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BILEVEL FIXED CHARGE TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS

DEEPA THIRWANI
Department of Mathematics, Hans Raj College, University of Delhi, Delhi

RITU ARORA
Department of Mathematics, Keshav Mahavidyalaya, University of Delhi, Delhi

Abstract

Bilevel programming is a two-stage optimization problem where the constraint region of the first level problem is implicitly determined by another optimization problem. In this paper, we consider the bilevel programming problem (BLPP) in which both the leader and the followers' problem are fixed charge Transportation problems. This paper is divided into two sections. In Section I of this paper, we restrict the total transportation flow to a known specified level and in Section II, the transportation flow is enhanced. Both the problems are converted into a standard fixed charge transportation problems. The algorithms based on the concept that the optimal solution of BLPP lies at an extreme point are presented to solve the problems which are illustrated with the help of an example.

Keywords: Bilevel programming problem, Non-convex optimization, Fixed charge transportation problem, Restricted Flow, Enhanced Flow.

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Introduction

Bilevel programming problem has been developed and studied by Bialas and Karwan [6, 7] in the year 1982, 1984; Candler and Townsley [10] in 1982; Bard [1, 2, 3] in the year 1983, 84, 92. The Bilevel programming problem can be thought of as a static version of Stackelberg leader follower game in which a stackelberg strategy is used by the leader, given the rational reaction of the follower.

The bilevel programming structure has also been used to model problems concerning traffic signal optimization [24], structural design [27], and genetic algorithms [13]. A subclass of linear programming problem is the Transportation problem. There are different types of transportation problems and the simplest of them was presented by Hitchcock in 1941 [16], along with a constructive solution and later independently by Koopman in 1947 [21]. F. Glover et al. [11] in 1974 considered a constrained transportation problem. D. Klingman et al. [20] in 1975 introduced a specialized method for solving a transportation problem with several additional constraints. Brigden [8], in 1974 considered the transportation problem with mixed constraints.

Fixed Charge Transportation Problem (FCTP)

The fixed charge transportation problem (FCTP) is a non-convex transportation problem. It was originally formulated by Dantzig and Hirsch [15] in 1954. K.G. Murty [23] solved the (FCTP) by ranking the extreme points. A number of branch and bound type methods for FCTP's have been developed by Barr et al. [4], Cabot and Erenguc [9], Gray [12], Kennington and Unger [17], Lamer and Wallace [22] and Palekar et al. [25]. Sandrock [26] gave a simple algorithm for solving a (FCTP), Basu et al. [5] gave an algorithm for solving a (FCTP), Thirwani et al. [28] discussed the (FCTP) with restricted flow. Khanna et al. [18,19] developed techniques for solving transportation problem when the flow is either restricted or enhanced.

As an application of Bilevel Transportation problem, consider a cold-drink manufacturer who has bottling plants located in different cities of India. The bottles are supplied from different factories to the whole sellers. It is a bilevel transportation problem in which the factories are at the upper level transporting bottles to the whole sellers which are at the lower level . The whole sellers then transport the bottles to the retailers.

SECTION I

When the total availability is not equal to the total demand, the some of the source and/ or destination constraints are satisfied as inequations. Sometimes situations may arise when one wishes to keep reserve stocks at sources, say for emergencies, thereby restricting the total transportation flow to a known specified level say P, where $P < \text{Min}\left(\sum_{i \in I} a_i, \sum_{j \in J} b_j\right)$.

Mathematical Formulation

A Bilevel Fixed charge Transportation Problem with restricted flow (BFTPR) is defined as

(BFTPR) : $\underset{X_1}{\text{Min}}\, Z_1 = c_1^T X_1 + c_2^T X_2 + F_1$

where X_2 solves, for a given X_1

$\underset{X_2}{\text{Min}}\, Z_2 = d_1^T X_1 + d_2^T X_2 + F_2$

subject to

$$\sum_{j \in J} x_{ij} \le a_i, \qquad i \in I$$

$$\sum_{i \in I} x_{ij} \le b_j, \qquad j \in J \qquad (1)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ij} = P < \text{Min}\left(\sum_{i \in I} a_i, \sum_{j \in J} b_j\right)$$

$$x_{ij} \geq 0, \forall\, i \in I, j \in J.$$

$$a_i > 0,\ b_j > 0,\ \ \forall\ i \in I,\ j \in J$$

where $c_1 = [c_{ij}] \qquad \forall\, i \in I_1 = \{1, 2, ..., m_1\};\ j \in J_1 = \{1, 2, ..., n_1\}$

$$c_2 = [c_{ij}] \qquad \forall\ i \in I_2 = \{m_1 + 1, ..., m\};\ \ j \in J_2 = \{n_1 + 1, ..., n\}$$

$$I = I_1 \cup I_2 = \{1, 2, ..., m\}$$

$$J = J_1 \cup J_2 = \{1, 2, ..., n\}$$

$$d_1 = [d_{ij}] \quad \forall\ i \in I_1;\ j \in J_1 \text{ and } d_2 = [d_{ij}];\ \forall\ i \in I_2;\ j \in J_2$$

$X_1 = [x_{ij}], \forall\, i \in I_1,\ j \in J_1$ and $X_2 = [x_{ij}]; \forall\, i \in I_2;\ j \in J_2$ are the variables controlled by the upper level and lower level problems respectively.

Here, $a_i (i \in I)$ are the amount of goods available at the ith origin and $b_j (j \in J)$ are the demands at the jth destinations. c_{ij} and d_{ij} ($i \in I$ and $j \in J$) are per unit costs of transportation of goods from the ith origin to the jth destination of the upper level and the lower level problems respectively.

$F_1 = \sum_{i \in I} F_i$ is the total fixed cost for the upper level problem and F_i is the fixed cost associated with origin i for the upper level problem and $F_2 = \sum_{i \in I} F_i'$ is the total fixed cost for the lower level problem and F_i' is the fixed cost associated with origin i for the lower level problem.

For formulation of F_i ($i \in I$), assume that F_i ($i \in I$) has p number of steps so that

$$F_i = \sum_{\ell=1}^{P} {}_i\, \delta_{i\ell} F_{i\ell},\ i \in I \text{ where}$$

$$\delta_{i\ell} = 1 \text{ if } \sum_{j=1}^{n} x_{ij} > A_{i\ell},\ i \in I,\ \ell \in L = \{1, 2, ..., p\}$$
$$= 0 \quad \text{otherwise.}$$

Here, $0 < A_{i1} < A_{i2} < ... < A_{ip}$

$A_{i1}, A_{i2}, ..., A_{ip}$ $(i \in I)$ are constants and $F_{i\ell}$ $(i \in I, \ell \in L)$ are fixed costs.

Similarly, for formulation of F_i' $(i \in I)$, assume that F_i' $(i \in I)$ has q number of steps, so that $F_i' = \sum_{k=1}^{q} \delta_{ik}' F_{ik}', \ i \in I$, where

$$\delta_{ik}' = 1 \text{ if } \sum_{j=1}^{n} x_{ij} > B_{ik}, \qquad i \in I,\ k \in K = \{1, 2, ..., q\}$$

$$= 0 \quad \text{otherwise.}$$

Here, $0 < B_{i1} < B_{i2} < ... < B_{iq}$

$B_{i1}, B_{i2}, ... B_{iq}$ $(i \in I)$ are constants and F_{ik}' $(i \in I,\ k \in K)$ are fixed costs.

Algorithmic Development for (BFTPR)

To solve the problem (BFTPR), we separate it into two problems, upper level fixed charge transportation problem with restricted flow (P1) and lower level fixed charge transportation problem with restricted flow (P2), defined as

(P1): $\underset{X_1}{\text{Min}}\, Z_1 = c_1^T X_1 + c_2^T X_2 + F_1$

subject to (1)

(P2): $\underset{X_2}{\text{Min}}\, Z_2 = d_1^T X_1 + d_2^T X_2 + F_2$, for a given X_1

subject to (1).

The flow constraint in the problem (BFTPR) implies that a total $(\sum_{i \in I} a_i - P)$ of source reserves has to be kept at the various sources and a total $(\sum_{j \in J} b_j - P)$ of destination slacks is to be retained at the various destinations. Therefore, an extra destination to receive the source reserves and an extra source to fill up the destination slacks are introduced. Hence, the Related Bilevel Fixed charge Transportation Problem with Restricted Flow for upper level and lower level problems are defined.

To solve the problem (P1) the related fixed charge transportation problem with restricted flow (RP1) is formulated with an additional supply point and an additional destination point.

(RP1): $\text{Min } Z_1 = c_1'^T Y_1 + c_2'^T Y_2 + F_1^1$

subject to

$$\sum_{j \in J'} y_{ij} = a_i', \quad i \in I' = I \cup \{m+1\}$$

$$\sum_{i \in I'} y_{ij} = b_j', \quad j \in J' = J \cup \{n+1\} \qquad (2)$$

$$y_{ij} \geq 0, \qquad i \in I',\ j \in J'$$

where $c_{ij}' = c_{ij}, \quad i \in I,\ j \in J$

$c'_{i,n+1} = c'_{m+1,j} = 0,\ i \in I,\ j \in J$

$c'_{m+1,n+1} = M$, where M is a large positive number.

$F_{m+1} = 0$,

$a_i' = a_i, \quad i \in I \qquad b_j' = b_j, \quad j \in J$

$a'_{m+1} = \left(\sum_{j \in J} b_j - P \right) \qquad b'_{n+1} = \left(\sum_{i \in I} a_i - P \right)$

Solve the problem (RP1) [5]. Since an additional source and an additional destination have been added, the method is such that it moves from an (m+1)×(n+1) fixed charge transportation problem and to an m×n fixed charge transportation problem.

On solving the related fixed charge transportation problem (RP1), let $Y^* = \{y^*_{ij}\},\ i \in I',\ j \in J'$ be the optimal solution. The optimal solution $\{x^*_{ij}\}$ to the fixed charge transportation problem with restricted flow (P1) is then derived from the following transformation

$$y_{ij} = x_{ij} \qquad i \in I,\ j \in J$$

$$y_{i,n+1} = a_i - \sum_{J} x_i\ , \qquad i \in I$$

$$y_{m+1,j} = b_j - \sum_{i \in I} x_{ij}\ , \qquad j \in J$$

$$y_{m+1,n+1} = 0.$$

Let the optimal solution of the upper level fixed charge transportation problem be denoted by $X^* = (X^*_1, X^*_2)$, with the value of the objective function as $Z_1{}^*$. Putting the value of $X_1 = X^*_1$ in the lower level fixed charge transportation problem (P2), its related problem (RP2) is formulated by the method explained above. Let $\hat{X}_2$ be its optimal solution with the value of the objective function as $\hat{Z}_2$. If $X^*_2 = \hat{X}_2$, then X^* is the optimal solution of the given problem (BFTPR).

If $X^*_2 \neq \hat{X}_2$, then find an alternate optimal solution of the problem (RP1). From this derive the solution of (P1). Let it be $X^{**} = (X^{**}_1, X^{**}_2)$, repeat the above process for $X_1 = X^{**}_1$. Let $\hat{\hat{X}}_2$ be the optimal solution of (P2).

If $X^{**}_2 = \hat{\hat{X}}_2$, then X^{**} is the optimal solution of given (BFTPR). If not, then test for other alternate solutions till we get the optimal solution of (BFTPR). This process must end in a finite number of steps because the solution of (BFTPR) lies at an extreme point which are finite in number.

If there does not exist an alternate solution of (RP1) which gives the solution of the given Bilevel programming Problem (BFTPR), then find the second best solution of (RP1) and repeat the above process to find the solution of (BFTPR).

Algorithm for Solving Bilevel Fixed Charge Transportation Problem with Restricted Flow

Step 1: Consider the Bilevel Fixed Charge Transportation Problem with Restricted Flow (BFTPR).

Step 2: Separate the problem (BFTPR) into two problems (P1) and (P2).

Step 3: Set k = 0, where k is the number of iterations in the algorithm.

Step 4: Set k = k + 1, k = 0, 1, 2, ...

Step 5: To solve (P1), formulate its related Fixed charge Transportation Problem (RP1), by introducing an additional row with availability $\left(\sum_{i\in I} a_i - P\right)$ and an additional column with demand $\left(\sum_{j\in J} b_j - P\right)$ respectively. Find a basic feasible solution of this problem with respect to the variable costs only. Applying the transformation, find the basic feasible solution of problem (P1).

Step 6: Find the corresponding fixed cost, Let it be denoted by $F_1^k(\text{current})$ where

$F_1^k(\text{current}) = \sum_{i=1}^{m} F_i$. Also, find $(A_{ij}^k)_1 = (c_{ij})_k \times (E_{ij})_k$, where $(c_{ij})_k = c_{ij} - u_i^k - v_j^k$, for all $(i, j) \notin B$, u_i^k, v_j^k are the dual variables for i = 1, ..., m + 2; j = 1, 2, ..., n + 2 and $(A_{ij}^k)_1$ is the change in the cost of the upper level that occurs on introducing a non-basic cell (i, j) with value $(E_{ij})_k$ into the basis by making reallocations.

Step 7(a) : Find $(F_{ij}^k)_1$ (Difference) = $(F_{ij}^k)_1$(NB) − $F_1{}^k$ (current) where $(F_{ij}^k)_1$ (NB) is the total fixed cost obtained on introducing the cell (i,j) into the basis.

(b): Find $(\Delta_{ij}^k)_1 = (F_{ij}^k)_1$ (Difference) $+ (A_{ij}^k)_1$, for all (i, j) $\notin$ B.

If all $(\Delta_{ij}^k)_1 \geq 0$, then go to step 8, otherwise find

$(\Delta_{pq})_1 = \text{Min}\{(\Delta_{ij}^k)_1 : (\Delta_{ij}^k)_1 < 0,\ (i, j) \notin B\}$.

Then, the cell (p, q) will enter into the basis. Go to step 6.

Step 8 : Find the optimal solution of the problem (P1) using the transformation. Let it be denoted by $X^* = (X_1^*, X_2^*)$.

Step 9: For a given $X_1 = X_1^*$, solve the problem (P2).

Formulate its related fixed charge transportation problem (RP2) and solve it by the method explained above. Let $\hat{X}_2$ be its optimal solution.

Step 10: If $X_2^* = \hat{X}_2$, then X^* is an optimal solution of the given problem (BFTPR).

If $X_2^* \neq \hat{X}_2$, find all possible alternate optimal solutions of (P1) and go to step 8.

The procedure is repeated till an optimal solution of Bilevel Fixed charge Transportation problem with Restricted Flow is obtained.

EXAMPLE: Consider the following Bilevel fixed charge transportation problem with Restricted Flow, as

(BFTPR): $$\underset{X_1}{\text{Minimize}}\, Z_1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}x_{ij} + \sum_{i=1}^{3} F_i$$

where X_2 solves

$$\underset{X_2}{\text{Minimize}}\, Z_2 = \sum_{i=1}^{3} \sum_{j=1}^{4} d_{ij}x_{ij} + \sum_{i=1}^{3} F_i'$$

subject to

$$\sum_{j=1}^{4} x_{1j} \le 65, \quad \sum_{j=1}^{4} x_{2j} \le 55$$

$$\sum_{j=1}^{4} x_{3j} \le 90, \quad \sum_{i=1}^{3} x_{i1} \le 85 \qquad (3)$$

$$\sum_{i=1}^{3} x_{i2} \le 35, \quad \sum_{i=1}^{3} x_{i3} \le 50$$

$$\sum_{i=1}^{3} x_{i4} \le 45$$

$x_{ij} \ge 0, \qquad i = 1, 2, 3, ; \quad j = 1, 2, 3,4.$

Here, $X_1 = (x_{11}, x_{12}, x_{13}, x_{14})$ are the variables controlled by the leader.

$X_2 = (x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34})$ are the variables controlled by the follower.

The above problem is separated into two problems.

The upper level problem (P1) is

$$\underset{X_1}{\text{Minimize}}\, Z_1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}x_{ij} + \sum_{i=1}^{3} F_i$$

subject to (3).

The lower level problem (P2) is

$$\underset{X_2}{\text{Minimize}}\, Z_2 = \sum_{i=1}^{3} \sum_{j=1}^{4} d_{ij} x_{ij} + \sum_{i=1}^{3} F_i'$$

subject to (3).

Table (1) and Table (2) give the values of variable cost c_{ij} (i = 1, 2,3 ; j = 1, 2, 3, 4) and d_{ij} (i = 1, 2, 3; j = 1, 2, 3, 4) for the upper level and lower level problems respectively.

i \ j	1	2	3	4	a_i
1	22	26	27	29	≤ 65
2	11	16	30	51	≤ 55
3	21	28	23	43	≤ 90
b_{ij}	≤ 85	≤ 35	≤ 50	≤ 45	

Table 1

i \ j	1	2	3	4	a_i
1	6	1	9	3	≤ 65
2	11	5	2	8	≤ 55
3	10	12	4	7	≤ 90
b_{ij}	≤ 85	≤ 35	≤ 50	≤ 45	

Table 2

The fixed costs for the upper level problem are

$F_{11} = 50$, $F_{12} = 25$ $F_{13} = 25$ $F_{14} = 20$

$F_{21} = 100$ $F_{22} = 50$ $F_{23} = 50$ $F_{24} = 25$

$F_{31} = 100$ $F_{32} = 50$ $F_{33} = 25$ $F_{34} = 25$

The total cost which is to be minimized is given by $\left(\sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij} + \sum_{i=1}^{3} F_i \right)$,

where $\sum_{l=1}^{3} \delta_{i\ell} F_{i\ell}$, $i = 1, 2, 3$

where $\delta_{i1} = 1$ if $\sum_{j=1}^{4} x_{ij} > 40$ for $i = 1, 2, 3$

$= 0$ otherwise

$\delta_{i2} = 1$ if $\sum_{j=1}^{4} x_{ij} > 60$ for $i = 1, 2, 3$

$= 0$ otherwise

$\delta_{i3} = 1$ if $\sum_{j=1}^{4} x_{ij} > 70$ for $i = 1, 2, 3$

$= 0$ otherwise

$\delta_{i4} = 1$ if $\sum_{j=1}^{4} x_{ij} > 80$ for $i = 1, 2, 3$

$= 0$ otherwise

The fixed costs for the lower level problem are

$F'_{11} = 50$ $F'_{12} = 50$ $F'_{13} = 25$ $F'_{14} = 20$

$F'_{21} = 25$ $F'_{22} = 25$ $F'_{23} = 20$ $F'_{24} = 10$

$F'_{31} = 20$ $F'_{32} = 20$ $F'_{33} = 10$ $F'_{34} = 5$

The total cost which is to be minimized is given by $\left(\sum_{i=1}^{3} \sum_{j=1}^{4} d_{ij} x_{ij} + \sum_{i=1}^{3} F'_i \right)$,

where $F'_i = \sum_{k=1}^{3} \delta'_{ik} F'_{ik}$, $i = 1, 2, 3$

where $\delta'_{i1} = 1$ if $\sum_{j=1}^{4} x_{ij} > 40$ for $i = 1, 2, 3$

$= 0$ otherwise

$$\delta'_{i2} = 1 \quad \text{if } \sum_{j=1}^{4} x_{ij} > 50 \quad \text{for } i = 1,2,3$$
$$= 0 \quad \text{otherwise}$$

$$\delta'_{i3} = 1 \quad \text{if } \sum_{j=1}^{4} x_{ij} > 60 \quad \text{for } i = 1,2,3$$
$$= 0 \quad \text{otherwise}$$

$$\delta'_{i4} = 1 \quad \text{if } \sum_{j=1}^{4} x_{ij} > 70 \quad \text{for } i = 1,2,3$$
$$= 0 \quad \text{otherwise}$$

Suppose for the upper level problem, the flow is enhanced to $P = 205 < \text{Min}\left(\sum_{i \in I} a_i, \sum_{j \in J} b_j\right)$. Introduce an additional source and an additional destination in Table (1) and form the corresponding related fixed charge transportation problem (RP1) for the upper level problem. Its initial basic feasible solution is given in Table (3).

i \ j	1	2	3	4	5	u_i
1	22, 3	26, (25)	27, 6	29, (35)	0, (5)	–2
2	11, (55)	16, –2	30, 17	51, 30	0, 8	–10
3	21, (30)	28, (10)	23, (50)	43, 12	0, –2	0
4	0, 10	0, 3	0, 8	0, (10)	M	–31
v_j	21	28	23	31	2	

Table 3

Using the transformation, the basic feasible solution for the upper level problem with fixed cost is given by

i \ j	1	2	3	4	F_1' (current)
1	22	26 (25)	27	29 (35)	50
2	11 (55)	16	30	51	100
3	21 (30)	28 (10)	23 (50)	43	200

Table 4

Calculate $(c_{ij})_1 = c_{ij} - u_i^1 - v_j^1 \quad \forall (i, j) \notin B$

and $(\Delta_{ij}^1)_1 = (F_{ij}^1)_1 \text{ (Difference)} + (A_{ij}^1)_1, \ \forall (i, j) \notin B$

which is given in Table 5.

(i,j)	(1,1)	(1,3)	(2,2)	(2,3)	(2,4)	(2,5)	(3,4)	(3,5)	(4,1)	(4,2)	(4,3)
$(A'_{ij})_1$	75	150	–20	850	300	40	120	-10	100	30	80
$(\Delta'_{ij})_1$	75	150	–20	850	300	65	120	15	100	30	80

Table 5

From Table 5, $(\Delta_{22}^1)_1 = -20$ is most negative. Therefore, (x_{22}) enters the basis. Proceeding as above, the optimal solution for the upper level problem so obtained is $X_1^* = (0, 25, 0, 35)$ & $X_2^* = (45, 10, 0, 0, 40, 0, 50, 0)$.

Putting the values of $X_1 = X_1^*$ in the lower level problem and solving by the same procedure as above, the optimal solution for the lower level problem is

$$\hat{X}_2 = (0, 10, 40, 0, 70, 0, 10, 10).$$

Since $X_2^* \neq \hat{X}_2$, find alternate optimal solutions for (P1). An alternate optimal solution for (P1) is $X_1^{**} = (25, 0, 0, 35)$ & $X_2^{**} = (20, 35, 0, 0, 40, 0, 50, 0)$.

Corresponding to X_1^{**}, the optimal solution or (P2) is

$$\hat{\hat{X}}_2 = (0, 35, 15, 0, 45, 0, 35, 10).$$

Since $X_2^{**} \neq \hat{\hat{X}}_2$, this is not the optimal solution for (BFTPR).

An alternate optimal solution for (P1) is $X_1^{***} = (25, 0, 0, 35)$ and $X_2^{***} = (0, 35, 20, 0, 60, 0, 30, 0)$.

Corresponding to X_1^{***}, the optimal solution for (P2) is

$\hat{\hat{\hat{X}}}_2 = (0, 35, 15, 0, 45, 0, 35, 10)$. Again, $X_2^{***} \neq \hat{\hat{\hat{X}}}_2$.

Another alternate solution for (P1) is $X_1^{****} = (30, 0, 0, 35)$ and $X_2^{****} = (0, 35, 20, 0, 55, 0, 30, 0)$.

The optimal solution for (P2) is $\hat{\hat{\hat{\hat{X}}}}_2 = (0, 35, 15, 0, 45, 0, 35, 10)$.

$$X_2^{****} \neq \hat{\hat{\hat{\hat{X}}}}_2$$

Find another alternate optimal solution for (P1) as $X_1^{*****} = (20, 0, 0, 45)$ and $X_2^{*****} = (0, 35, 20, 0, 55, 0, 30, 0)$. For (P2) optimal solution is $\hat{\hat{\hat{\hat{\hat{X}}}}}_2 = (0, 35, 15, 0, 55, 0, 35, 0)$. $X_2^{*****} \neq \hat{\hat{\hat{\hat{\hat{X}}}}}_2$

An alternate optimal solution for (P1) is $X_1^{******} = (20, 0, 0, 45)$ and $X_2^{******} = (0, 35, 15, 0, 55, 0, 35, 0)$. For (P2) optimal solution is $\hat{\hat{\hat{\hat{\hat{\hat{X}}}}}}_2 = (0, 35, 15, 0, 55, 0, 35, 0)$.

Since, $X_2^{******} = \hat{\hat{\hat{\hat{\hat{\hat{X}}}}}}_2$, this is the optimal solution for (BFTPR).

Hence, the optimal solution for (BFTPR) is

(20, 0, 0, 45, 0, 35, 15, 0, 55, 0, 35, 0) with $Z_1 = 4715$ and $Z_2 = 1150$.

SECTION II

When the total availability is not equal to the total demand, then some of the source and/or destination constraints are satisfied as inequations. Sometimes situations may arise when because of the extra demand in the market, the total flow needs to be enhanced, compelling some of the factories to increase their productions in order to meet this extra demand. The total flow from the factories in the market is now increased by the amount of extra demand.

Let $P > \text{Max}\left(\sum_{i\in I} a_i, \sum_{j\in J} b_j\right)$ be the enhanced flow.

The Bilevel Fixed charge Transportation Problem with Enhanced Flow (BFTPE) is defined as

(BFTPE) : $\underset{X_1}{\text{Min}}\, Z_1 = c_1^T X_1 + c_2^T X_2 + F_1$

where X_2 solves, for a given X_1

$\underset{X_2}{\text{Min}}\, Z_2 = d_1^T X_1 + d_2^T X_2 + F_2$

subject to

$$\sum_{j\in J} x_{ij} \geq a_i, \qquad i \in I$$

$$\sum_{i\in I} x_{ij} \geq b_j, \qquad j \in J \qquad (1)$$

$$\sum_{i\in I} \sum_{j\in J} x_{ij} = P > \text{Max}\left(\sum_{i\in I} a_i, \sum_{j\in J} b_j\right)$$

$x_{ij} \geq 0$, $\forall$ $i \in I$, $j \in J$.

$a_i > 0,\ b_j > 0,\ \ \forall\ i \in I,\ j \in J$

The symbols defined in the above problem are same as defined in Section I.

Algorithmic Development for (BFTPE)

To solve the problem (BFTPE), we separate it into two problems, upper level fixed charge transportation problem with enhanced flow (P3) and lower level fixed charge transportation problem with enhanced flow (P4), defined as

(P3): $$\min_{X_1} Z_1 = c_1^T X_1 + c_2^T X_2 + F_1$$

subject to (4).

(P4): $$\min_{X_2} Z_2 = d_1^T X_1 + d_2^T X_2 + F_2, \text{ for a given } X_1$$

subject to (4).

In order to deal with the flow constraint in the problem (BFTPE), the Related Bilevel Fixed charge Transportation Problem with Enhanced Flow for upper level and lower level problems are defined. The problems are formulated by adding a fictitious factory with availability $\left(P - \sum_{i \in I} a_i\right)$ and a fictitious destination with demand equal to $\left(P - \sum_{j \in J} b_j\right)$. Hence, to solve the problem (P3) the related fixed charge transportation problem with enhanced flow (RP3) is formulated with an additional supply point and an additional destination point.

(RP3): $$\text{Min } Z_1 = c_1'^T Y_1 + c_2'^T Y_2 + F_1^1$$

subject to

$$\sum_{j \in J'} y_{ij} = a_i', \quad i \in I' = I \cup \{m+1\}$$

$$\sum_{i \in I'} y_{ij} = b_j', \quad j \in J' = I \cup \{n+1\} \qquad (5)$$

$$y_{ij} \geq 0, \qquad i \in I',\ j \in J'$$

where $c'_{ij} = c_{ij}, \quad i \in I,\ j \in J$

$$c'_{i,n+1} = \underset{j \in J}{\text{Min}}\, c_{ij} \qquad c'_{m+1,j} = \underset{i \in I}{\text{Min}}\, c_{ij}$$

$c'_{m+1,n+1} = M$, where M is a large positive number.

$$a'_i = a_i, \quad i \in I \qquad b'_j = b_j, \quad j \in J$$

$$a'_{m+1} = P - \sum_{i \in I} a_i \qquad b'_{n+1} = P - \sum_{j \in J} b_j$$

$$F_{m+1} = 0$$

The optimal solution of (BFTPE) is obtained by solving the related problem (RP3) as (RP1), explained in Section I.

Algorithm for Solving Bilevel Fixed Charge Transportation Problem with Enhanced Flow

Step 1: Consider the Bilevel Fixed Charge Transportation Problem with Enhanced Flow (BFTPE).

Step 2: Separate the problem (BFTPE) into the problems (P3) and (P4).

Step 3: Set k = 0, where k is the number of iterations in the algorithm.

Step 4: Set k = k + 1, k = 0, 1, 2, ...

Step 5: To solve (P3), formulate its related fixed charge transportation problem (RP3), by introducing an additional row with availability $\left(P - \sum_{i \in I} a_i\right)$ and an additional column with availability $\left(P - \sum_{j \in J} b_j\right)$. respectively. Find a basic feasible solution of this problem with respect to the variable costs only. Applying the transformation, find the basic feasible solution of problem (P3).

Step 6: Find the corresponding fixed cost, Let it be denoted by $F_1^k(\text{current})$ where

$F_1^k(\text{current}) = \sum_{i=1}^{m} F_i$. Also, find $(A_{ij}^k)_1 = (c_{ij})_k \times (E_{ij})_k$,

where $(c_{ij})_k = c_{ij} - u_i^k - v_j^k$, for all $(i, j) \notin B$, u_i^k, v_j^k are the dual variables for i = 1, ..., m + 2; j = 1, 2, ..., n + 2 and $(A_{ij}^k)_1$ is the change in the cost of the upper level that occurs on introducing a non-basic cell (i, j) with value $(E_{ij})_k$ into the basis by making reallocations.

Step 7(a) : Find $(F_{ij}^k)_1$ (Difference) = $(F_{ij}^k)_1$(NB) − $F_1{}^k$ (current) where $(F_{ij}^k)_1$ (NB) is the total fixed cost obtained on introducing the cell (i,j) into the basis.

(b): Find $(\Delta_{ij}^k)_1 = (F_{ij}^k)_1\ (\text{Difference}) + (A_{ij}^k)_1$, for all (i, j) $\notin$ B. If all $(\Delta_{ij}^k)_1 \geq 0$, then go to step 8, otherwise find

$(\Delta_{pq})_1 = \text{Min}\{(\Delta_{ij}^k)_1 : (\Delta_{ij}^k)_1 < 0,\ (i, j) \notin B\}$.

Then, the cell (p, q) will enter into the basis. Go to step 6.

Step 8 : Find the optimal solution of the problem (P3) using the transformation. Let it be denoted by $X^* = (X_1^*, X_2^*)$.

Step 9: For a given $X_1 = X_1^*$, solve the problem (P4).

Formulate its related fixed charge transportation problem (RP4) and solve it by the method explained above. Let $\hat{X}_2$ be its optimal solution.

Step 10: If $X_2^* = \hat{X}_2$, then X^* is an optimal solution of the given problem (BFTPE).

If $X_2^* \neq \hat{X}_2$, find all possible alternate optimal solutions of (P3) and go to step 8.

The procedure is repeated till an optimal solution of Bilevel Fixed charge Transportation problem with Enhanced Flow is obtained.

EXAMPLE: Consider the following Bilevel fixed charge transportation problem with Enhanced Flow, as

(BFTPE): $$\underset{X_1}{\text{Minimize}}\, Z_1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij} + \sum_{i=1}^{3} F_i$$

where X_2 solves

$$\underset{X_2}{\text{Minimize}}\, Z_2 = \sum_{i=1}^{3} \sum_{j=1}^{4} d_{ij} x_{ij} + \sum_{i=1}^{3} F_i'$$

subject to

$$\sum_{j=1}^{4} x_{1j} \geq 65, \quad \sum_{j=1}^{4} x_{2j} \geq 60$$

$$\sum_{j=1}^{4} x_{3j} \geq 90, \quad \sum_{i=1}^{3} x_{i1} \geq 85 \qquad (6)$$

$$\sum_{i=1}^{3} x_{i2} \geq 40, \quad \sum_{i=1}^{3} x_{i3} \geq 50$$

$$\sum_{i=1}^{3} x_{i4} \geq 45$$

$x_{ij} \geq 0$, $\quad$ i = 1, 2, 3, ; j = 1, 2, 3,4.

Here, $X_1 = (x_{11}, x_{12}, x_{13}, x_{14})$ are the variables controlled by the leader.

$X_2 = (x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34})$ are the variables controlled by the follower.

The above problem is separated into two problems.

The upper level problem (P3) is

$$\underset{X_1}{\text{Minimize}}\, Z_1 = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij} + \sum_{i=1}^{3} F_i$$

subject to (6).

The lower level problem (P4) is

$$\underset{X_2}{\text{Minimize}}\, Z_2 = \sum_{i=1}^{3} \sum_{j=1}^{4} d_{ij} x_{ij} + \sum_{i=1}^{3} F_i'$$

subject to (6).

Table (1) and Table (2) give the values of variable cost c_{ij} (i = 1, 2,3 ; j = 1, 2, 3, 4) and d_{ij} (i = 1, 2, 3; j = 1, 2, 3, 4) for the upper level and lower level problems respectively.

i \ j	1	2	3	4	a_i
1	24	28	29	31	≥65
2	13	18	26	20	≥ 60
3	23	30	33	28	≥90
b_{ij}	≥ 85	≥40	≥ 50	≥ 45	

Table 1

i \ j	1	2	3	4	a_i
1	7	2	10	4	≥65
2	1	3	9	7	≥ 60
3	4	12	10	8	≥ 90
b_{ij}	≥ 85	≥40	≥ 50	≥ 45	

Table 2

The fixed costs for the upper level problem are

$F_{11} = 50$, $F_{12} = 50$ $F_{13} = 25$ $F_{14} = 25$

$F_{21} = 100$ $F_{22} = 50$ $F_{23} = 50$ $F_{24} = 25$

$F_{31} = 100$ $F_{32} = 50$ $F_{33} = 50$ $F_{34} = 50$

The total cost which is to be minimized is given by $\left(\sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij} + \sum_{i=1}^{3} F_i \right)$,

where $\sum_{l=1}^{3} \delta_{i\ell} F_{i\ell}$, $i = 1, 2, 3$

where $\delta_{i1} = 1$ if $\sum_{j=1}^{5} x_{ij} > 50$ for $i = 1, 2, 3$

$= 0$ otherwise

$\delta_{i2} = 1$ if $\sum_{j=1}^{5} x_{ij} > 60$ for $i = 1, 2, 3$

$= 0$ otherwise

$\delta_{i3} = 1$ if $\sum_{j=1}^{5} x_{ij} > 70$ for $i = 1, 2, 3$

$= 0$ otherwise

$\delta_{i4} = 1$ if $\sum_{j=1}^{5} x_{ij} > 80$ for $i = 1, 2, 3$

$= 0$ otherwise

The fixed costs for the lower level problem are

$F'_{11} = 50$ $F'_{12} = 50$ $F'_{13} = 25$ $F'_{14} = 20$

$F'_{21} = 50$ $F'_{22} = 25$ $F'_{23} = 25$ $F'_{24} = 20$

$F'_{31} = 50$ $F'_{32} = 50$ $F'_{33} = 25$ $F'_{34} = 20$

The total cost which is to be minimized is given by $\left(\sum_{i=1}^{3} \sum_{j=1}^{4} d_{ij} x_{ij} + \sum_{i=1}^{3} F'_i \right)$,

where $F'_i = \sum_{k=1}^{3} \delta'_{ik} F'_{ik}$, $i = 1, 2, 3$

where $\delta'_{i1} = 1$ if $\sum_{j=1}^{5} x_{ij} > 60$ for $i = 1, 2, 3$

$= 0$ otherwise

$\delta'_{i2} = 1$ if $\sum_{j=1}^{5} x_{ij} > 70$ for $i = 1, 2, 3$

$= 0$ otherwise

$\delta'_{i3} = 1$ if $\sum_{j=1}^{5} x_{ij} > 80$ for $i = 1, 2, 3$

$= 0$ otherwise

$\delta'_{i4} = 1$ if $\sum_{j=1}^{5} x_{ij} > 90$ for $i = 1, 2, 3$

$= 0$ otherwise

Proceeding as explained in the algorithm, the optimal solution for (BFTPE) is (0, 15, 50, 0, 45, 25, 0, 0, 45, 0, 0,45) with Z_1 = 5200 and Z_2 = 1190.

Conclusions: The algorithm moves from one extreme point to another extreme point in (BFTPR) as well as in (BFTPE). Since the extreme points are finite in numbers, therefore, the procedure must end in a finite number of steps and the optimal solution of both the problems (BFTPR) and (BFTPE) lies at an extreme point.

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