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A bio-economic model of fishery where prices depend on harvest

Youssef ELFOUTAYENI[a,b], Mohamed KHALADI[a]

[a]Mathematical Populations Dynamics Laboratory
Department of Mathematics, Faculty of Sciences Semlalia
University Cadi Ayyad Marrakech Morocco
and UMI UMMISCO, IRD - UPMC, France
khaladi@ucam.ac.ma

[b]Computer Sciences Department, School of Engineering
and Innovation, Private University of Marrakech
youssef_foutayeni@yahoo.fr

Abstract: Most bio-economic models do not take into account the variational of the price of fish population. Usually, the existing models consider that the prices of the fish populations are constants. In this work, we will take that the price of fish population depends on quantity harvested; for this we propose to define a bio-economic model that merges a model of competition and a model of prey-predator of three fish populations. More specifically, we assume that on the one hand, the evolution of the first and second fish population is described by a density dependent model taking into account the competition between fish populations which compete with each other for space or food; on the other hand, the evolution of the second and third fish population is described by a Lotka-Volterra model. The objective of this work is to maximize the income of the fishing fleet that exploits the three fish populations, but we have to respect two constraints, the first one is the sustainable management of the resources and the second one is the preservation of the biodiversity. The existence of the steady states and its stability are studied using eigenvalue analysis. The problem of determining the equilibrium point that maximizes the income is then solved by using the linear complementarity problem. Finally, some numerical simulations are discussed.

Keywords: Bioeconomic model, Price depends on quantity, Fish populations in competition, Prey-predator model, Sustainable management of the resources, preservation of the biodiversity, Linear complementarity problem.

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1 Introduction

In recent years the bio-economic modelling of the exploitation of biological resources such as fisheries has gained importance, we cite for example in the work of P. Auger et al.[1] with which the authors have given a mathematical model of artificial pelagic multisite fisheries; the model is a stock–effort dynamical model of a fishery subdivided into artificial fishing sites such as fish-aggregating devices (FADs) or artificial habitats (AHs); the objective of their work is to investigate the effects of the number of sites on the global activity of the fishery.

An other example is also that R. Mchich et al.[21] who in their work have presented a stock-effort dynamical model of a fishery subdivided on several fishing zones; the stock corresponds to a fish population moving between different zones, on which they are harvested by fishing fleets.

Many mathematical models have been developed to describe the dynamics of fisheries, we can refer for example to S. Charles et al.[2], M. Haddon[18] and T. J. Quinn et al.[19], and many other works included economic factors (see the books by C.W. Clark[7] and Y. Cohen[9] and the works of F. H. Clarke and G. R. Munro[8] and N. Raïssi[24]).

In this context, Y. Elfoutayeni et al.[12] who in their work have defined a bio-economic equilibrium model for several fishermen who catch two fish species; in this work, the authors have showed that the existence of the steady states and its stability are studied using eigenvalue analysis; the problem of determining the equilibrium point that maximizes the profit of each fisherman is solved by using linear complementarity problem and finally the authors have given some numerical simulations to illustrate the results.

An other important example in this context is also that Y. Elfoutayeni et al.[13] who in their work have defined a bio-economic equilibrium model for 'n' fishermen who catch three species, these species compete with each other for space or food; the natural growth of each species is modeled using a logistic law; the objective of their work is to calculate the fishing effort that maximizes the profit of each fisherman at biological equilibrium by using the generalized Nash equilibrium problem.

An other important example in this context is also that Y. Elfoutayeni et al.[14] who in their work have presented a bio-economic model for several fish populations taking into consideration the fact that the prices of fish populations vary according to the quantity harvested. These fish populations compete with each other for space or food. The natural growth of each one is modeled using a logistic law. The objective of this work is multiple, it consists in defining the mathematical model; studying the existence and stability of the equilibrium point; calculating the fishing effort that maximizes the income of the fishing fleet exploiting all fish populations.

K. S. Chaudhuri ([3],[4]) has studied the combined harvesting of two competing species from the standpoint of bio-economic harvesting and has discussed dynamic optimization of the harvest policy. K. S. Chaudhuri et al.[5] have studied combined harvesting of a prey-predator community with some

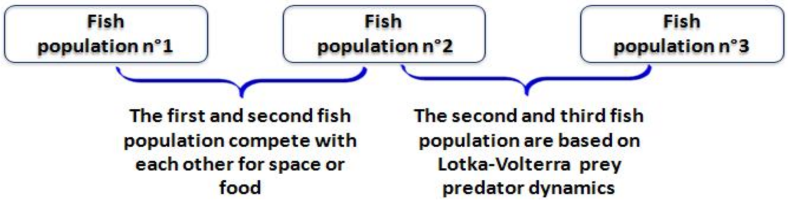

Figure 1: The model with three fish populations: the first and second one are in competition, the second and third one are prey-predator.

prey hiding in refuges.

All of the bio-economic models mentioned above consider that the prices of fish populations are constants.

In the present paper, we propose to define a bio-economic model that merges a model of competition and a model of prey-predator of three fish populations. Specifically, we assume that on the one hand, the evolution of the first and second fish population is described by a density dependent model taking into account the competition between fish populations which compete with each other for space or food (see P. F. Verhulst[25]); on the other hand, the evolution of the second and third fish population is described by a Lotka-Volterra model (see figure 1).

Most bio-economic models do not take into account the variational of the price of fish population. Usually, the existing models consider that the prices of the fish populations are constants. In this work, we will take that the price of fish population depends on quantity harvested; specifically we assumed that the price of the fish population increases with decreasing harvest and the price of the fish population decreases with the increase of the harvest, but the minimum price is equal to a fixed positive constant. More precisely we take that $p_i(H_i) = \frac{a_i}{H_i} + p_{0i}$ where a_i and p_{0i} are positive parameters given for all $i = 1, 2, 3$.

The objective of the fishing fleet is to maximize its income at biological equilibrium, but we have to respect two constraints, the first one is the sustainable management of the resources, the second one is the preservation of the biodiversity. With all these considerations, our problem leads to a convex quadratic problem.

The paper is organized as follows. In the next section, we present the mathematical model which consist in a system of three ordinary differential equations, the first equation describes the natural growth of the first fish population and competition between the first and second fish population; the second equation describes the natural growth of the second fish population, competition with the first fish population and a prey of the third fish population; the third equation describes the natural growth of the third fish population as a predator of the second fish population. The existence of the steady states of this system and its stability are studied using eigenvalue analysis and we define a bio-economic equilibrium model for the three fish populations exploited by a fishing fleet. In section 3 we prove that the resolution of bio-economic equilibrium model of the three fish populations is equivalent to solving a convex quadratic problem and then we show that the latter problem has a unique solution. In section 4 we give a numerical simulation of the mathematical model and discussion of the results; finally we give a conclusion and some potential perspectives in section 5.

2 A dynamic model of fish populations

In this work we consider three fish populations. Let $B_i(t)$ be the fish density of fish population $i = 1, 2, 3$ at time t. In this model, it is assumed that

the first fish population grows according to a logistic equation with growth rate r_1 and carrying capacity K_1, this fish population competes with the second one as follows (see G. F. Gause[16])

$$\dot{B}_1 = r_1 B_1 (1 - \frac{B_1}{K_1}) - c_{12} B_1 B_2 \tag{1}$$

where c_{12} is the coefficient of competition between the first and second fish population.

The second fish population grows according to a logistic equation with growth rate r_2 and carrying capacity K_2, this fish population competes with the first one and it is a prey of the third one as follows

$$\dot{B}_2 = r_2 B_2 (1 - \frac{B_2}{K_2}) - c_{21} B_1 B_2 - \alpha B_2 B_3 \tag{2}$$

where c_{21} is the coefficient of competition between the second and first fish population and α is the predation rate coefficient.

The third fish population is the predator of the second one as follows

$$\dot{B}_3 = -B_3(\beta - \gamma B_2) \tag{3}$$

where β is the predator mortality rate and γ is the reproduction rate of predator (see Figure 1).

In order to ensure the existence and stability of the locally asymptotically stable of the three fish populations we assume that

$$\begin{cases} r_1 - c_{12} K_1 > 0 \\ r_2 - c_{21} K_2 > 0 \\ 1 - \frac{c_{12}}{r_1} \frac{\beta}{\gamma} > 0 \\ r_1 r_2 K_2 \gamma - r_1 r_2 \beta - c_{21} r_1 K_1 K_2 \gamma + c_{12} c_{21} K_1 K_2 \beta > 0 \end{cases} \tag{4}$$

Under these assumptions, the evolution of the biomass of the three fish populations is modelled by the following equations

$$\begin{cases} \dot{B}_1 = r_1 B_1 (1 - \frac{B_1}{K_1}) - c_{12} B_1 B_2 \\ \dot{B}_2 = r_2 B_2 (1 - \frac{B_2}{K_2}) - c_{21} B_1 B_2 - \alpha B_2 B_3 \\ \dot{B}_3 = -B_3(\beta - \gamma B_2) \end{cases} \tag{5}$$

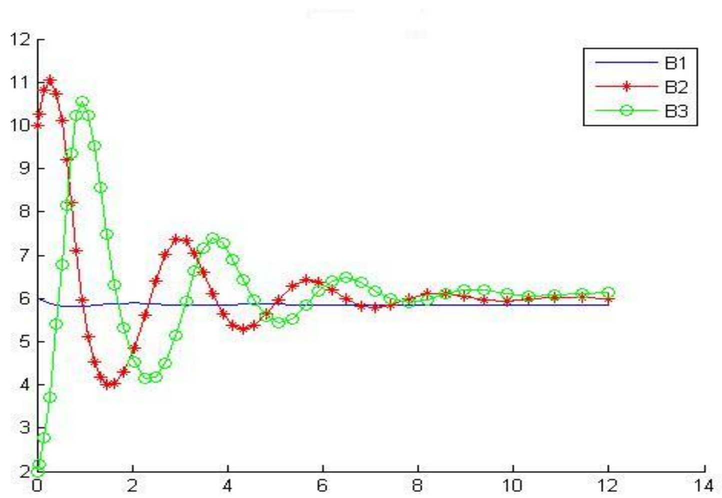

Figure 2: Figure illustrates the evolution of the fish populations in the case $r_1 = 2.5$; $r_2 = 2.7$; $K_1 = 6$; $K_2 = 20$; $c_{12} = 0.01$; $c_{21} = 0.001$; $\alpha = 0.3$; $\beta = 3$and $\gamma = 0.5$. The initial conditions are 6, 2and 10of the first, second and third population respectively. The three fish populations converge to their steady states $B^* = (5.86; 6.00; 6.28)$in finite time.

The steady states of the system of equations (5) are obtained by solving the equations

$$\begin{cases} B_1[r_1(1-\frac{B_1}{K_1}) - c_{12}B_2] = 0 \\ B_2[r_2(1-\frac{B_2}{K_2}) - c_{21}B_1 - \alpha B_3] = 0 \\ -B_3(\beta - \gamma B_2) = 0 \end{cases} \tag{6}$$

The system (6) has six equilibria, only one of them can give coexistence of the three fish populations, in this case the bio-masses are positive; this solution is the point $P(B_1^*, B_2^*, B_3^*)$ where

$$\begin{cases} B_1^* = K_1(1-\frac{c_{12}}{r_1}\frac{\beta}{\gamma}) \\ B_2^* = \frac{\beta}{\gamma} \\ B_3^* = \frac{r_1r_2K_2\gamma - r_1r_2\beta - c_{21}r_1K_1K_2\gamma + c_{21}c_{12}K_1K_2\beta}{\alpha r_1 K_2 \gamma} \end{cases} \tag{7}$$

Now we will prove a result which gives the stability of the point $P(B_1^*, B_2^*, B_3^*)$ given by (7), exactly, the steady state $P(B_1^*, B_2^*, B_3^*)$ is locally asymptotically stable (see figure 2).

Proposition 1 *Under the conditions* (4) *the steady state* $P(B_1^*, B_2^*, B_3^*)$ *is locally asymptotically stable.*

Proof. The variational matrix of the system (5) at $P(B_1^*, B_2^*, B_3^*)$ is

$$J = \begin{bmatrix} -r_1\frac{B_1^*}{K_1} & -c_{12}B_1^* & 0 \\ -c_{21}B_2^* & -r_2\frac{B_2^*}{K_2} & -\alpha B_2^* \\ 0 & \gamma B_3^* & 0 \end{bmatrix}$$

The characteristic polynomial is given by

$$P(\lambda) = a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$$

where

$$\begin{cases} a_0 = 1 > 0 \\ a_1 = r_2\frac{B_2^*}{K_2} + r_1\frac{B_1^*}{K_1} > 0 \\ a_2 = \alpha\gamma B_2^* B_3^* + \frac{B_1^* B_2^*}{K_1 K_2}(r_1 r_2 - c_{21}c_{12}K_1K_2) > 0 \\ a_3 = r_1\frac{B_1^*}{K_1}\alpha\gamma B_2^* B_3^* > 0 \end{cases}$$

Moreover

$$\begin{aligned} a_1a_2 - a_0a_3 &= (r_2\frac{B_2^*}{K_2} + r_1\frac{B_1^*}{K_1})[\alpha\gamma B_2^* B_3^* + \frac{B_1^* B_2^*}{K_1K_2}(r_1r_2 - c_{21}c_{12}K_1K_2)] \\ &\quad - r_1\frac{B_1^*}{K_1}\alpha\gamma B_2^* B_3^* \\ &= r_2\frac{B_2^*}{K_2}a_2 + r_1\frac{B_1^*}{K_1}\alpha\gamma B_2^* B_3^* + r_1\frac{B_1^*}{K_1}\frac{B_1^* B_2^*}{K_1K_2}(r_1r_2 - c_{21}c_{12}K_1K_2) \\ &\quad - r_1\frac{B_1^*}{K_1}\alpha\gamma B_2^* B_3^* \\ &= r_2\frac{B_2^*}{K_2}a_2 + r_1\frac{B_1^*}{K_1}\frac{B_1^* B_2^*}{K_1K_2}(r_1r_2 - c_{21}c_{12}K_1K_2) \end{aligned}$$

Using the fact that by (4) we have $a_1a_2 - a_0a_3 > 0$ and therefore by Routh-Hurwitz Stability Criterion we have $P(B_1^*, B_2^*, B_3^*)$ is locally asymptotically stable. ■

Now let x_i be the harvesting effort used to harvest fish population i and let q_i be the catchability coefficient of fish population i (defined as the fraction of the population fished by an effort unit (see A. Laurec et al.[20])). The evolution of the biomass changes through time can be expressed as

$$\begin{cases} \dot{B}_1 = r_1B_1(1 - \frac{B_1}{K_1}) - c_{12}B_1B_2 - H_1 \\ \dot{B}_2 = r_2B_2(1 - \frac{B_2}{K_2}) - c_{21}B_1B_2 - \alpha B_2B_3 - H_2 \\ \dot{B}_3 = -B_3(\beta - \gamma B_2) - H_3 \end{cases} \tag{8}$$

where $H_i = q_ix_iB_i$ (see M. B. Schaefer[23]) is the harvest of fish population i. The biomasses at biological equilibrium are the solutions of the system

$$\begin{cases} r_1(1 - \frac{B_1}{K_1}) - c_{12}B_2 - q_1x_1 = 0 \\ r_2(1 - \frac{B_2}{K_2}) - c_{21}B_1 - \alpha B_3 - q_2x_2 = 0 \\ -(\beta - \gamma B_2) - q_3x_3 = 0 \end{cases} \tag{9}$$

The solution of this system is given by $B = -Ax + B^*$ where

$$A = \begin{bmatrix} K_1\frac{q_1}{r_1} & 0 & K_1\frac{c_{12}q_3}{r_1\gamma} \\ 0 & 0 & -\frac{q_3}{\gamma} \\ -\frac{c_{21}K_1q_1}{\alpha r_1} & \frac{q_2}{\alpha} & \frac{r_2q_3r_1 - c_{21}K_1K_2c_{12}q_3}{\alpha K_2\gamma r_1} \end{bmatrix}$$

Expression of the total effort cost: Let c_i be the constant cost per unit of harvesting effort of fish population i. We shall assume that $(TC) = x^TCx$,

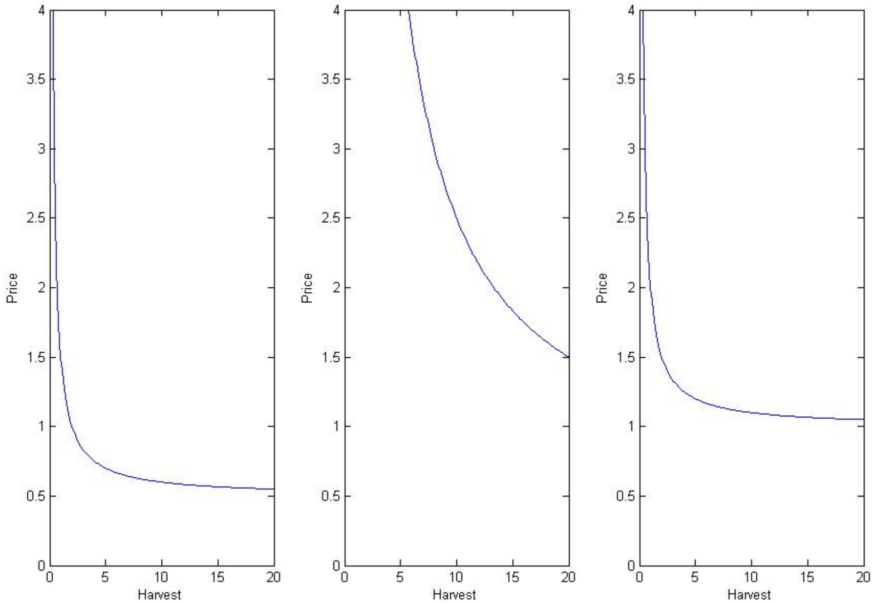

Figure 3: Price of the fish population increases with decreasing harvest $p(H) = \frac{a}{H} + p_0$. The first figure illustrates the price with parameters as follows $a = 1$and $p_0 = 0.5$. The second figure illustrates the price with parameters as follows $a = 20$and $p_0 = 0.5$. The first figure illustrates the price with parameters as follows $a = 1$and $p_0 = 1$.

where (TC) is the total effort cost; the superscript T denotes the transpose and C is the 3×3 diagonal matrix with $C_{ii} = c_i$ for all i and $C_{ij} = 0$ for all $i \neq j$.

Expression of the total revenue: We shall assume, in keeping with many standard fisheries models (e.g., the model of H. S. Gordon[17], and W. C. Clark et al.[6]), that the total revenue (TR) of fishing fleet from harvest the three fish populations is defined as $(TR) = p_1H_1 + p_2H_2 + p_3H_3$, where p_i is the price of fish population i.

In this work we assumed that the price of the fish population increases with decreasing harvest and the price of the fish population decreases with the increase of the harvest but the minimum price is equal to a fixed positive constant. More precisely we take $p_i(H_i) = \frac{a_i}{H_i} + p_{\min i}$ where a_i and $p_{\min i}$ are positive parameters given (see figure 3). Under these more realistic assumptions we have

$$\begin{aligned}(TR) &= p_1H_1 + p_2H_2 + p_3H_3 \\ &= (\frac{a_1}{H_1} + p_{01})H_1 + (\frac{a_2}{H_2} + p_{02})H_2 + (\frac{a_3}{H_3} + p_{03})H_3 \\ &= p_{01}H_1 + p_{02}H_2 + p_{03}H_3 + a \\ &= p_{01}q_1x_1B_1 + p_{02}q_2x_2B_2 + p_{03}q_3x_3B_3 + a \\ &= p_{01}q_1x_1(-a_{11}x_1 - a_{12}x_2 - a_{13}x_3 + B_1^*) \\ &\quad + p_{02}q_2x_2(-a_{21}x_1 - a_{22}x_2 - a_{23}x_3 + B_2^*) \\ &\quad + p_{03}q_3x_3(-a_{31}x_1 - a_{32}x_2 - a_{33}x_3 + B_3^*) + a \\ &= -x^T(p_0qAx) + x^T(p_0qB^*) + a\end{aligned}$$

where $a = a_1 + a_2 + a_3$ and

$$p_0qA = \begin{bmatrix} p_{01}q_1a_{11} & p_{01}q_1a_{12} & p_{01}q_1a_{13} \\ p_{02}q_2a_{21} & p_{02}q_2a_{22} & p_{02}q_2a_{23} \\ p_{03}q_3a_{31} & p_{03}q_3a_{32} & p_{03}q_3a_{33} \end{bmatrix}.$$

We note that in this work the product of two vectors $x \in IR^n$ and $y \in IR^n$ is the vector $xy \in IR^n$ defined by $xy := (x_1y_1, ..., x_ny_n)^T$.

Expression of the profit: The profit for the fishing fleet is equal to total revenue (TR) minus total cost (TC), in other words, this profit is represented by the following function

$$\pi(x) = -x^T(p_0qA + C)x + x^T(p_0qB^*) + a. \tag{10}$$

The objective is to maximize fishing fleet's income but we must respect two constraints, the first one is the preservation of the biodiversity of fish populations $B = -Ax + B^* \geq B_0$ where B_0 is a positive constant given (in the numerical simulations we will take $B_0 = 10\%B^*$); the second one is the positivity of the fishing effort $x \geq 0$. With all these considerations, our problem leads to the following problem

$$\begin{cases} \max \pi(x) = -x^T(p_0qA + C)x + x^T(p_0qB^*) + a \\ \text{subject to} \quad Ax \leq B^* - B_0 \\ \qquad\qquad\qquad x \geq 0 \end{cases} \tag{11}$$

3 Resolution of the mathematical model

It is clear that the matrix $Q = p_0qA + C$ associated with the optimization problem obtained is not symmetric positive definite to say that this problem has a unique solution; by cons we can show that this matrix is a $P - matrix$ (Recall that a matrix M is called $P - matrix$ if all of its principal minors are positive).

Theorem 2 *The problem*

$$\begin{cases} \max \pi(x) = -x^TQx + x^T(p_0qB^*) + a \\ \textit{subject to} \quad Ax \leq B^* - B_0 \\ \qquad\qquad\qquad x \geq 0 \end{cases}$$

has a unique solution.

Proof. To prove this result it suffices to show that the matrix Q

$$Q = \begin{bmatrix} p_{01}q_1K_1\frac{q_1}{r_1} + c_1 & 0 & p_{01}q_1K_1\frac{c_{12}q_3}{r_1\gamma} \\ 0 & c_2 & -p_{02}q_2\frac{q_3}{\gamma} \\ -p_{03}q_3\frac{c_{21}K_1q_1}{\alpha r_1} & p_{03}q_3\frac{q_2}{\alpha} & p_{03}q_3\frac{r_2q_3r_1-c_{21}K_1K_2c_{12}q_3}{\alpha K_2\gamma r_1} + c_3 \end{bmatrix}$$

is $P-matrix$.

Now if we note by $(Q_i)_{i=1,2,3}$ the submatrix of Q, we obtain
$\det(Q_1) = p_{01}q_1K_1\frac{q_1}{r_1} + c_1 > 0$;
$\det(Q_2) = (p_{01}q_1K_1\frac{q_1}{r_1} + c_1)c_2 > 0$ and
$\det(Q_3) = (p_{01}q_1K_1\frac{q_1}{r_1} + c_1)[c_2(p_{03}q_3^2\frac{r_1r_2-c_{12}c_{21}K_1K_2}{\alpha K_2\gamma r_1} + c_3) + p_{03}q_3\frac{q_2}{\alpha}p_{02}q_2\frac{q_3}{\gamma}]$
$-p_{03}q_3\frac{c_{21}K_1q_1}{\alpha r_1}(-c_2p_{01}q_1K_1\frac{c_{12}q_3}{r_1\gamma}) > 0$.

So the matrix Q is $P-matrix$ and therefore the problem (11) admits one and only one solution (see Y. Elfoutayeni[15]). ■

The theorem proves that the existence and uniqueness x^* that maximizes the profit $\pi(x) = -x^T(p_0qA + C)x + x^T(p_0qB^*) + a$.

We note that the problem (11) is equivalent to (see Y. Elfoutayeni[15]) what is called a Linear Complementarity Problem $LCP(M,\tilde{q})$ (we recall that the linear complementarity problem $LCP(M,\tilde{q})$ is to find a vector z in IR^n satisfying $z^T(Mz+\tilde{q}) = 0$, $Mz+\tilde{q} \geqslant 0$, $z \geqslant 0$, where $M = (m_{ij}) \in \mathrm{IR}^{n\times n}$ and $\tilde{q} \in \mathrm{IR}^n$ are given).

In this cas

$$M = \begin{bmatrix} 2p_0qA + 2C & A^T \\ -A & 0 \end{bmatrix}$$

and

$$\tilde{q} = \begin{pmatrix} c - p_0qB^* \\ B^* - B_0 \end{pmatrix}$$

For solving the linear complementarity problem $LCP(M,\tilde{q})$ we can demonstrate that the matrix M is $P-matrix$ and we will use the following result:

A linear complementarity problem $LCP(M,\tilde{q})$ has a unique solution for every $\tilde{q}$ if and only if M is a $P-matrix$ (For demonstration we can see K. G. Murty[22]).

Note that each matrix symmetric positive definite is $P-matrix$, but the reverse is not always true.

To calculate x^* solution of (11) we can use the methods of Y. Elfoutayeni and M. Khaladi([10],[11]) because of their speed of convergence.

4 Numerical simulations and the discussion of the results

In this section, we assign numerical values to the parameters of the system (5) and compute some simulations using those values.

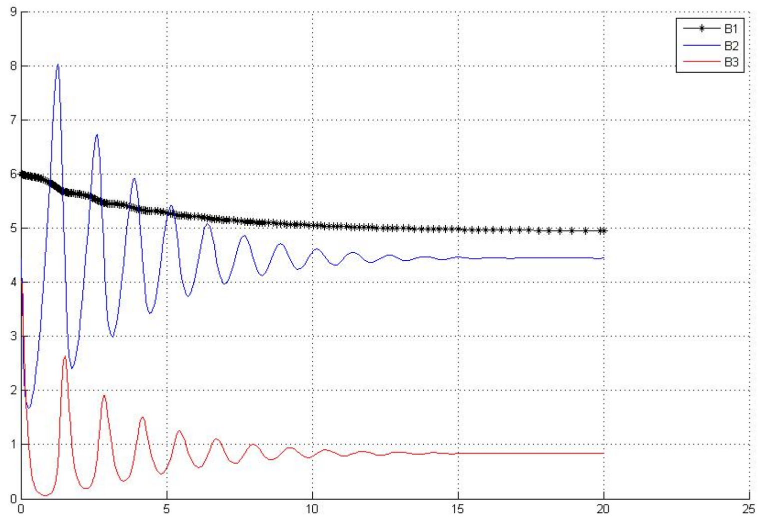

Figure 4: The evolution of the three fish populations with the increasing time.

In order to ensure the existence and stability of the locally asymptotically stable of the three fish populations let us consider the parameters of the model system (5) as $r_1 = 0.25$; $r_2 = 2.7$; $K_1 = 6$; $K_2 = 20$; $c_{12} = 0.01$; $c_{21} = 0.001$; $\alpha = 2.5$; $\beta = 12$ and $\gamma = 2.7$.

Then in this case the steady states is $B^* = (4.93; 4.44; 0.84)$.

The initial conditions are 6, 5 and 4 of the first, second and third fish population respectively. Then it is observed from the figure 4 and 5 that $P(B_1^*, B_2^*, B_3^*)$ is locally asymptotically stable and the three fish populations converge to their steady states in finite time.

Now let us consider the economic parameters such as $a_1 = 0.1$; $a_2 = 0.2$; $a_3 = 0.3$; $p_{01} = 0.15$; $p_{02} = 2$; $p_{03} = 1$; $q_1 = 0.4$; $q_2 = 0.2$; $q_3 = 0.3$; $c_1 = 0.01$; $c_2 = 0.2$; $c_3 = 0.3$ and $B_0 = 10\%B^* = (0.49; 0.44; 0.08)$. In this case the fishing effort which maximizes the profit of fishing fleet is $x^* = (0.25; 4.47; 0.57)$ and the profit is $\pi^* = 4, 69$.

Now we'll see how changes in the minimum price of each fish population can affect the profit of fishing fleet: as well as an increase in minimum price leads to an increase in profit of the fishing fleet as shown in the following table.

$p_{\min 1}$	00.15	00.65	01.15	01.65	02.15	02.65	03.15	03.65
$p_{\min 2}$	02.00	02.50	03.00	03.50	04.00	04.50	05.00	05.50
$p_{\min 3}$	01.00	01.50	02.00	02.50	03.00	03.50	04.00	04.50
Profit	04, 69	07.13	10.10	13.60	17.61	22.03	26.49	30.96

Table 1: An increase in minimum price leads to an increase in fishing fleet.

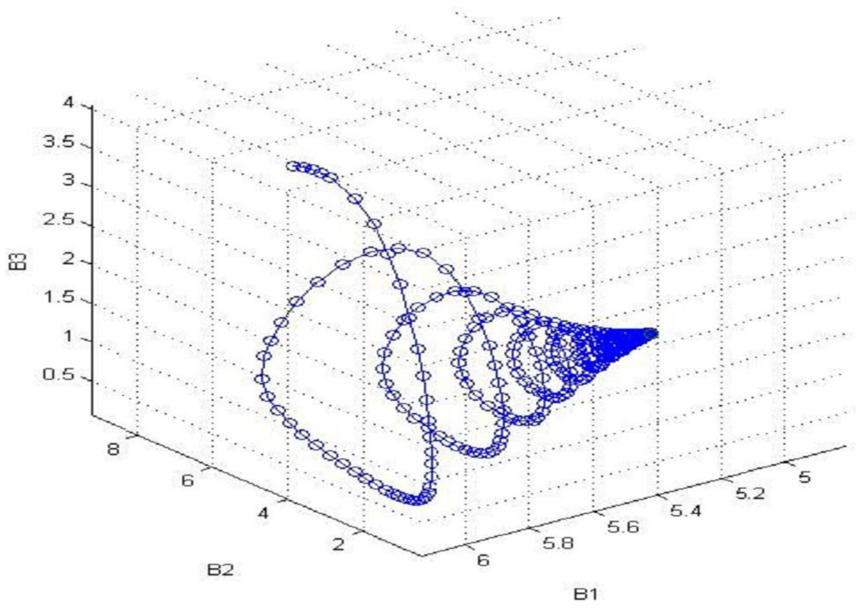

Figure 5: Phase space trajectories of the three fish populations.

5 Conclusion and perspectives

In this paper we have developed a bio-economic model that combines a model of competition and a model of prey-predator of three fish populations. In this model we have assumed that on the one hand, the evolution of the first and second fish population is described by a density dependent model taking into account the competition between fish populations which compete with each other for space or food; on the other hand, the evolution of the second and third fish population is described by a Lotka-Volterra model. In this work we have calculated the fishing effort which maximizes the income of the fishing fleet that exploits the three fish populations subject to two constraints, the first one is the sustainable management of the resources and the second one is the preservation of the biodiversity of the fish populations. The existence of the steady states and its stability are studied using eigenvalue analysis. Finally, some numerical simulations are given to illustrate the results obtained. We note that to get the numerical simulations, we used Matlab.

As a perspective we intend to generalize the results obtained by considering a bio-economic model that combines several fish populations between competition model and prey-predator model and the price of each population is variable.

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