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Continuous Review Inventory Model with Crashing Cost under Service Level Constraint and Probabilistic Fuzzy Numbers

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Abstract: Mixture shortage continuous review inventory model with varying holding cost have been studied, under service level constraint when lead time is reduction by the lead time crashing cost. Then the optimal lead time and the optimal order quantity are determined in this case. Hence the same model is developed when the average demand per year and the backorder fraction are triangular fuzzy numbers and the optimal policy of the fuzzy model is derived. To get these optimal policies of the crisp and the fuzzy model under the service level constraint, we provide an algorithm which gives the minimum expected annual total cost under the constraint. An illustrative example is used to determine the optimal policy of the models in the crisp and the fuzzy case according to the algorithm. The sign distance method used to defuzzify the average demand per year and the backorder fraction.
Keywords: Inventory, Service Level Constraint, Fuzziness, Lead Time Crashing Cost.

1. Introduction

In order to control the lead time, we can reduce it by the crashing cost; since lead time usually consist of the following components: order preparation, order transit, supplier lead time, delivery time, and set up time as in [Tersine, 1982], while most of authors [Abuo El Ata et al., 2002, Elwakeel, 2006, Hadley and Whitin, 1963, Montgomery et al., 1973, Ouyang and Wu, 1996] dealing with the lead time as a given parameter or a random variable which mean that it is not under control. By shorting the lead time, we can lower the safety stock, reduce the loss caused by stockout, improve the service level to the customer, which increasing the competitive ability in business. [Ben-Daya and Abdul Raouf, 1994] present a continuous review inventory model by considering both order quantity and lead time as a decision variables with two different form of the lead time crashing cost. They determined the optimal lead time and the optimal order quantity which minimize the sum of the order cost, the inventory holding cost and the lead time crashing cost but the shortage is not allowed.

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[Ouyang and Wu, 1996] developed model with one form of lead time crashing cost of the Ben-Daya's models by adding the stockout cost (mixture shortage). [Ouyang and Wu, 1997] have extended the continuous review inventory model by adding the lead time crashing cost but instead of having a stockout cost term in the expected annual total cost, they adding to the model a service level constraint, which implies that the stockout level per cycle is bounded. Service level measures the performance of a system. Certain goals are defined and the service level gives the percentage to which they should be achieved. In recent years, some authors have begun dealing with the inventory models under fuzziness like [Chang et al., 2004, Chiang et al., 2005, Vijayan and Kumaran, 2007, Yao and Chiang, 2003] in order to minimize the optimal expected annual total cost.

In this article our aim is developed the model of [Ouyang and Wu, 1997] by considering the expected inventory holding cost as varying holding cost when the lead time demand follow the normal distribution and the model is under service level constraint, and hence obtain the optimal lead time and the optimal order quantity which minimize the expected annual total cost, then fuzzify both the average demand per year and the backorder fraction as a triangular fuzzy numbers, where we defuzzified it by the signed distance method. Follow that the determinations of the optimal order quantity and the optimal lead time in the fuzzy case. An algorithm is presented to determine the minimum expected annual total cost for the crisp and the fuzzy model which is illustrative by numerical example using the mathematica program V.5.

2. Assumptions and Notations

2.1 Assumptions:

1. Continuous review inventory model with varying holding cost is considered.
2. Backorder cost is dependent of time.
3. Let the model is under a service level constraint.
4. γ is a fraction of unsatisfied demand that will be backorder while the remaining fraction $(1-\gamma)$ is completely lost $0<\gamma<1$.
5. Lead time L is deterministic, while the demand $\overline{D}$ is a continuous random variable, and the lead time demand X is normally distributed with mean $\mu\, L$ and standard deviation $\sigma\sqrt{L}$.
6. The reorder point $r=\mu L+k\sigma\sqrt{L}$ (see [Tersine, 1982]) where k is the safety factor and satisfies $P(X>r)=P(Z>k)=q,\ Z$ present the standard normal random variable and q present the allowable stockout probability during L.
7. The lead time L has *n* mutually independent components. The *ith* component has a minimum duration a_i , normal duration b_i and a crashing cost per unit time c_i. Further we assume that $c_1 \le c_2 \le \cdots \le c_n$.
8. The components of lead time are crashed one at a time starting with the component of least c_i and so on.
9. If we let $L_0 \equiv \sum_{i=1}^{n} b_i$ and L_i be the length of lead time with components $1,2,\cdots,i.$ crashed to their minimum duration, then $L_i=\sum_{i=1}^{n} b_i-\sum_{j=1}^{i}(b_j-a_j)$, $1,2,\cdots,i.$ and the lead time crashing cost $C(L)$ for given $L\in(L_i,L_{i-1})$ is given by: $C(L)=c_i(L_{i-1}-L)+\sum_{j=1}^{i-1}c_j(b_j-a_j)$.

2.2. List of notations:

Q	the decision variable presenting the order quantity per cycle
L	the decision variable presenting the lead time
c_o	the order cost per unit per cycle
c_h	the holding cost per unit per cycle
$c_h\ (Q)$	the varying holding cost per unit per cycle $= c_h\ Q^{\beta}$
β	a constant real number selected to provide the best fit of estimated cost function
a	proportion of demands which are not met from stock Hence, $1 - a$ is the service level.

3. Model Analysis

Service level is used in supply chain management and in inventory management to measure the performance of inventory systems. Several definitions of service levels are used in the literature as well as in practice. These may differ not only with respect to their scope and to the number of considered products but also with respect to the time interval they are related to. This service level $1 - a$ is equal to the probability that an arbitrary demand unit is delivered without delay.

In order to consider the reorder point $r = \mu L + k\sigma\sqrt{L}$, and the lead time demand has a normal probability density function(p.d.f.) $f(x)$ with mean μL and standard deviation $\sigma\sqrt{L}$ and by assuming that the shortages are allowed, then the expected shortage at the end of the cycle is given by:

$$\bar{S}(r) = \int_{r}^{\infty}(x - r)f(x)dx = \sigma\ \sqrt{L}\ \Psi(k)$$

Where $\Psi(k) = \phi(k) - k[1 - \Phi(k)]$, φ denote the standard normal probability density function (p.d.f.), and ϕ is the cumulative distribution function (c.d.f.). Thus the expected number of backorder per cycle is $\gamma\ \bar{S}(r)$ while the expected number of lost sales per cycle is$(1 - \gamma)\ \bar{S}(r)$. Therefore the expected annual total cost can be developed as follows:

$$E(Total\ Cost)\ =\ E\ (Order\ Cost)\ +\ E\ (Holding\ Cost) + E\ (Lead\ time\ crashing\ Cost)$$

Our aim is to minimize the expected annual total cost $E[TC(Q, L)]$ under the service level constraint, which it can be written in the following form:

$$E[TC(Q,L)]\ = c_o\frac{\bar{D}}{Q} + c_hQ^{\beta}[\frac{Q}{2} + k\sigma\ \sqrt{L} + (1-\gamma)\sigma\ \sqrt{L}\ \Psi(k)]\ +\ \frac{\bar{D}}{Q}C(L)$$

Subject to: $$\frac{\bar{S}(r)}{Q} \le a$$

This model is a non-linear programming problem, it can be verified that the Kuhn-Tucker conditions are not satisfied. To solve this kind of problem, we first ignore the service level constraint and take the partial derivatives of $E[TC(Q,L)]$with respect to Q and L in each time interval (L_i, L_{i-1}). Let us first rewritten the problem as follows:

$$G(Q,L) = c_o\frac{\bar{D}}{Q} + c_h Q^{\beta}[\frac{Q}{2} + k\sigma\sqrt{L} + (1-\gamma)\sigma\sqrt{L}\,\Psi(k)] + \frac{\bar{D}}{Q}C(L) \tag{3.1}$$

Subject to:

$$\frac{\sigma\sqrt{L}\,\Psi(k)}{Q} \le a \tag{3.2}$$

To find the optimal values $Q^*$and L^* which minimize equation (3.1) under the constraint (3.2), equating with zero the first partial derivatives with respect to Q and L respectively, we obtain:

$$(1+\beta)c_h Q^{\beta+2} + 2\beta c_h Q^{\beta+1}[k + (1-\gamma)\Psi(k)]\sigma\sqrt{L} - 2\bar{D}[c_o + C(L)] = 0 \tag{3.3}$$

And,

$$\sigma c_h Q^{\beta+1}[k + (1-\gamma)\Psi(k)] - 2\bar{D}c_i\sqrt{L} = 0 \tag{3.4}$$

It is clear that, for fixed L ,

$$\frac{\partial^2 G(Q,L)}{\partial Q^2} = 2\frac{c_o\bar{D}}{Q^3} + \frac{1}{2}\beta(1+\beta)c_h Q^{\beta-1} + 2\frac{\bar{D}}{Q^3}C(L) + \beta(\beta-1)c_h Q^{\beta-2}[k\sigma\sqrt{L} + (1-\gamma)\sigma\sqrt{L}\,\Psi(k)] > 0$$

Which mean that the expected annual total cost is convex in Q . While for fixed Q :

$$\frac{\partial^2 G(Q,L)}{\partial L^2} = -\frac{1}{4}c_h Q^{\beta}[k + (1-\gamma)\,\Psi(k)]\sigma L^{-\frac{3}{2}} < 0$$

Which is concave in L in the interval $(L_i, L_{i-1}]$ for any given safety factor k, where $\Psi(k) > 0$. Therefore, for fixed Q, the minimum total expected annual cost will occur at the end points of the interval $[L_i, L_{i-1}]$ and the optimal solution must obey the service level constraint.

We can obtain the expected annual total cost mathematically by solving equations (3.3) and (3.4) to get the optimal values of Q^* and L^* as illustrative in the numerical example.

4. Fuzzy Mixture Inventory Model under Service Level Constraint

According to [Chang et al., 2004], because of various uncertainties, the annual average demand may have a little fluctuation, especially, in a perfect competitive market, where it is difficult for the decision maker to assess the annual average demand by a crisp value $\bar{D}$, but easier to determine it by an interval $[\bar{D}-\delta_1, \bar{D}+\delta_2]$ where δ_1 and δ_2 are determined by the decision maker. Therefore, corresponding to that interval, we set the following triangular fuzzy number:

$$\tilde{D} = (\bar{D}-\delta_1, \bar{D}, \bar{D}+\delta_2) \tag{4.1}$$

Where δ_1 and δ_2 should satisfy the conditions $0 < \delta_1 < \bar{D}$ and $0 < \delta_2$. (See [Chang et al., 2004])

Now to use $\bar{D}$ we should defuzzify it by using the signed distance method and hence we need the following definitions:

Definition 1. For the fuzzy set $\widetilde{D}(\alpha) = \{x: m_{\widetilde{D}}(x) \geq \alpha\}$ where $\alpha \in [0,1]$ is called the *α-cut* of $\widetilde{D}$. $\widetilde{D}(\alpha)$ is a non-empty bounded closed interval contained in the set of real numbers and it can be denoted by $\widetilde{D}(\alpha) = [\widetilde{D}_v(\alpha), \widetilde{D}_u(\alpha)]$, where $\widetilde{D}_v(\alpha)$ and $\widetilde{D}_u(\alpha)$ are respectively the left and right limits of $\widetilde{D}(\alpha)$ and are usually known as the left and right *α-cuts* of $\widetilde{D}$.

[Yao and Chiang, 2003] present the signed distance of $\widetilde{D}_v(\alpha)$ and $\widetilde{D}_u(\alpha)$ measured from 0 by $d_0\big(\widetilde{D}_v(\alpha), 0\big) = \widetilde{D}_v(\alpha)$ and $d_0\big(\widetilde{D}_u(\alpha), 0\big) = \widetilde{D}_u(\alpha)$, respectively. Therefore, we may define the signed distance from $[(\widetilde{D}_v(\alpha), \widetilde{D}_u(\alpha)); \alpha]$ to $\tilde{0}$ as follows:

Definition 2. For each$\alpha \in [0,1]$, the crisp interval $[\widetilde{D}_v(\alpha), \widetilde{D}_u(\alpha)]$ and the level fuzzy interval $[(\widetilde{D}_v(\alpha), \widetilde{D}_u(\alpha)); \alpha]$ are in one to one correspondence. Therefore, we may define the signed distance from $[(\widetilde{D}_v(\alpha), \widetilde{D}_u(\alpha)); \alpha]$ to $\tilde{0}$ as:

$$d\left([(\tilde{D}_v(\alpha), \tilde{D}_u(\alpha)); \alpha], \tilde{0}\right) = d_0\left([\tilde{D}_v(\alpha), \tilde{D}_u(\alpha)], 0\right) = \frac{1}{2}\left(\tilde{D}_v(\alpha) + \tilde{D}_u(\alpha)\right).$$

Since $\widetilde{D}_v(\alpha)$ and $\widetilde{D}_u(\alpha)$ exist and are integrable for $\alpha \in [0,1]$, as in [Yao and Wu, 2000] we have that

$$d(\tilde{D}, \tilde{0}) = \frac{1}{2}\int_0^1 (\tilde{D}_v(\alpha) + \tilde{D}_u(\alpha))\,d\alpha$$

Also, as the same way, we can fuzzify the backorder fraction of the demand during the stock out period as the following triangular fuzzy number:

$$\tilde{\gamma} = (\gamma - \delta_3, \gamma, \gamma + \delta_4) \tag{4.2}$$

Where δ_3 and δ_4 are determined by the decision maker and should satisfy the conditions $0 < \delta_3 < \tilde{\gamma}$ and $0 < \delta_4$.

From definition 1; the lift and right limit *α-cuts* of $\widetilde{D}$ and $\tilde{\gamma}$ respectively are:

$$\tilde{D}_v(\alpha) = \bar{D} - (1-\alpha)\delta_1, \quad \tilde{D}_u(\alpha) = \bar{D} + (1-\alpha)\delta_2 \tag{4.3}$$
$$\gamma_v(\alpha) = \gamma - (1-\alpha)\delta_3, \quad \gamma_u(\alpha) = \gamma + (1-\alpha)\delta_4$$

Hence, employ the method of the signed distance to defuzzify $\widetilde{D}$, we get:

$$d(\tilde{D},\tilde{0}) = \bar{D} + \frac{1}{4}(\delta_2 - \delta_1) > 0 \tag{4.4}$$

Similarly, using the method of the signed distance to defuzzify $\tilde{\gamma}$, we obtain:

$$d(\tilde{\gamma},\tilde{0}) = \gamma + \frac{1}{4}(\delta_4 - \delta_3) > 0 \tag{4.5}$$

When $\bar{D}$ and γ in equation (3.1) (the expected annual total cost) are fuzzified to be $\widetilde{D}$ and $\tilde{\gamma}$ as described in equations (4.1), (4.2) respectively, we can rewrite the expected annual total cost with fuzziness as in the crisp case by the following:

$$\tilde{G}_{(Q,L)}(\tilde{D},\tilde{\gamma}) = c_h Q^{\beta}[\frac{Q}{2} + k\sigma\sqrt{L} + (1-\tilde{\gamma})\sigma\sqrt{L}\,\Psi(k)] + \frac{1}{Q}[c_o + C(L)]\tilde{D} \tag{4.6}$$

Subject to:

$$\frac{\sigma\sqrt{L}\,\Psi(k)}{Q} \leq a \tag{4.7}$$

From definition 1; and for $\alpha \in [0,1]$ we can obtain the lift and right limit α *-cuts* of $G_{(Q,L)}(\widetilde{D},\tilde{\gamma})$ respectively by the form:

$$\tilde{G}_{(Q,L)}(\tilde{D},\tilde{\gamma})_v(\alpha) = c_h Q^{\beta}[\frac{Q}{2} + k\sigma\sqrt{L} + (1-\gamma_v(\alpha))\sigma\sqrt{L}\,\Psi(k)] + \frac{1}{Q}[c_o + C(L)]\bar{D}_v(\alpha) \tag{4.8}$$

And,

$$\tilde{G}_{(Q,L)}(\tilde{D},\tilde{\gamma})_u(\alpha) = c_h Q^{\beta}[\frac{Q}{2} + k\sigma\sqrt{L} + (1-\gamma_u(\alpha))\sigma\sqrt{L}\,\Psi(k)] + \frac{1}{Q}[c_o + C(L)]\bar{D}_u(\alpha) \tag{4.9}$$

From definition 2, and equations (4.4) , (4.5); the fuzzy expected annual total cost can defuzzify by using the signed distance method as following:

$$d(\tilde{G}(\tilde{D},\tilde{\gamma}),\tilde{0}) = c_h Q^{\beta}[\frac{Q}{2} + k\sigma\sqrt{L} + (1-m_2)\sigma\sqrt{L}\,\Psi(k)] + \frac{1}{Q}[c_o + C(L)]\,m_1 \tag{4.10}$$

Subject to:

$$\frac{\sigma\sqrt{L}\,\Psi(k)}{Q} \leq a$$

Where: $\quad m_1 = \bar{D} + \frac{1}{4}(\delta_2 - \delta_1), \quad m_2 = \gamma + \frac{1}{4}(\delta_4 - \delta_3),$

The defuzzified value $d(G(\widetilde{D},\tilde{\gamma}),\tilde{0})$ consider the estimate of fuzzy cost function which is given in the equation (4.6), similarly as in the crisp case, to solve this primal function in equation (4.10) and derived the optimal values Q^*, L^* which is convex in Q

for fixed L, first ignore the service level constraint , then equating to zero each of the corresponding first partial derivatives of equation (4.10) with respect to Q and L respectively hence we get the following equations:

$$(1+\beta)c_h Q^{\beta+2} + 2\beta c_h Q^{\beta+1}[k+(1-m_2)\Psi(k)]\sigma\sqrt{L} - 2m_1[c_o + C(L)] = 0 \quad (4.11)$$

And,

$$\sigma c_h Q^{\beta+1}[k+(1-m_2)\Psi(k)] - 2m_1 c_i \sqrt{L} = 0 \quad (4.12)$$

5. Numerical Examples

To illustrative the model in both crisp and fuzzy case we can establish the following algorithm to find the optimal lead time L^* and the optimal order quantity Q^* which minimize the expected annual total cost.

Algorithm:

Step 1: Set β which you want to determined the minimum total cost at it.

Step 2: For each L_i, $i = 0,1,2,\ldots n$.compute Q_i from the equation (3.3) for the crisp case and equation.(4.11) for the fuzzy case.

Step 3: For each pair (Q_i, L_i), compute the corresponding expected annual total cost $G(Q_i, L_i)$, $i = 0,1,2,\ldots n$..

Step 4: Determined $\min_{i=0,1,2,\ldots,n} G(Q_i, L_i)$, and let $G(Q_s, L_s) = \min_{i=0,1,2,\ldots,n} G(Q_i, L_i)$

Step 5: For a given safety factor k, check the constraint,

(i.e.) check if $\dfrac{\sigma\sqrt{L_s}\Psi(k)}{Q_s}$ holds or not.

Step 6: If the constraint holds then, the solution $G(Q_s, L_s)$ is the optimal solution, hence stop and change the value of β then repeat steps 2 to 6 to determined the optimal solution for all values of β . Otherwise go to step 7.

Step 7: If the constraint is not holds then find another pair of (Q_i, L_i) which give the next minimum value of the expected annual total cost and check the constraint for this pair , if it holds then, this another pair of (Q_i, L_i) is the optimal solution. Otherwise, continue to find the solution until it satisfies the service level constraint. If all solution do not satisfy the service level constraint given in equation (3.2), then this inventory model has no feasible solution.

The example will clarify this procedure.

The example: Let us consider an inventory system with the data: $\overline{D}$=1400 units/year, c_o=200$ per order, c_h=20$ per unit per year, the backorder fraction will be γ =0.8,

k = 0.845 , σ =7 units/weeks, and the lead time has three components with data shown in Table 1 and the value of lead time crashing cost shown in Table 2. The service level $1-a$ =0.975 (i.e.) the proportion of demands which are not met from stock is 0.025.

Table 1: Lead time data

Lead Time component i	Normal Duration b_i (days)	Minimum Duration a_i (days)	Unit Crashing Cost c_i ($/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Table 2: Lead time crashing cost

Lead Time Component i	Lead Time L_i in weeks	Lead Time Crashing Cost $C(L_i)$	The Expected Shortage $\overline{S}(r)$
0	8	0	2.1818
1	6	5.6	1.8895
2	4	22.4	1.5428
3	3	57.4	1.3361

To establish Q^* and L^* which minimize the expected annual total cost for both the crisp and the fuzzy model using the above algorithm, and hence we can summarize the optimal results of the crisp model in Table 3.

Table 3: The results of the crisp model

β	L_i	Q_i	$E[TC(Q^*,L^*)]$
0.1	6	125.69	4810.34
0.2	6	95.7	6130.9
0.3	4	77.79	7771.02
0.4	4	61.96	9518.2
0.5	3	53.83	12186.4

Also for the fuzzy model take δ_1=200, δ_2=100, δ_3=0.2, and δ_4=0.1,Then the optimal inventory policy by using the algorithm will shown in Table 4.(see all results in Tables A.1 and A.2 in Appendix).

Table 4: The results of the fuzzy model

β	L_i	Q_i	$E[TC(Q^*,L^*)]$
0.1	6	124.6	4770.8
0.2	6	94.9	6079.31
0.3	4	77.16	7702.1
0.4	4	61.47	9432.12
0.5	3	53.41	12071.3

By Comparing between The Crisp Model and The Fuzzy Model we find, at β =0.5 we find the crisp value of the expected total cost [Table 3] is 12186.4 $ and for the fuzzy one [Table 4] is 12071.3 $ with service level constraint which mean that we can obtain the minimum value of the optimal expected annual total cost by using the fuzzy system and so on for every value of β , and that minimization is the aim of our work.

6. Conclusion

This article studied a continuous review inventory model where both lead time and the order quantity are considered as the decision variables. An optimal policy of mixture shortage inventory model with varying holding cost under service level constraint has been obtained. By applying the fuzzy sets theory we determined the optimal total expected annual cost under the constraint when both the average demand per year and the backorder fraction are fuzzified as the triangular fuzzy numbers, which are defuzzified by the signed distance method. The two models of the crisp and the fuzzy case are illustrated by numerical examples.

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Appendix: Tables A.1, A.2 show the algorithm to find the optimal policy under the service level constraint

TableA.1: All results of the crisp model

β	i	L_i	Q_i	$E[TC(Q^*, L^*)]$	a
0.1	0	8	123.83	4822.03	0.018
	1	L^*=6	Q^*=125.69	4810.34	0.015
	2	4	130.76	4905.12	0.012
	3	3	140.4	5213.38	0.009
0.2	0	8	94.14	6162.65	0.023
	1	L^*=6	Q^*=95.70	6130.90	0.019
	2	4	99.67	6235.26	0.015
	3	3	106.88	6627.49	0.012
0.3	0	8	73.22	7723.62	0.029
	1	6	74.58	7662.48	0.0253
	2	L^*=4	Q^*=77.79	7771.02	0.02
	3	3	83.36	8257.75	0.016
0.4	0	8	58.06	9512.78	0.0376
	1	6	59.27	9411.96	0.032
	2	L^*=4	Q^*=61.96	9518.20	0.0249
	3	3	66.39	10109.9	0.0201
0.5	0	8	46.82	11533.3	0.0466
	1	6	47.93	11382.2	0.0394
	2	4	50.22	11479.2	0.0307
	3	L^*=3	Q^*=53.83	12186.4	0.0248

The asterisk symbol (*) is used to identify the optimal values

Table A.2: All results of the fuzzy model

β	i	L_i	Q_i	$E[TC(Q^*, L^*)]$	a
0.1	0	8	122.76	4783.25	0.018
	1	L^*=6	Q^*=124.60	4770.80	0.015
	2	4	129.63	4863.67	0.012
	3	3	139.19	5168.44	0.01
0.2	0	8	93.35	6112.02	0.023
	1	L^*=6	Q^*=94.9	6079.31	0.0199
	2	4	98.83	6181.18	0.016
	3	3	105.99	6568.73	0.013
0.3	0	8	72.61	7658.99	0.03
	1	6	73.96	7596.71	0.026
	2	L^*=4	Q^*=77.16	7702.1	0.02
	3	3	82.7	8182.76	0.016
0.4	0	8	57.58	9431.83	0.038
	1	6	58.79	9329.72	0.032
	2	L^*=4	Q^*=61.47	9432.12	0.025
	3	3	65.86	10016.1	0.02
0.5	0	8	46.44	11433.5	0.047
	1	6	47.55	11281	0.04
	2	4	49.83	11373.6	0.031
	3	L^*=3	Q^*=53.41	12071.3	0.025