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ON SMARANDACHE ts CURVES OF BIHARMONIC $\mathcal{S}$-CURVES ACCORDING TO SABBAN FRAME IN HEISENBERG GROUP HEIS^3

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ABSTRACT. In this paper, we study Smarandache **ts** curves according to Sabban frame in the Heisenberg group Heis^3. Finally, we find explicit parametric equations of Smarandache **ts** curves according to Sabban Frame.

1. Introduction

A smooth map $\phi : N \longrightarrow M$ is said to be biharmonic if it is a critical point of the bienergy functional:

$$E_2(\phi) = \int_N \frac{1}{2} |\mathcal{T}(\phi)|^2 \, dv_h,$$

where $\mathcal{T}(\phi) := \mathrm{tr}\nabla^{\phi} d\phi$ is the tension field of ϕ.

The Euler–Lagrange equation of the bienergy is given by $\mathcal{T}_2(\phi) = 0$. Here the section $\mathcal{T}_2(\phi)$ is defined by

$$\mathcal{T}_2(\phi) = -\Delta_\phi \mathcal{T}(\phi) + \mathrm{tr}R(\mathcal{T}(\phi), d\phi)\, d\phi, \tag{1.1}$$

and called the bitension field of ϕ. Non-harmonic biharmonic maps are called proper biharmonic maps.

This study is organised as follows: Firstly, we study Smarandache **ts** curves according to Sabban frame in the Heisenberg group Heis^3. Finally, we find explicit parametric equations of Smarandache **ts** curves according to Sabban Frame.

2. The Heisenberg Group Heis^3

Heisenberg group Heis^3 can be seen as the space $\mathbb{R}^3$ endowed with the following multipilcation:

$$(\overline{x}, \overline{y}, \overline{z})(x, y, z) = (\overline{x} + x, \overline{y} + y, \overline{z} + z - \frac{1}{2}\overline{x}y + \frac{1}{2}x\overline{y}) \tag{2.1}$$

Heis^3 is a three-dimensional, connected, simply connected and 2-step nilpotent Lie group.

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The Riemannian metric g is given by

$$g = dx^2 + dy^2 + (dz - xdy)^2.$$

The Lie algebra of Heis^3 has an orthonormal basis

$$\mathbf{e}_1 = \frac{\partial}{\partial x}, \quad \mathbf{e}_2 = \frac{\partial}{\partial y} + x\frac{\partial}{\partial z}, \quad \mathbf{e}_3 = \frac{\partial}{\partial z}, \tag{2.2}$$

for which we have the Lie products

$$[\mathbf{e}_1, \mathbf{e}_2] = \mathbf{e}_3, \quad [\mathbf{e}_2, \mathbf{e}_3] = [\mathbf{e}_3, \mathbf{e}_1] = 0$$

with

$$g(\mathbf{e}_1, \mathbf{e}_1) = g(\mathbf{e}_2, \mathbf{e}_2) = g(\mathbf{e}_3, \mathbf{e}_3) = 1.$$

We obtain

$$\begin{aligned} \nabla_{\mathbf{e}_1}\mathbf{e}_1 &= \nabla_{\mathbf{e}_2}\mathbf{e}_2 = \nabla_{\mathbf{e}_3}\mathbf{e}_3 = 0, \\ \nabla_{\mathbf{e}_1}\mathbf{e}_2 &= -\nabla_{\mathbf{e}_2}\mathbf{e}_1 = \frac{1}{2}\mathbf{e}_3, \\ \nabla_{\mathbf{e}_1}\mathbf{e}_3 &= \nabla_{\mathbf{e}_3}\mathbf{e}_1 = -\frac{1}{2}\mathbf{e}_2, \\ \nabla_{\mathbf{e}_2}\mathbf{e}_3 &= \nabla_{\mathbf{e}_3}\mathbf{e}_2 = \frac{1}{2}\mathbf{e}_1. \end{aligned} \tag{2.3}$$

The components $\{R_{ijkl}\}$ of R relative to $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ are defined by

$$R_{ijk} = R(\mathbf{e}_i, \mathbf{e}_j)\mathbf{e}_k, \quad R_{ijkl} = R(\mathbf{e}_i, \mathbf{e}_j, \mathbf{e}_k, \mathbf{e}_l) = g\left(R(\mathbf{e}_i, \mathbf{e}_j)\mathbf{e}_l, \mathbf{e}_k\right).$$

The non vanishing components of the above tensor fields are

$$R_{121} = \tfrac{3}{4}\mathbf{e}_2, \quad R_{131} = -\tfrac{1}{4}\mathbf{e}_3, \quad R_{122} = -\tfrac{3}{4}\mathbf{e}_1,$$
$$R_{232} = -\tfrac{1}{4}\mathbf{e}_3, \quad R_{133} = \tfrac{1}{4}\mathbf{e}_1, \quad R_{233} = \tfrac{1}{4}\mathbf{e}_2,$$

and

$$R_{1212} = -\tfrac{3}{4}, \quad R_{1313} = R_{2323} = \tfrac{1}{4}.$$

3. Biharmonic $\mathcal{S}$–Helices According To Sabban Frame In The Heisenberg Group Heis^3

Let $\gamma : I \longrightarrow Heis^3$ be a non geodesic curve on the Heisenberg group Heis^3 parametrized by arc length. Let $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ be the Frenet frame fields tangent to the Heisenberg group Heis^3 along γ defined as follows:

$\mathbf{T}$ is the unit vector field γ' tangent to γ, N is the unit vector field in the direction of $\nabla_{\mathbf{T}}\mathbf{T}$ (normal to γ), and $\mathbf{B}$ is chosen so that $\{\mathbf{T}, \mathbf{N}, \mathbf{B}\}$ is a positively oriented orthonormal basis. Then, we have the following Frenet formulas:

$$\begin{aligned} \nabla_{\mathbf{T}}\mathbf{T} &= \kappa\mathbf{N}, \\ \nabla_{\mathbf{T}}\mathbf{N} &= -\kappa\mathbf{T} + \tau\mathbf{B}, \\ \nabla_{\mathbf{T}}\mathbf{B} &= -\tau\mathbf{N}, \end{aligned} \tag{3.1}$$

where κ is the curvature of γ and τ is its torsion,

$$g(\mathbf{T}, \mathbf{T}) = 1, \ g(\mathbf{N}, \mathbf{N}) = 1, \ g(\mathbf{B}, \mathbf{B}) = 1,$$
$$g(\mathbf{T}, \mathbf{N}) = g(\mathbf{T}, \mathbf{B}) = g(\mathbf{N}, \mathbf{B}) = 0.$$

In the rest of the paper, we suppose everywhere

$$\kappa \neq 0 \text{ and } \tau \neq 0.$$

Now we give a new frame different from Frenet frame. Let $\alpha : I \longrightarrow \mathbb{S}^2_{Heis^3}$ be unit speed spherical curve. We denote σ as the arc-length parameter of α . Let us denote $\mathbf{t}(\sigma) = \alpha'(\sigma)$, and we call $\mathbf{t}(\sigma)$ a unit tangent vector of α. We now set a vector $\mathbf{s}(\sigma) = \alpha(\sigma) \times \mathbf{t}(\sigma)$ along α. This frame is called the Sabban frame of α on the Heisenberg group Heis^3. Then we have the following spherical Frenet-Serret formulae of α :

$$\begin{aligned} \nabla_{\mathbf{t}}\alpha &= \mathbf{t}, \\ \nabla_{\mathbf{t}}\mathbf{t} &= -\alpha + \kappa_g \mathbf{s}, \\ \nabla_{\mathbf{t}}\mathbf{s} &= -\kappa_g \mathbf{t}, \end{aligned} \tag{3.2}$$

where κ_g is the geodesic curvature of the curve α on the $\mathbb{S}^2_{Heis^3}$ and

$$\begin{aligned} g(\mathbf{t},\mathbf{t}) = 1,\ g(\alpha,\alpha) = 1,\ g(\mathbf{s},\mathbf{s}) = 1, \\ g(\mathbf{t},\alpha) = g(\mathbf{t},\mathbf{s}) = g(\alpha,\mathbf{s}) = 0. \end{aligned}$$

With respect to the orthonormal basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, we can write

$$\begin{aligned} \alpha &= \alpha_1\mathbf{e}_1 + \alpha_2\mathbf{e}_2 + \alpha_3\mathbf{e}_3, \\ \mathbf{t} &= t_1\mathbf{e}_1 + t_2\mathbf{e}_2 + t_3\mathbf{e}_3, \\ \mathbf{s} &= s_1\mathbf{e}_1 + s_2\mathbf{e}_2 + s_3\mathbf{e}_3. \end{aligned} \tag{3.3}$$

To separate a biharmonic curve according to Sabban frame from that of Frenet-Serret frame, in the rest of the paper, we shall use notation for the curve defined above as biharmonic $\mathcal{S}$-curve.

Lemma 3.1. *$\alpha : I \longrightarrow \mathbb{S}^2_{Heis^3}$ is a biharmonic $\mathcal{S}$-curve if and only if*

$$\begin{aligned} \kappa_g &= \text{constant} \neq 0, \\ 1 + \kappa_g^2 &= -[\frac{1}{4} - s_3^2] + \kappa_g[-\alpha_3 s_3], \\ \kappa_g^3 &= -\alpha_3 s_3 - \kappa_g[\frac{1}{4} - \alpha_3^2]. \end{aligned} \tag{3.4}$$

Then the following result holds.

Theorem 3.2. ([9]) *All of biharmonic $\mathcal{S}$-curves in $\mathbb{S}^2_{Heis^3}$ are helices.*

Theorem 3.3. ([9]) *Let* $\alpha : I \longrightarrow \mathbb{S}^2_{Heis^3}$ *be a unit speed non-geodesic biharmonic* $\mathcal{S}$*-curve. Then, the position vector of* α *is*

$$\begin{aligned}\alpha(\sigma) &= [-\frac{\sin^2\mathcal{E}}{\mathcal{V}}\cos[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_2]\mathbf{e}_1+[\frac{\sin^2\mathcal{E}}{\mathcal{V}}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_3]\mathbf{e}_2\\ &+[\cos\mathcal{E}\sigma-\frac{\mathcal{V}\sigma+\mathcal{M}_1}{2\mathcal{V}^2}\sin^4\mathcal{E}-\frac{\sin 2[\mathcal{M}\sigma+\mathcal{M}_1]}{4\mathcal{V}^2}\sin^4\mathcal{E}\\ &-[\frac{\sin^2\mathcal{E}}{\mathcal{V}}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_3][-\frac{\sin^2\mathcal{E}}{\mathcal{V}}\cos[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_2]\\ &+\frac{\mathcal{M}_2}{\mathcal{V}}\sin^3\mathcal{E}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_4]\mathbf{e}_3,\end{aligned} \tag{3.5}$$

where $\mathcal{M}_1,\mathcal{M}_2,\mathcal{M}_3,\mathcal{M}_4$ *are constants of integration and*

$$\mathcal{M}=(\frac{\sqrt{1+\kappa_g^2}}{\sin\mathcal{E}}-\cos\mathcal{E}) \text{ and } \mathcal{V}=\sqrt{1+\kappa_g^2}-\frac{1}{2}\sin 2\mathcal{E}.$$

4. Smarandache ts Curves Of Biharmonic $\mathcal{S}$-Curves According To Sabban Frame In The Heisenberg Group Heis^3

Definition 4.1. *Let* $\alpha : I \longrightarrow \mathbb{S}^2_{Heis^3}$ *be a unit speed regular curve in the Heisenberg group* $Heis^3$ *and* $\{\alpha,\mathbf{t},\mathbf{s}\}$ *be its moving Bishop frame. Smarandache* **ts** *curves are defined by*

$$\gamma_{\mathbf{ts}}=\frac{1}{\sqrt{1+2\kappa_g^2}}(\mathbf{t}+\mathbf{s}). \tag{4.1}$$

Theorem 4.2. *Let* $\alpha : I \longrightarrow \mathbb{S}^2_{Heis^3}$ *be a unit speed non-geodesic biharmonic* $\mathcal{S}$*-curve* $\gamma_{\mathbf{ts}}$ *its Smarandache* **ts** *curve. Then, the position vector of Smarandache* **ts** *curve is*

$$\begin{aligned}\gamma_{\mathbf{ts}}(\sigma) &= \frac{1}{\sqrt{1+2\kappa_g^2}}[\sin\mathcal{E}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\frac{1}{\kappa_g}[\sin\mathcal{E}\cos[\mathcal{M}\sigma+\mathcal{M}_1](\mathcal{M}+\cos\mathcal{E})\\ &-\frac{\sin^2\mathcal{E}}{\mathcal{V}}\cos[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_2]]\mathbf{e}_1+\frac{1}{\sqrt{1+2\kappa_g^2}}[\sin\mathcal{E}\cos[\mathcal{M}\sigma+\mathcal{M}_1]\\ &+\frac{1}{\kappa_g}[-\sin\mathcal{E}\sin[\mathcal{M}\sigma+\mathcal{M}_1](\mathcal{M}+\cos\mathcal{E})+\frac{\sin^2\mathcal{E}}{\mathcal{V}}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_3]]\mathbf{e}_2\\ &+\frac{1}{\sqrt{1+2\kappa_g^2}}[\cos\mathcal{E}+\frac{1}{\kappa_g}[\cos\mathcal{E}\sigma-\frac{\mathcal{V}\sigma+\mathcal{M}_1}{2\mathcal{V}^2}\sin^4\mathcal{E}-\frac{\sin 2[\mathcal{M}\sigma+\mathcal{M}_1]}{4\mathcal{V}^2}\sin^4\mathcal{E}\\ &-[\frac{\sin^2\mathcal{E}}{\mathcal{V}}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_3][-\frac{\sin^2\mathcal{E}}{\mathcal{V}}\cos[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_2]\\ &+\frac{\mathcal{M}_2}{\mathcal{V}}\sin^3\mathcal{E}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_4]\mathbf{e}_3,\end{aligned} \tag{4.2}$$

where $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4$ *are constants of integration and*

$$\mathcal{M} = (\frac{\sqrt{1+\kappa_g^2}}{\sin\mathcal{E}} - \cos\mathcal{E}) \text{ and } \mathcal{V} = \sqrt{1+\kappa_g^2} - \frac{1}{2}\sin 2\mathcal{E}. \tag{4.3}$$

Proof. From definition of $\mathcal{S}$-helix, we obviously obtain

$$\mathbf{t} = \sin\mathcal{E}\sin[\mathcal{M}\sigma + \mathcal{M}_1]\mathbf{e}_1 + \sin\mathcal{E}\cos[\mathcal{M}\sigma + \mathcal{M}_1]\mathbf{e}_2 + \cos\mathcal{E}\mathbf{e}_3. \tag{4.4}$$

We can easily verify that

$$\nabla_{\mathbf{t}}\mathbf{t} = (t_1' + t_2 t_3)\mathbf{e}_1 + (t_2' - t_1 t_3)\mathbf{e}_2 + t_3'\mathbf{e}_3. \tag{4.5}$$

Since, we immediately arrive at

$$\begin{aligned}\nabla_{\mathbf{t}}\mathbf{t} &= \sin\mathcal{E}\cos[\mathcal{M}\sigma + \mathcal{M}_1](\mathcal{M} + \cos\mathcal{E})\mathbf{e}_1 \\ &\quad - \sin\mathcal{E}\sin[\mathcal{M}\sigma + \mathcal{M}_1](\mathcal{M} + \cos\mathcal{E})\mathbf{e}_2.\end{aligned}$$

Obviously, we also obtain

$$\begin{aligned}\mathbf{s}(\sigma) &= \frac{1}{\kappa_g}[\sin\mathcal{E}\cos[\mathcal{M}\sigma + \mathcal{M}_1](\mathcal{M} + \cos\mathcal{E}) - \frac{\sin^2\mathcal{E}}{\mathcal{V}}\cos[\mathcal{M}\sigma + \mathcal{M}_1] + \mathcal{M}_2]\mathbf{e}_1 \\ &\quad + \frac{1}{\kappa_g}[-\sin\mathcal{E}\sin[\mathcal{M}\sigma + \mathcal{M}_1](\mathcal{M} + \cos\mathcal{E}) + \frac{\sin^2\mathcal{E}}{\mathcal{V}}\sin[\mathcal{M}\sigma + \mathcal{M}_1] + \mathcal{M}_3]\mathbf{e}_2 \\ &\quad + \frac{1}{\kappa_g}[\cos\mathcal{E}\sigma - \frac{\mathcal{V}\sigma + \mathcal{M}_1}{2\mathcal{V}^2}\sin^4\mathcal{E} - \frac{\sin 2[\mathcal{M}\sigma + \mathcal{M}_1]}{4\mathcal{V}^2}\sin^4\mathcal{E} \\ &\quad - [\frac{\sin^2\mathcal{E}}{\mathcal{V}}\sin[\mathcal{M}\sigma + \mathcal{M}_1] + \mathcal{M}_3][-\frac{\sin^2\mathcal{E}}{\mathcal{V}}\cos[\mathcal{M}\sigma + \mathcal{M}_1] + \mathcal{M}_2] \\ &\quad + \frac{\mathcal{M}_2}{\mathcal{V}}\sin^3\mathcal{E}\sin[\mathcal{M}\sigma + \mathcal{M}_1] + \mathcal{M}_4]\mathbf{e}_3,\end{aligned} \tag{4.6}$$

where

$$\mathcal{M} = (\frac{\sqrt{1+\kappa_g^2}}{\sin\mathcal{E}} - \cos\mathcal{E}) \textit{ and } \mathcal{V} = \sqrt{1+\kappa_g^2} - \frac{1}{2}\sin 2\mathcal{E}.$$

Substituting (4.4) and (4.6) in (4.1) we have (4.3), which completes the proof.

Corollary 4.3. *Let* $\alpha : I \longrightarrow \mathbb{S}^2_{Heis^3}$ *be a unit speed non-geodesic biharmonic* $\mathcal{S}$*-curve* $\gamma_{\mathbf{ts}}$ *its Smarandache* **ts** *curve. Then, the parametric equations of*

Smarandache **ts** *curve are*

$$x_{\mathbf{ts}}(\sigma)=\frac{1}{\sqrt{1+2\kappa_g^2}}[\sin\mathcal{E}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\frac{1}{\kappa_g}[\sin\mathcal{E}\cos[\mathcal{M}\sigma+\mathcal{M}_1](\mathcal{M}+\cos\mathcal{E})$$
$$-\frac{\sin^2\mathcal{E}}{\mathcal{V}}\cos[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_2]],$$

(4.7)

$$y_{\mathbf{ts}}(\sigma)=\frac{1}{\sqrt{1+2\kappa_g^2}}[\sin\mathcal{E}\cos[\mathcal{M}\sigma+\mathcal{M}_1]+\frac{1}{\kappa_g}[-\sin\mathcal{E}\sin[\mathcal{M}\sigma+\mathcal{M}_1](\mathcal{M}+\cos\mathcal{E})$$
$$+\frac{\sin^2\mathcal{E}}{\mathcal{V}}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_3]],$$

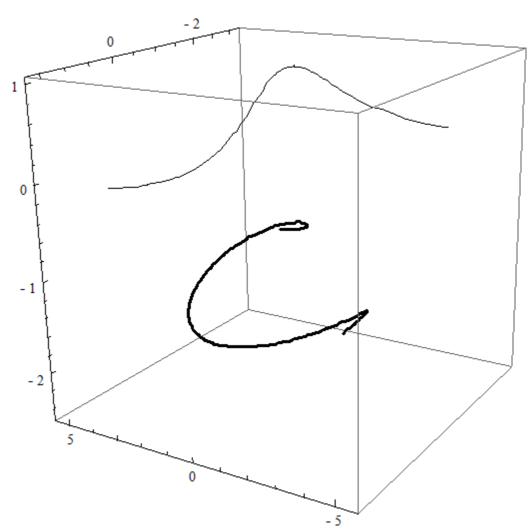
$$z_{\mathbf{ts}}(\sigma)=\frac{1}{\sqrt{1+2\kappa_g^2}}[\cos\mathcal{E}+\frac{1}{\kappa_g}[\cos\mathcal{E}\sigma-\frac{\mathcal{V}\sigma+\mathcal{M}_1}{2\mathcal{V}^2}\sin^4\mathcal{E}-\frac{\sin 2[\mathcal{M}\sigma+\mathcal{M}_1]}{4\mathcal{V}^2}\sin^4\mathcal{E}$$
$$-[\frac{\sin^2\mathcal{E}}{\mathcal{V}}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_3][-\frac{\sin^2\mathcal{E}}{\mathcal{V}}\cos[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_2]$$
$$+\frac{\mathcal{M}_2}{\mathcal{V}}\sin^3\mathcal{E}\sin[\mathcal{M}\sigma+\mathcal{M}_1]+\mathcal{M}_4],$$

where $\mathcal{M}_1,\mathcal{M}_2,\mathcal{M}_3,\mathcal{M}_4$ *are constants of integration and*

$$\mathcal{M}=(\frac{\sqrt{1+\kappa_g^2}}{\sin\mathcal{E}}-\cos\mathcal{E})\ \textit{and}\ \mathcal{V}=\sqrt{1+\kappa_g^2}-\frac{1}{2}\sin 2\mathcal{E}.$$

Proof. Substituting (2.1) to (4.2), we have (4.7) as desired.

If we use Mathematica both unit speed non-geodesic biharmonic $\mathcal{S}$-curve and its Smarandache **ts** curve, we have



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