of the fish populations we assume that $\frac{r_i}{Ki} - \sum_{j\neq i}^{n} c_{ij} > 0$ for all $i = 1, ..., n$.

2.2 The steady states of the system

The steady states of the system of equations (1) are obtained by solving the equations

$$\begin{cases} r_1 b_1^*(1 - \frac{b_1^*}{K_1}) - c_{12}b_1^*b_2^*... - c_{1n}b_1^*b_n^* = 0 \\ r_2 b_2^*(1 - \frac{b_2^*}{K_2}) - c_{21}b_1^*b_2^*... - c_{2n}b_2^*b_n^* = 0 \\ .. \qquad\qquad\qquad\qquad .. \\ .. \qquad\qquad\qquad\qquad .. \\ r_n b_n^*(1 - \frac{b_n^*}{K_n}) - c_{n1}b_1^*b_n^*... - c_{nn-1}b_{n-1}^*b_n^* = 0 \end{cases} \tag{2}$$

This system is equivalent to $B^*(-Cb^* + r) = 0$ where $B^* = diag(b^*)$ (that is to say that B^* is the $n \times n$ diagonal matrix with $B^*_{ii} = b^*_i$ for all i and $B^*_{ij} = 0$ for all $i \neq j$) and

$$C = \begin{bmatrix} \frac{r_1}{K_1} & c_{12} & ... & c_{1n} \\ c_{21} & \frac{r_2}{K_2} & ... & c_{2n} \\ ... & ... & ... & ... \\ c_{n1} & c_{n2} & ... & \frac{r_n}{K_n} \end{bmatrix} \tag{3}$$

It is to note that this system has several solutions, only one of them can give the coexistence of the fish populations; this solution is given by the proposition

Proposition 1 : *The system $Cb^* = r$ has one solution.*

Proof. : To prove this proposition it is enough to show that the matrix C is non-singular.

Using the fact that $\frac{r_i}{Ki} - \sum_{j\neq i}^{n} c_{ij} > 0$ for all $i = 1, ..., n$, the matrix C is a strictly diagonally dominant matrix, let's assume C is singular, that is, $\lambda = 0 \in \sigma(C)$, then, by Gershgorin's circle theorem, an index i exists such that $|c_{ii}| = |\lambda - c_{ii}| \leqslant \sum_{j\neq i}^{n} |c_{ij}|$ which is in contrast with a strictly diagonally dominance definition. ■

This proposition shows that the point $P(b^*)$ where $b^* = C^{-1}r$ is the steady state of the system (1) that ensures the coexistence of the fish populations. We note that the biological model is meaningful only insofar as the bio-masses of the fish populations are positive, then we must have $b^* > 0$.

The variational matrix of the system at the steady state $P(b^*)$ is

$$J = -\begin{bmatrix} J_{11} & c_{12}b_1^* & ... & c_{1n}b_1^* \\ c_{21}b_2^* & J_{22} & ... & c_{2n}b_2^* \\ ... & ... & ... & ... \\ c_{n1}b_n^* & c_{n2}b_n^* & ... & J_{nn} \end{bmatrix} \tag{4}$$

where $J_{ii} = r_i(1 - \frac{2b_i^*}{K_i}) - \sum_{j\neq i}^{n} c_{ij}b_i^*$ for all $i = 1, ..., n$. Using the fact that by (2) we have $r_i(1 - \frac{2b_i^*}{K_i}) - \sum_{j\neq i}^{n} c_{ij}b_i^* = -r_i\frac{b_i^*}{K_i}$ for all $i = 1, ..., n$; then

$$J = -\begin{bmatrix} \frac{r_1}{K_1}b_1^* & c_{12}b_1^* & ... & c_{1n}b_1^* \\ c_{21}b_2^* & \frac{r_2}{K_2}b_2^* & ... & c_{2n}b_2^* \\ ... & ... & ... & ... \\ c_{n1}b_n^* & c_{n2}b_n^* & ... & \frac{r_n}{K_n}b_n^* \end{bmatrix} \tag{5}$$

and therefore $J = -B^*C$.

Now we will prove a result which gives the stability of the point $P(b^*)$, more precisely, the steady state $P(b^*)$ is locally asymptotically stable. To demonstrate this result, we will show that the real parts of the eigenvalues of the matrix C are positive and therefore the real parts of the eigenvalues of the matrix J are strictly negative; to do so, we must show the following two lemmas

Lemma 2 *: Let $M \in IR^{n\times n}$ be a diagonally dominant matrix with real positive diagonal then the real parts of the eigenvalues of the matrix M are positive.*

Proof. : Let λ_M be an eigenvalue of the matrix M, we have for all $cte \in IR$

$$|\mathrm{Re}(\lambda_M) - cte| \leqslant \sqrt{(\mathrm{Re}(\lambda_M) - cte)^2 + \mathrm{Im}(\lambda_M)^2}$$

then

$$|\mathrm{Re}(\lambda_M) - cte| \leqslant |\lambda_M - cte| \tag{6}$$

where $\mathrm{Re}(\lambda_M)$ and $\mathrm{Im}(\lambda_M)$ are the real part and imaginary part of the eigenvalue of the matrix M respectively.

Using the fact that by (6) and according to a Gerschgörin-Hadamard theorem we have for all $i = 1, ..., n$

$$\begin{aligned}
|\mathrm{Re}(\lambda_M) - m_{ii}| &\leqslant |\lambda_M - m_{ii}| \\
&\leqslant \sum_{j \neq i}^{n} |m_{ij}| \\
&< m_{ii}
\end{aligned}$$

then $\mathrm{Re}(\lambda_M) > 0$. ■

Lemma 3 *: Let $M \in IR^{n \times n}$ be a diagonally dominant matrix with real positive diagonal then the real parts of the eigenvalues of the matrix DM are positive where D is the $n \times n$ diagonal matrix with $D_{ii} = d_i > 0$ for all i and $D_{ij} = 0$ for all $i \neq j$.*

Proof. : Let $N = DM$ and λ_N be an eigenvalue of the matrix N, we have for all $i = 1, ..., n$

$$\begin{aligned}
|\lambda_N - n_{ii}| &\leqslant \sum_{j \neq i}^{n} |n_{ij}| \\
&< |n_{ii}| \\
&= |d_i m_{ii}| \\
&= d_i m_{ii} \\
&= n_{ii}
\end{aligned}$$

then $\mathrm{Re}(\lambda_N) > 0$. ■

Now we show that

Theorem 4 *: The steady state $P(b^*)$ is locally asymptotically stable*

Proof. : According to the previous lemmas, the real parts of the eigenvalues of the matrix C are positive. Moreover the real parts of the eigenvalues of the matrix $J = -B^*C$ are strictly negative and therefore the steady state $P(b^*)$ is locally asymptotically stable. ■

3 Bio-economic model of fishery where prices depend on harvest

Now, we introduce the fishing by reducing the rate of fish population growth by the amount (Schafer, 1954)

$$H_i = q_i x_i b_i \tag{7}$$

where x_i is the fishing effort to exploit a fish population i and q_i is the catchability coefficient of fish population i, defined as the fraction of the population fished by an effort unit (Gulland, 1983 and Laurec, 1981); in this paper this parameter is assumed to be constant. Bio-mass changes through time can be expressed as

$$\begin{cases} db_1/dt = r_1 b_1(1-\frac{b_1}{K_1}) - \sum_{i\neq 1}^{n} c_{1i} b_1 b_i - q_1 x_1 b_1 \\ db_2/dt = r_2 b_2(1-\frac{b_2}{K_2}) - \sum_{i\neq 2}^{n} c_{2i} b_2 b_i - q_2 x_2 b_2 \\ .. \qquad\qquad\qquad .. \\ .. \qquad\qquad\qquad .. \\ db_n/dt = r_n b_n(1-\frac{b_n}{K_n}) - \sum_{i\neq n}^{n} c_{ni} b_n b_i - q_n x_n b_n \end{cases} \tag{8}$$

When the fish populations are at biological equilibrium, i.e., the variation of the bio-mass of each fish population is zero, thus losses by natural and fishing mortalities are compensated by the fish population increase due to individual growth and recruitment. The system can be defined as

$$\begin{cases} r_1(1-\frac{b_1}{K_1}) = \sum_{i\neq 1}^{n} c_{1i} b_i + q_1 x_1 \\ r_2(1-\frac{b_2}{K_2}) = \sum_{i\neq 2}^{n} c_{2i} b_i + q_2 x_2 \\ .. \qquad\qquad .. \\ .. \qquad\qquad .. \\ r_n(1-\frac{b_n}{K_n}) = \sum_{i\neq n}^{n} c_{ni} b_i + q_n x_n \end{cases} \tag{9}$$

thus, the solutions of this system as a function of the fishing effort are given by

$$b = -Ax + b^* \tag{10}$$

where $A = C^{-1}Q$ and $Q = diag(q)$.

3.1 Total Cost function

We shall assume, in accordance with many standard fisheries models (e.g., the models of Clark, 1975 and Gordon, 1954), that

$$(TC) =< c, x > \tag{11}$$

where (TC) is the total costs for harvesting and c is the harvesting cost per effort.

3.2 Total revenue function

There exist many different variables that affect the fish price; in this work, we will consider that the price of the fish population depends on the quantity harvested; specifically we assumed that the price of the fish population increases with the decreasing harvest and the price of the fish population decreases with the increase of the harvest, but the minimum price is equal to a fixed positive constant. More precisely, we take that $p_i(H_i) = \frac{a_i}{H_i} + p_{0i}$ where a_i and p_{0i} are given positive parameters for all $i = 1, ..., n$. Under these more realistic assumptions we have

$$\begin{aligned}(TR) &= p_1H_1 + p_2H_2 ... + p_nH_n \\ &= (\frac{a_1}{H_1} + p_{01})H_1 + (\frac{a_2}{H_2} + p_{02})H_2 ... + (\frac{a_n}{H_n} + p_{0n})H_n \\ &= p_{01}H_1 + p_{02}H_2 ... + p_{0n}H_n + \sum_{i=1}^{n} a_i \\ &=< p_0, QXb > + \sum_{i=1}^{n} a_i \\ &=< x, P_0Q(-Ax + b^*) > + \sum_{i=1}^{n} a_i\end{aligned}$$

so

$$(TR) =< x, -P_0QAx + P_0Qb^* > + \sum_{i=1}^{n} a_i \tag{12}$$

where (TR) is the total revenue; p_i is the price per unit bio-mass of the fish population i and $P_0 = diag(p_0)$.

3.3 Net economic revenue function

The profit for the fishing fleet $\pi(x)$ is equal to total revenue (TR) minus total cost (TC), in other words, the profit for the fishing fleet is represented by the following function

$$\pi(x) = (TR) - (TC).$$

It follows from (11) and (12) that

$$\pi(x) =< x, -P_0QAx + P_0Qb^* - c > + \sum_{i=1}^{n} a_i. \tag{13}$$

3.4 Bio-economic optimization

The objective is to maximize fishing fleet's income but we must respect two constraints, the first one is the preservation of the biodiversity of fish populations $b = -Ax + b^* \geqslant b_0$ where b_0 is a given positive constant; the second one is the positivity of the fishing effort $x \geq 0$. With all these considerations, our problem leads to the following problem

$$\begin{cases} \max \pi(x) =< x, -P_0QAx + P_0Qb^* - c > + \sum_{i=1}^{n} a_i \\ \text{subject to} \\ \qquad\qquad Ax \leqslant b^* - b_0 \\ \qquad\qquad x \geqslant 0 \end{cases} \tag{14}$$

Now we give the result which shows the existence and uniqueness of this problem.

Lemma 5 *: The problem* (14) *is equivalent to*

$$\begin{cases} \max \pi_b =< b, -P_0Cb + P_0Cb^* + C^TQ^{-1}c > + \sum_{i=1}^n a_i - < Cb^*, Q^{-1}c > \\ \textit{subject to} \\ \qquad\qquad Cb \leqslant Cb^* \\ \qquad\qquad b \geqslant b_0 \end{cases} \tag{15}$$

Proof. : Using the fact that

$$\begin{cases} \pi(x) =< x, -P_0QAx + P_0Qb^* - c > + \sum_{i=1}^n a_i \\ x = A^{-1}(b^* - b) \\ A^{-1} = Q^{-1}C \\ Ax = b^* - b \end{cases}$$

we have

$$\begin{aligned} \pi(x) &=< x, -P_0QAx + P_0Qb^* - c > + \sum_{i=1}^n a_i \\ &=< A^{-1}(b^* - b), -P_0Q(b^* - b) + P_0Qb^* - c > + \sum_{i=1}^n a_i \\ &=< Q^{-1}C(b^* - b), P_0Qb - c > + \sum_{i=1}^n a_i \\ &=< C(b^* - b), P_0b - Q^{-1}c > + \sum_{i=1}^n a_i \\ &=< b, -P_0Cb + P_0Cb^* + C^TQ^{-1}c > + \sum_{i=1}^n a_i - < Cb^*, Q^{-1}c > . \end{aligned}$$

■

Now we show that the problem (15) has one solution.

Theorem 6 *: The problem* (15) *has one and only one solution.*

Proof. : To prove this theorem it is enough to put $M = \frac{1}{2}(C + C^T)$ and show that M is a symmetric positive definite matrix. Now let (λ, v) be an eigenvalue and an eigenvector of the matrix C, it is clear to show that (λ, v) is an eigenvalue and an eigenvector of the matrix C^T, that is to say $Cv = \lambda v$ and $C^Tv = \lambda v$ and therefore $Mv = \lambda v$ i.e.: (λ, v) is an eigenvalue and an eigenvector of the matrix M.

Moreover, since the matrix M is a symmetric then λ is real and using the fact that $\mathrm{Re}(\lambda) > 0$ we have $\lambda > 0$ and therefore M is a symmetric positive definite matrix. ■

The previous theorem shows that the problem (14) has one and only one solution noted x^* which represents the point that maximizes the income of the fishing fleet. We note that to calculate x^* we can show that the problem (14) is equivalent to a Linear Complementarity Problem (Elfoutayeni, 2011c) and we use the methods of Y. Elfoutayeni and M. Khaladi (2010, 2012a) because of their speed of convergence.

4 Conclusion

In this paper we have presented a bio-economic model for several fish populations. In this model we have assumed that the evolution of the fish populations is described by a density dependent model taking into account the competition between fish populations which compete with each other for space or food. The natural growth of each fish population is modeled using a logistic law. On the other hand, we have assumed that the prices of fish populations vary according to the quantity harvested. In this work we have shown that the steady state is locally asymptotically stable using an eigenvalue analysis and calculated the fishing effort that maximizes the income of the fishing fleet exploiting all fish populations at bio-economic equilibrium subject to two constraints, the first one is the sustainable management of the resources and the second one is the preservation of the biodiversity of the fish populations.

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