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Two Notes About Rolling Estimates

Reza Habibi
Department of Statistics, Central Bank of Iran

Abstract. This paper proposes two notes rolling estimates. These estimates are used in rolling analysis to detect instabilities of a time series. At first, it is seen that, under the null hypothesis of no change point, rolling estimates constitute a moving average process. Then, we find that using a rolling estimates in change point test statistics gives better results, in practice.

Keywords: Change point; Covariance Structure; Moving average; Rolling analysis; Time series

1 Introduction. The constancy of parameters of a time series model over time is too important assumption, in practice. The rolling analysis is useful technique to monitor a time series. It is done by estimating parameters over a rolling window with fixed length during the given sample. We describe the method as follows.

Let $\{X_t\}_{t\geq 1}$ be independent observations come form a statistical distribution indexed by parameter(s) $\theta \in \Theta$. Let $X_1^n = (X_1, ..., X_n)$ and $\widehat{\theta} = T(X_1^n)$ be a suitable estimator of θ; e.g. maximum likelihood or least square estimates. Let $l \geq 1$ be an integer number and define the rolling estimates given by

$$\widehat{\theta}_k = T(X_t^{t+l-1}),\ t = 1, 2, ..., n+l-1.$$

If $\{\widehat{\theta}_t\}_{t\geq 1}$ do not differ then the parameter θ is fixed over time. However, when a change has occurred in θ, plotting $(t, \widehat{\theta}_t)$ will detect this shift. Rolling analysis is studied by Alexander (2001) and Zivot and Wang (2006).

It is easy to see that $\{\widehat{\theta}_t\}_{t\geq 1}$ is a stationary and l-correlated process, that is $\text{cor}(\widehat{\theta}_t, \widehat{\theta}_{t+h}) = 0$, for $|h| > l$. Following Brockwell and Davis (2002), it has a unique $MA(l)$ representation as

$$\widehat{\theta}_t = \mu_\theta + \sum_{j=1}^{l} \alpha_j Z_{t-j},$$

where $\mu_\theta = E(\widehat{\theta}_t)$ and Z_t are $\text{WN}(0, \sigma^2)$. Therefore,

$$\gamma(h) = \text{cov}(\widehat{\theta}_t, \widehat{\theta}_{t+h}) = \begin{cases} \sigma^2 \sum_{j=0}^{l-|h|} \alpha_j \alpha_{j+|h|} & |h| \leq l \\ 0 & |h| > l, \end{cases}$$

[1]Reza Habibi is staff of Central Bank of Iran. He has a PhD in Statistics form Shiraz U.
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with $\alpha_0 = 1$. An estimator for μ_θ is $\overline{\widehat{\mu}}$ which is unbiased-consistent estimator with variance

$$(1/n) \sum_{h=-n}^{n} (1 - \frac{|h|}{n})\gamma(h).$$

The moment method estimate is obtained by solving

$$\mu_\theta = \overline{\widehat{\mu}} = (1/n) \sum_{t=1}^{n} \widehat{\theta}_t.$$

The Newton-Raphson may be applied to derive it (see Gelman(1994)). A 95% confidence bands for μ_θ is given by

$$\overline{\widehat{\mu}} \pm 1.96\sqrt{\frac{v}{n}},$$

where $v = \sum_{|h|<\infty} \gamma(h)$. The $\gamma(h)$ is estimated by empirical auto-covariance of $\widehat{\theta}_t$ defined by

$$\widehat{\gamma}(h) = (1/n) \sum_{t=1}^{n-|h|} (\widehat{\theta}_t - \overline{\widehat{\mu}})(\widehat{\theta}_{t+|h|} - \overline{\widehat{\mu}}).$$

The unknown parameters $\{\alpha_j, j \geq 1\}$ are estimated using a suitable estimation method such as maximum likelihood method or Yule-Walker approach (see Brockwell and Davis (2002)). Statistical software R has useful package for performing time series analyses as well as computing these estimators of unknown parameters. Two criteria for measuring the difference between $\widehat{\theta}_t$'s are the range and variance of them, that is

$$(1/n) \sum_{t=1}^{n|} (\widehat{\theta}_t - \overline{\widehat{\mu}})^2 \text{ and } \max_{1\leq t\leq n} \widehat{\theta}_t - \min_{1\leq t\leq n} \widehat{\theta}_t.$$

Example 1. Let X_k, $k = 1, 2, ..., 1000$. Let $l = 3$ and compute the rolling means. By moving average modeling, one can see that, for example, $\widehat{\rho}(i) = 0.0367,\ 0.0214,\ -0.0072$ for $i = 1, 2, 5$, respectively. The estimate of σ^2 (the variance of white noise process) is 1.462. One can also see that the variance of rolling means $\overline{X}_t$ is 0.972 which indicates that there is no change point.

Remark 1. Here, we suppose that there exists a change point at θ. Then the MA model with parameters $(\mu_\theta, \{\alpha_j\}_{j=1}^{l}, \sigma^2)$ shifts to a MA averages with new parameters $(\mu'_\theta, \{\alpha'_j\}_{j=1}^{l}, \sigma'^2)$. Therefore, methods for change point detection in time series model (e.g Hansen (1992)) may be applied for change point detection. One should note that using a rolling estimate (instead of original data) in change point test statistics, we guarantee that the variances of observations are reduced and therefore, this method detects the change point much better. The following example shows this fact.

Example 2. Suppose that the 500 original data (X_i) come form normal distribution with common variance 2 and mean changes from 0 to 2 after 300-th observations. Here, using a Monte Carlo method with $M = 1000$ repetitions, we compute the $P(\widehat{k}_i = k)$ for change point estimator $\widehat{k}_i$ of i-th method ($i = 1, 2$). Here, $\widehat{k}_1$ is the minimizer of CUSUM process defined by $\sum_{i=1}^{k}(X_i - \overline{X})$, for $k = 1, 2, ..., 499$. And, $\widehat{k}_2$ is the minimizer of CUSUM process obtained substituting X_i with rolling means (with $l = 3$), $\overline{X}_t$, $t = 1, 2, ..., 497$. The $\overline{X}$ is replaced with the mean of $\overline{X}_t$'s. The following Tables gives the results. It is seen that $\widehat{k}_2$ proposes a better concentrations on $k_0 = 300$. Our studies (are not given here) shows that (using rolling estimates) the results are much better when the variance 2 gets large.

Table 1: Probability mass of cusum estimator with X_i's

k	≤ 295	296	297	298	299	300	≥ 301
$P(\widehat{k}_1 = k)$	0.2	0.1	0.1	0.05	0.15	0.15	0.25

Table 2: Probability mass of cusum estimator with rolling means

k	≤ 295	296	297	298	299	300	≥ 301
$P(\widehat{k}_1 = k)$	0.2	0.05	0.1	0.1	0.1	0.35	0.1

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