Problem on optimal distribution of induced microwave by heating probe at the tumor site in hyperthermia

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Abstract

In this paper we have investigated analytically the optimal distribution of time dependent point heating power inserted in the tumour site [Deng and liu, 2002] of the tissue described by bio-heat equation so as to attain desired temperature of the tumour at the end of time of operation of the process under constant surface cooling temperature using conjugate gradient method [Loulou and Scott, 2002].
Here the temperature of the tissue against the length of the tissue at different total times of operation of the process due to calculated distribution of heating power is numerically evaluated for investigation of desired tumour temperature.

Keywords: Bio – heat equation, conjugate gradient method, optimal control, tumour, heating power.

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Notations:

c	=	specific heat of tissue, J/kg ^{o}C
h	=	heat transfer coefficient between the skin and the ambient air, $Wm^{-2}/{}^{0}C$
k	=	thermal conductivity of tissue, $W\,m^{-1}/{}^{0}C$
L	=	thickness of the plate, m
x_1	=	Point of location of the tumour, m
χ	=	temperature, ^{0}C
χ_a	=	arterial temperature, ^{0}C
χ_0	=	initial temperature, ^{0}C
u(t)	=	temperature of the surrounding medium, ^{0}C
χ^*	=	desired temperature to be attained, ^{0}C
T	=	Total time of the process, s
t_1	=	switching time, s
Q(t)	=	optimal heat generation rate due to volumetric heating, Wm^{-3}
ρ	=	density of tissue, Kg m^{-3}
δ	=	dirac – delta function.
ω	=	product of flow and heat capacity of blood, W m^{-3} / ^{0}C
Q_m	=	rate of metabolic heat generation, Wm^{-3}

1. Introduction

The determination of temperature distributions throughout the biological tissue by solving simple Pennes' bio-heat equation, so as to elevate the temperature of tumour to therapeutic value avoiding the overheating of the healthy tissue, are important issues of investigation in hyperthermia. In course of this investigation, the mathematical models of patient's anatomy and patient's tolerance due to the effect of heating power deposition patterns carry vital points of consideration. Computer simulation have more potentiality for determination of the optimal power of the applied heat source and also surface cooling temperature which generally can be regarded as direct control input variables. The location of the tumour cells in the tissue as well as blood flow rate is also needful for the purpose of investigating problems on optimization.

[Deng and Liu, 2002] have performed several closed form of analytical solutions in bio-heat transfer problems, with transient heating on the skin surface or inside biological bodies by inserting a heating probe at the tumour region, using Green's function method. [Dhar and Sinha, 1989] investigated analytically optimal temperature control in hyperthermia by controlling artificial surface cooling.

[Wagter, 1986] studied an optimization procedure to calculate transient temperature profiles in plane tissue by multiple electro-magnetic applicatiors. Analytical study by [Butkovasky, 1969] had carried the fundamentals of optimal control problems in distributed parameter system. [Dhar and Sinha, 1988] had considered analytically an optimal control problem so as to attain a desired temperature throughout the tissue by induced heat source at least possible time.

A computational technique for fast hyperthermia temperature optimization using finite element method was presented by [Das, Clegg and Samulski, 1999].[Kowalsk and Jin, 2003] conducted a study on cost minimization problem in space by feedback control system applying electro-magnetic phased –array. [Loulou and Scott, 2002] investigated a thermal dose optimization problem in hyperthermia with the aid of conjugate gradient method.

An optimization problem on diseased tissue by generating heat with the aid of alternating magnetic field was investigated in [Bagaria and Johnson, 2005].

In course of investigation on optimization of radio-immunotherapy (RIT) interactions with hyperthermia, the combination of local hyperthermia with RIT has been discussed in [Kinuya et al., 2004]. [Szasz and Vincze, 2006] proposed a generalization of Pennes- equation inducing the entire energy balance where the new paradigm could be a theoretical basis of the empirical dose-construction for oncological hyperthermia.

The article of [Rapoport et al., 2009] described the study of targeted chemotherapeutic intervention on solid tumors by means of ultrasound.

In the work of [Liu and Chen, 2009] analyzed the temperature rise behaviors in biological tissues during hyperthermia treatment within the dual-phase-lag model, which accounted the effect of local non-equilibrium on the thermal behavior.

The aim of the investigation presented in [Shih et al., 2008] was to consider the feasibility of the heating on the tumor periphery using high intensity focused ultrasound during thermal surgery.

[Dhar et al., 2010] investigated analytically a distributed optimal control problem by achieving desired temperature of the tumor due to induced microwave in a homogeneous tissue by Conjugate gladiate method.

In this paper, an analytical investigation of the optimal distribution of time dependent point heating power inserted in the tumour site [Deng and liu,2002] so as to attain desired temperature of the tumour at the end of operation of the process under constant surface cooling temperature was carried out with the aid of bioheat equation using conjugate gradient method [loulou and Scott, 2002] under calculus of variation.

The distribution of heating power for different values of total time of operation of the process have been calculated numerically. The temperature of the tissue along the length of the tissue at diffent times due to calculated distribution of heating power have also been worked out.

2 Mathematical Analysis

The one dimensional bio-heat equation for a point conducting heating probe at $x = x_1$ [Deng and Liu,2002] can be written as,

$$\rho c\frac{\partial \chi}{\partial t} = k\frac{\partial^2 \chi}{\partial x^2} + \omega(\chi_a - \chi) + Q(t)\delta(x - x_1) + Qm \tag{1}$$

Boundary condition :

$$k\frac{\partial \chi}{\partial x} = h\{\chi - u(t)\}\, on\, x = 0$$

$$\chi = \chi_a\, on\, x = L \tag{2}$$

Initial Condition:

$$\chi(x,o) = \chi_0 \tag{3}$$

We would like to attain the desired temperature χ^* at the point $x = x_1$, where the tumour is located, at the end of total time T of the process by controlling optimally rate of generation of heat Q(t) induced by conducting heating probe at $x = x_1$ [Deng and Liu,2002].

Thus the functional [Dhar and Sinha, 1989 ; Wagter, 1986]

$$\frac{1}{2}\int_0^L \left\{\chi^* - \chi(x,T)\right\}^2 \delta(x - x_1)dx \tag{4}$$

is to be minimized.

The first term designates the square deviation of the temperature χ^* from $\chi(x,t)$ at $x = x_1$.

Let us write a function J, given by [ButKovasky, 1969 ; Loulou and Scott, 2002]

$$J = -\frac{1}{2}\int_0^L \left\{\chi^* - \chi(x,T)\right\}^2 \delta(x - x_1)dx$$

$$+\int_0^L\int_o^T \psi(x,t)\left\{\frac{k}{\rho c}\frac{\partial^2 \chi}{\partial x^2} + \frac{\omega}{\rho c}(\chi_a - \chi) + \frac{1}{\rho c}Q(t)\delta(x - x_1) + \frac{Q_m}{\rho c} - \frac{\partial}{\partial t}\chi\right\}dxdt \tag{5}$$

where $\psi(x,t)$ is the auxiliary function.

By considering Q_m as constant, the first variation of the function J can be written as,

$$\delta J = \int_0^L \left\{\chi^* - \chi(x,T)\right\}\delta(x - x_1)\delta\chi(x,T)dx$$

$$+\frac{k}{\rho c}\int_o^T \psi(L,t)\delta\chi_x(L,t)dt + \frac{1}{\rho c}\int_o^T \left\{k\frac{\partial \psi}{\partial x}(o,t) - h\psi(o,t)\right\}\delta\chi(o,t)dt$$

$$+\frac{h}{\rho c}\int_o^T \psi(o,t)\delta u(t)dt - \frac{k}{\rho c}\int_o^T \frac{\partial}{\partial x}\psi(L,t)\delta\chi(L,t)dt + \frac{k}{\rho c}\int_o^T\int_o^L \frac{\partial^2}{\partial x^2}\psi(x,t)\delta\chi(x,t)dxdt$$

$$-\frac{\omega}{\rho c}\int_o^L\int_o^T \psi(x,t)\delta\chi(x,t)dxdt + \frac{1}{\rho c}\int_o^T\int_o^L \psi(x,t)\delta(x - x_1)\delta Q(t)dxdt$$

$$+\int_0^L\int_o^T \frac{\partial \psi(x,t)}{\partial t}\delta\chi(x,t)dxdt - \int_o^L \psi(x,T)\delta\chi(x,T)dx$$

$$+\int_0^L \psi(x,o)\delta\chi(x,o)\,dx$$

with the help of equations (2) and (3). By assuming δJ to vanish for any $\delta\chi_x(L,t),\delta\chi(x,t),\delta\chi(o,t),\delta\chi(x,T),\delta Q(t)$, $\delta u(t)$ and taking $\delta\chi(x,o),\delta\chi(L,t)$ both equal to Zero, a system of auxiliary function $\psi(x,t)$ is obtained as,

$$\frac{\partial\psi}{\partial t}+\frac{k}{\rho c}\frac{\partial^2\psi}{\partial x^2}=\frac{\omega}{\rho c}\psi. \tag{7}$$

$$k\frac{\partial\psi}{\partial x}=h\psi \;\; on\, x=0 \tag{8}$$

$$\begin{aligned}&\psi(x,t)=0\, on\, x=L\\&\psi(x,T)=\{\chi^*-\chi(x,T)\}\delta(x-x_1)\end{aligned} \tag{9}$$

and the optimal value of the controls Q(t) and u(t) stand,

$$\begin{aligned}&Q(t)=\frac{1}{\rho c}\int_0^L\psi(x,t)\delta(x-x_1)dx=\frac{1}{\rho c}\psi(x_1,t)\\&u(t)=Sign\,\psi(o,t),\end{aligned} \tag{10}$$

Here the conjugate gradient method with the aid of calculus of variation have been used [Butkovasky 1969 ; Loulou, T. and Scott, E.P. 2002]. Considering $\chi_1(x,t)=\chi\;(x,t)-\chi_a$ and expressing $\chi_1(x,t)$ in Finite Sine Transform, given by,

$$\overline{\chi}_{1n}(t)=\int_0^L\chi_1(x,t)\sin p_n(L-x)dx \tag{11}$$

and

$$\chi_1(x,t)=\sum_{n=1}^{\infty}\overline{\chi}_{1n}(t)\times\frac{2\sin p_n(L-x)}{L-\dfrac{\sin 2p_nL}{2p_n}} \tag{12}$$

where p_n are positive, real roots of the equation,

$$p\,cot(pL)=\frac{-h}{k} \tag{13}$$

the equation (1) with the help of equations (2), (3) and (13) stands,

$$\frac{d}{dt}\overline{\chi}_{1n}(t)+\alpha_{1n}\overline{\chi}_{1n}(t)=\alpha_{3n}Q(t)Sinp_n(L-x_1)+\alpha_{4n}+\alpha_{5n};\, n=1,2,3,\cdots \tag{14}$$

Where,

$$\begin{aligned}&\alpha_{1n}=\frac{1}{\rho c}\{kp_n^2+\omega\},\\&\alpha_{4n}=\frac{h}{\rho c}\{u(t)-\chi_a\}\sin p_nL,\\&\alpha_{3n}=\frac{1}{\rho c}\\&\alpha_{5n}=\frac{1}{\rho c}\left(\frac{1-\cos p_nL}{p_n}\right)Q_m\end{aligned} \tag{15}$$

Finally we get,

$$\chi(x,t) = \chi_a + \sum_{n=1}^{\infty} \overline{\chi}_{1n}(t) \times R_n(x) \tag{16}$$

The solution of equation (14) with the help of equation (15) stands,

$$\overline{\chi}_{1n}(t) = [(\chi_o - \chi_a)\left(\frac{1-\cos p_n L}{p_n}\right) + \frac{h}{\rho c}\sin p_n L \int_0^t \{u(\xi) - \chi_a\} e^{\alpha_{1n}\xi} d\xi + \left(\frac{1-Cosp_n L}{p_n}\right)\frac{1}{\rho c} Q_m \int_0^t e^{\alpha_{1n}\xi} d\xi$$
$$+ \frac{Sinp_n(L-x_1)}{\rho c}\int_0^t Q(\xi) e^{\alpha_{in}\xi} d\xi] \times e^{-\alpha_{1n}t};\ n = 1,2,3,\cdots \tag{17}$$

where

$$R_n(x) = \frac{2\sin p_n(L-x)}{L - \frac{\sin 2p_n L}{2p_n}} \tag{18}$$

The corresponding solution of equation (7) with the help of equations (8) and (9) can be written as, with the help of earlier Finite Transform,

$$\psi(x,t) = \sum_{m=1}^{\infty} \overline{\psi}_m(t)\, R_m(x) \tag{19}$$

where

$$\overline{\psi}_m(t) = \{(\chi^* - \chi_a) - \sum_{n=1}^{\infty} \overline{\chi}_{1n}(T) \times R_n(x_1)\} Sinp_m(L-x_1) \times e^{-\alpha_{1m}(T-t)} \tag{20}$$

for p_m are roots of the equation (13).

Considering u(t) as constant, the value of optimal control Q(t) can be obtained from equation (10) with the help of equations (15), (16),(17), (18) , (19) and (20).

3 Results and Discussions

Data used in computation are given as follows

c	=	3770 J kg^{-1} $^0C^{-1}$
ρ	=	998 Kgm^{-3}
k	=	.5 Wm^{-1} $^0C^{-1}$
h	=	6 Wm^{-2} $^0C^{-1}$
χ_a	=	37^0C
χ^*	=	43^0C
L	=	.01 m,
x_1	=	.006m
ω	=	3000 Wm^{-3} $^0C^{-1}$
Q_m	=	33800 Wm^{-3}
χ_0	=	25^0C
T	=	600s, 800s, 1000s
u(t)	=	20^oC

The value of Q(t) (Wm^{-3}) can be obtained from equation (10) with a the help of equations (17),(18), (19) and (20) since $\chi(t)$ can be calculated by equations (16), (17) and (18).

Computational Algorithm:

(i) Take a set of values of time (t_1, t_2, ..., t_r = T). Asume initial guess of Q(0).
(ii) Obtain $\chi(x_1, t_1)$ with the help of equations (16) and (17), by Simpon's rule of integration
(iii) Repeat this operation to obtain $\chi(x_1, t_r = T)$
(iv) Obtain auxiliary function $\psi(x_1, t_{r-1})$ with the help of equaitons (19) and (20).
(v) After obtaining $\psi(x_1, t_{r-1})$ complete Q(t_{r-1}) by equation (10).
(vi) The exact value of Q(t_{r-1}) is obtained by improving initial Q(0) (mentioned (i)) with the aid of simulation so that the increment of Q(t_{r-1}) is satisfied within a relative error of 10^{-3}.
(vii) Repeat the procedure from (i) to (v) to compute Q(t_i) (i=r – 2, r – 3, …. 2,1) after obtaining the exact values of Q(t_{r-1}) and Q(0).
(viii) Thus an exact set of values Q(t_i) (i = 1,2 ………, r – 1, T) together with Q(0) are obtained where Q(T) is calculated with the help of equations (19) and (20) which approaches to the vlaue Zero. [Vide the condition of the problem given in equation (9)].

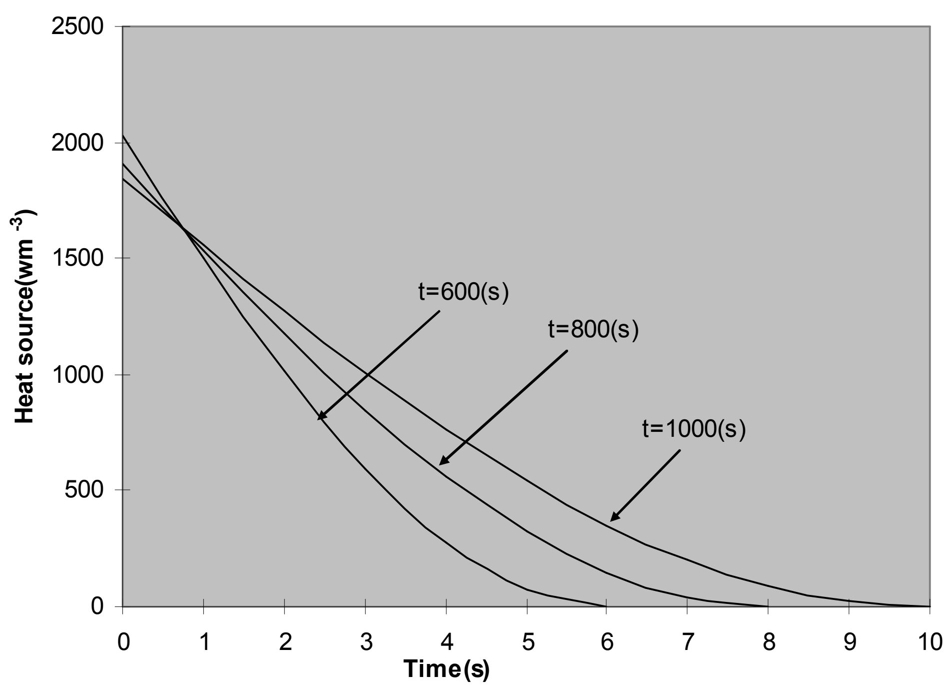

Fig 1: Distribution of applied heat source Q(t)(wm^{-3}) versus time(s) for different total time of the process T=600s,T=800s,T=1000s.

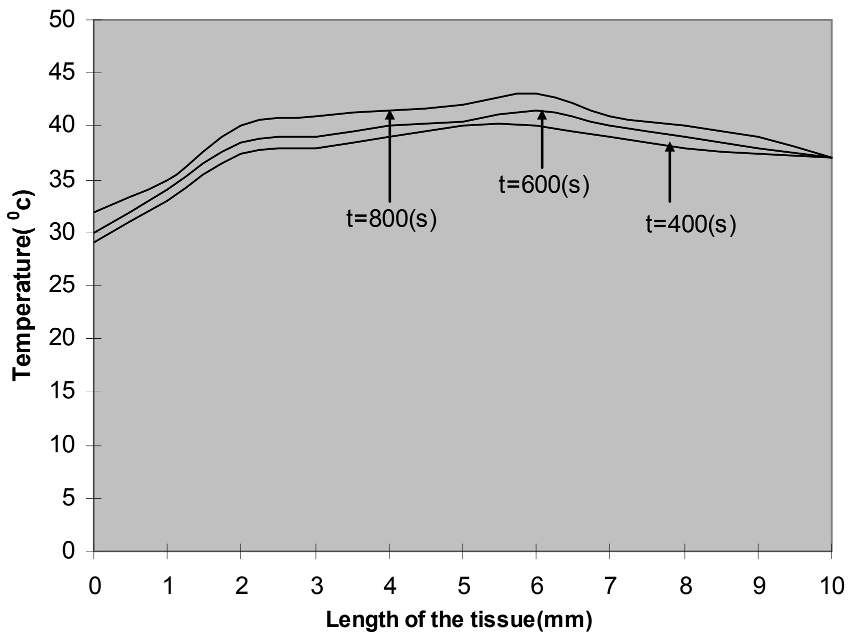

Fig 3: Temperature(^{0}c) of the tissue along the length of the tissue(mm) for heat source Q(t)(wm^{-3}) in Fig 1 at T=800s.

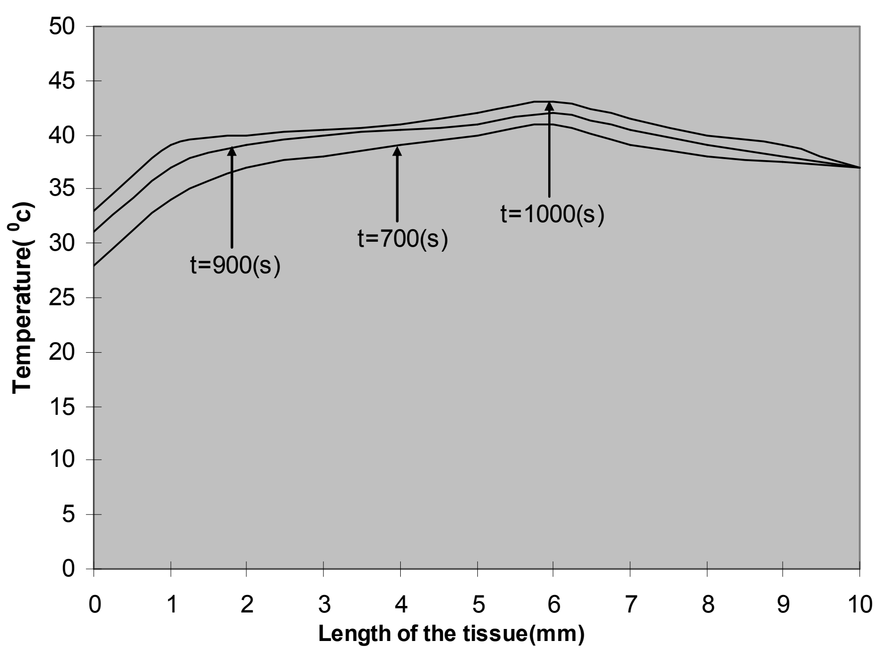

Fig 4: Temperature(^{0}c) of the tissue along the length of the tissue(mm) for heat source Q(t)(wm^{-3}) in Fig 1 at T=1000s.

Fig 1 displays the optimal distribution of time dependent point heating power Q(t) (Wm^{-3}) versus time for different total time of operation of the process T = 600s, 800s and 1000s respectively. It is seen that Q(t) (Wm^{-3}) is maximum at the staring time of operation and decreases rapidly till the end of the process to the value zero for T = 600s, 800s and 1000s respectively. Further it is observed the starting value of Q(t) decreases with the increase of total time of opertation of the process.

Fig 2, Fig 3 and Fig 4 depict the distribuion of temperature of the tissue at different times for various total time of operation of the porcess T = 600s, 800s and 1000 respectively due to the application of calculated distributions of Q(t) (Wm^{-3}) given in Fig 1. Here it is observed in Fig2, Fig 3 and Fig 4 that the temperature of the tissue increases steadily till it attains the tumour temperature 43^{o}C at the location of tumour x_1=.006m at the end of the process T(s) and after that it decreases rapidly to 37^{o}C (Arterial temperature). Thus overall, it is seen that the temperaute of healthy tissue are not been overheated avoiding it's damage and so, most possibly, this analytical study conforms one of basic concept of hyperthermia treatment.

4. Conclusion:

This analytical study will provide a focusing aspect for further developments in case of different times of operation of the process and different points of location of the tumour having various length of tumour.

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