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Fuzzy linear programming with interval linear programming approach

M. Allahdadi, H. Mishmast Nehi*

Faculty of Mathematics,

University of Sistan and Baluchestan, Zahedan, Iran

hmnehi@hamoon.usb.ac.ir

Abstract

In this paper we deal with solving Fuzzy Linear Programming (FLP) problem by Interval Linear Programming (ILP) approach. Firstly, we convert FLP problem to ILP problem by $\alpha-$cuts and in general case, we determine ILP on the basis of α. Then we will show that Tong-Shaocheng method for finding the worst value of objective function encounter a difficulty for solving problems with equality constraints.

Key words: Fuzzy linear programming; Interval linear programming; interval coefficient; interval systems.

1 Introduction

In fuzzy decision making problems, the concept of maximizing decision was proposed by Bellman and Zadeh. This concept was adopted to linear programming problems by Zimmermann. Fuzzy linear programming problems was formulated by Negoita and Dubois and Prade.

In this paper, we convert fuzzy linear programming to interval linear programming by $\alpha-$cut method. Then, we solve this problem by tong-shaocheng method and show that this method encounter a difficulty for solving problems with equality constraints.

*Corresponding author

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In this paper we want to extend the method of solving ILP problems presented by Ramadan[2] for minimize and maximize objective function subject to equality and inequality constraints. Furthermore, we will solve examples for it. In Section 2 we will introduce fuzzy sets and Tong-Shaocheng method for solving ILP problems with nonnegative variables. In Sections 3 we will solve ILP problems with $\geq$, $\leq$ and $=$ constraints. In Section 4 we will convert FLP problem to ILP problem. In Section 5 we will present two examples and their solutions.

2 Definitions and preliminaries

In this section we give some definitions and preliminaries in which needed in next sections.

Definition 2.1. Let X denote a universal set. A fuzzy subset $\widetilde{A}$ of X is defined as a set of ordered pairs of element x and grade $\mu_{\widetilde{A}}(x)$ and is written

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) \ : \ x \in X\}$$

where $\mu_{\widetilde{A}}(x)$ is membership function from X to $[0,1]$.

Definition 2.2. The $\alpha-$cut set of a fuzzy set $\widetilde{A}$ is defined as an ordinary set A_α where

$$A_\alpha = \{x : \mu_{\widetilde{A}}(x) \geq \alpha\} \quad c\ \alpha \in [0,1]$$

Definition 2.3. A fuzzy number $\widetilde{A} = (a,b,c,d)$ is a trapezoidal fuzzy number if

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & x \leq a, x \geq b \\ \frac{x-a}{b-a} & a < x < b \\ 1 & b \leq x \leq c \\ \frac{x-d}{c-d} & c < x < d \end{cases}$$

We can illustrate trapezoidal fuzzy number $\widetilde{A} = (a,b,c,d)$ as an interval $[\underline{m}_\alpha, \overline{m}_\alpha]$, where

$$\underline{m}_\alpha = a + \alpha(b-a)$$
$$\overline{m}_\alpha = d - \alpha(d-c)$$

Definition 2.4. An interval number X is generally represented as $[\underline{X}, \overline{X}]$ where $\underline{X} \leq \overline{X}$. If $\underline{X} = \overline{X}$, then X will be degenerate.

The extension of ordinary arithmetic to intervals is known as interval arithmetic. For a detailed discussion of this topic, refer to [5]. Let X° and X^{si} be as follows:

X°: Variables set which are not associated with an interval coefficient anywhere in the ILP problem and can be sign-restricted or unrestricted.

X^{si}: Variables set which are sign-restricted and are associated with at least one interval coefficient in ILP problem.

Definition 2.5. An ILP problem define as

$$\begin{aligned} \min \quad & Z = \sum_{j=1}^{n} [\underline{c}_j, \overline{c}_j] x_j \\ s.t. \quad & \sum_{j=1}^{n} [\underline{a}_{ij}, \overline{a}_{ij}] x_j \geq [\underline{b}_i, \overline{b}_i] \qquad i = 1, 2, ..., m \\ & x_j \in (X^{\circ} \cup X^{si}) \qquad j = 1, 2, ..., n \end{aligned} \tag{1}$$

We state Problem (1) as characteristic version

$$\begin{aligned} \min \quad & Z = \sum_{j=1}^{n} c_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a_{ij} x_j \geq b_i \qquad i = 1, 2, ..., m \\ & x_j \in (X^{\circ} \cup X^{si}) \end{aligned} \tag{2}$$

where $c_j \in [\underline{c}_j, \overline{c}_j]$, $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$ and $b_i \in [\underline{b}_i, \overline{b}_i]$.

In particular case when variables of ILP Problem (1) be non-negative, we will have model (3) and Theorem (1) introduces the best and worst values of objective function by Tong-Shaocheng method.

$$\begin{aligned} \min \quad & Z = \sum_{j=1}^{n} [\underline{c}_j, \overline{c}_j] x_j \\ s.t. \quad & \sum_{j=1}^{n} [\underline{a}_{ij}, \overline{a}_{ij}] x_j \geq [\underline{b}_i, \overline{b}_i] \qquad i = 1, 2, ..., m \\ & x_j \geq 0 \qquad j = 1, 2, ..., n \end{aligned} \tag{3}$$

Theorem 2.1. The best and worst optimum values of objective function for ILP problem (3) obtain by solving following problems respectively.

$$\begin{aligned} \min \quad & \underline{Z} = \sum_{j=1}^{n} \underline{c}_j x_j \\ s.t. \quad & \sum_{j=1}^{n} \overline{a}_{ij} x_j \geq \underline{b}_i \qquad i = 1, 2, ..., m \\ & x_j \geq 0 \qquad j = 1, 2, ..., n \end{aligned} \tag{4}$$

$$\begin{aligned} \min \quad & \overline{Z} = \sum_{j=1}^{n} \overline{c}_j x_j \\ s.t. \quad & \sum_{j=1}^{n} \underline{a}_{ij} x_j \geq \overline{b}_i \qquad i = 1, 2, ..., m \\ & x_j \geq 0 \qquad j = 1, 2, ..., n \end{aligned} \tag{5}$$

proof: See [2].

For equality constraints we have

$$\sum_{j=1}^{n}[\underline{a}_j,\overline{a}_j]x_j=[\underline{b},\overline{b}]$$

Therefore

$$\sum_{j=1}^{n}[\underline{a}_j,\overline{a}_j]x_j\geq[\underline{b},\overline{b}]\quad,\quad\sum_{j=1}^{n}[\underline{a}_j,\overline{a}_j]x_j\leq[\underline{b},\overline{b}]$$

Hence

$$\sum_{j=1}^{n}[\underline{a}_j,\overline{a}_j]x_j\geq[\underline{b},\overline{b}]\quad,\quad\sum_{j=1}^{n}[-\overline{a}_j,-\underline{a}_j]x_j\geq[-\overline{b},-\underline{b}]$$

Then due to Theorem 2.1 the best and worst values of objective function will obtain by solving the following problems respectively:

$$\begin{array}{llll}
\min & \underline{Z}=\sum_{j=1}^{n}\underline{c}_jx_j & \min & \overline{Z}=\sum_{j=1}^{n}\overline{c}_jx_j \\
s.to & \sum_{j=1}^{n}\overline{a}_{ij}x_j\geq\underline{b}_i \quad i=1,2,...,m & s.to & \sum_{j=1}^{n}\underline{a}_{ij}x_j\geq\overline{b}_i \quad i=1,2,...,m \\
 & \sum_{j=1}^{n}\underline{a}_{ij}x_j\leq\overline{b}_i \quad i=1,2,...,m & & \sum_{j=1}^{n}\overline{a}_{ij}x_j\leq\underline{b}_i \quad i=1,2,...,m \\
 & x_j\geq 0 \quad j=1,2,...,n & & x_j\geq 0 \quad j=1,2,...,n
\end{array} \tag{6}$$

3 Solving interval linear programming

In this section the method for solving interval linear programming is proposed[6]. The proposed method is presented in differents cases as $\geq$, $\leq$ and $=$ constraints.

3.1 Solving ILP problems with $\geq$ constraints

In this subsection first, we introduce the largest and smallest feasible regions for inequality constraints as $\geq$. Then we state the best and worst values of objective function for it.

Theorem 3.1. Let x_j belong to $X^{\circ}\cup X^{si}$ for all j. Then for interval inequality

$$\sum_{j=1}^{n}[\underline{a}_j,\overline{a}_j]x_j\geq[\underline{b},\overline{b}],$$

Where for all j, $x_j \in (X^\circ \cup X^{si})$, $\sum_{j=1}^{n} a'_j x_j \geq \underline{b}$ and $\sum_{j=1}^{n} a''_j x_j \geq \overline{b}$ are the largest and smallest feasible regions respectively.

where

$$a'_j = \begin{cases} \overline{a}_j & x_j \geq 0 \\ \underline{a}_j & x_j \leq 0 \end{cases}, \quad a''_j = \begin{cases} \underline{a}_j & x_j \geq 0 \\ \overline{a}_j & x_j \leq 0 \end{cases}.$$

Proof: See [2].

Theorem 3.2. For ILP Problem (1) the best and worst optimum obtain by solving the following problems respectively.

$$\begin{aligned} \min \quad & \underline{Z} = \sum_{j=1}^{n} c'_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a'_{ij} x_j \geq \underline{b}_i \qquad i = 1, 2, ..., m \end{aligned} \tag{7}$$

$$\begin{aligned} \min \quad & \overline{Z} = \sum_{j=1}^{n} c''_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a''_{ij} x_j \geq \overline{b}_i \qquad i = 1, 2, ..., m \end{aligned} \tag{8}$$

where

$$a'_{ij} = \begin{cases} \overline{a}_{ij} & x_j \geq 0 \\ \underline{a}_{ij} & x_j \leq 0 \end{cases} \qquad a''_{ij} = \begin{cases} \underline{a}_{ij} & x_j \geq 0 \\ \overline{a}_{ij} & x_j \leq 0 \end{cases}$$

$$c'_j = \begin{cases} \underline{c}_j & x_j \geq 0 \\ \overline{c}_j & x_j \leq 0 \end{cases} \qquad c''_j = \begin{cases} \overline{c}_j & x_j \geq 0 \\ \underline{c}_j & x_j \leq 0 \end{cases}$$

Proof. See [6].

Theorem 3.3. If the objective function of problem (1) is changed to " max ", then the best and worst optimum values obtain by solving the following problems respectively.

$$\begin{aligned} \max \quad & \overline{Z} = \sum_{j=1}^{n} c''_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a'_{ij} x_j \geq \underline{b}_i \qquad i = 1, 2, ..., m \end{aligned}$$

$$\begin{aligned} \max \quad & \underline{Z} = \sum_{j=1}^{n} c'_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a''_{ij} x_j \geq \overline{b}_i \qquad i = 1, 2, ..., m \end{aligned}$$

where a'_{ij}, a''_{ij}, c'_j and c''_j are as defined in Theorem 3.2.
Proof. See [6].

3.2 Solving ILP problem with $\leq$ constraints

In this subsection we introduce the largest and smallest feasible regions for inequality constraints as $\leq$. Then we state the best and worst values of objective function for it.
Theorem 3.4. Let x_j belong to $X^\circ \cup X^{si}$ for all j. Then for interval inequality

$$\sum_{j=1}^{n}[\underline{a}_j, \overline{a}_j]x_j \leq [\underline{b}, \overline{b}],$$

$\sum_{j=1}^{n} a''_j x_j \leq \overline{b}$ and $\sum_{j=1}^{n} a'_j x_j \leq \underline{b}$ are the largest and smallest feasible regions respectively, where

$$a'_j = \begin{cases} \overline{a}_j & x_j \geq 0 \\ \underline{a}_j & x_j \leq 0 \end{cases}, \quad a''_j = \begin{cases} \underline{a}_j & x_j \geq 0 \\ \overline{a}_j & x_j \leq 0 \end{cases}.$$

Proof. See [6].
Theorem 3.5. For ILP problem

$$\begin{aligned} \min \quad & Z = \sum_{j=1}^{n}[\underline{c}_j, \overline{c}_j]x_j \\ s.t. \quad & \sum_{j=1}^{n}[\underline{a}_{ij}, \overline{a}_{ij}]x_j \leq [\underline{b}_i, \overline{b}_i] \qquad i = 1, 2, ..., m \\ & x_j \in (X^\circ \cup X^{si}) \end{aligned} \tag{9}$$

the best and worst optimum values obtain by solving the following problems respectively.

$$\begin{aligned} \min \quad & \underline{Z} = \sum_{j=1}^{n} c'_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a''_{ij} x_j \leq \overline{b}_i \qquad i = 1, 2, ..., m \end{aligned} \tag{10}$$

$$\begin{aligned} \min \quad & \overline{Z} = \sum_{j=1}^{n} c''_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a'_{ij} x_j \leq \underline{b}_i \qquad i = 1, 2, ..., m \end{aligned} \tag{11}$$

where $a'_{ij}, a''_{ij}, c'_j, c''_j$ are as defined in Theorem 3.2.
Proof. See [6].
Theorem 3.6. If the objective function of Problem (9) is changed to " max ", then the best

and worst optimum values obtain by solving the following linear programming problems respectively.

$$\begin{aligned} \max \quad & \overline{Z} = \sum_{j=1}^{n} c''_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a''_{ij} x_j \le \bar{b}_i \qquad i = 1, 2, ..., m \end{aligned}$$

$$\begin{aligned} \max \quad & \underline{Z} = \sum_{j=1}^{n} c'_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a'_{ij} x_j \le \underline{b}_i \qquad i = 1, 2, ..., m \end{aligned}$$

where a'_{ij}, a''_{ij}, c'_j and c''_j are as defined in Theorem 3.2.

Proof. See [6].

3.3 Solving ILP problem with equality constraints

In this subsection we introduce the largest and smallest feasible regions for equality constraints. Then we state the best and worst values of objective function for it.

Theorem 3.7. the interval equaity constraint

$$\sum_{j=1}^{n} [\underline{a}_j, \overline{a}_j] x_j = [\underline{b}, \overline{b}]$$

can be considered as the following two inequality constraints

$$\sum_{j=1}^{n} a'_j x_j \ge \underline{b} \ , \ \ \sum_{j=1}^{n} a''_j x_j \le \overline{b}$$

such that they define a convex region in which every point could satisfy some charactristic formula of the original interval equality constraint by an appropriate choice of fixed values for the interval coefficients where a'_j and a''_j are as defined in Theorem 3.2.

Proof. See [6].

Theorem 3.8. For ILP problem

$$\begin{aligned} \min \quad & Z = \sum_{j=1}^{n} [\underline{c}_j, \overline{c}_j] x_j \qquad (12) \\ s.t. \quad & \sum_{j=1}^{n} [\underline{a}_{ij}, \overline{a}_{ij}] x_j = [\underline{b}_i, \overline{b}_i] \qquad i = 1, 2, ..., m \\ & x_j \in (X^{\circ} \cup X^{si}) \end{aligned}$$

the best optimum value obtains by solving the following problem

$$\begin{aligned} \min \quad & \underline{Z} = \sum_{j=1}^{n} c'_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a'_{ij} x_j \geq \underline{b}_i \qquad i = 1, 2, ..., m \\ & \sum_{j=1}^{n} a''_{ij} x_j \leq \overline{b}_i \qquad i = 1, 2, ..., m \\ & x_j \in X^{si} \end{aligned} \tag{13}$$

and the worst value of objective function obtains by solving one of the following two linear programming problems.

$$\begin{aligned} \min \quad & \overline{Z}_1 = \sum_{j=1}^{n} c''_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a'_{ij} x_j = \underline{b}_i \qquad i = 1, 2, ..., m \\ & x_j \in X^{si} \end{aligned} \tag{14}$$

$$\begin{aligned} \min \quad & \overline{Z}_2 = \sum_{j=1}^{n} c''_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a''_{ij} x_j = \overline{b}_i \qquad i = 1, 2, ..., m \\ & x_j \in X^{si} \end{aligned} \tag{15}$$

where $a'_{ij}, a''_{ij}, c'_j, c''_j$ are as defined in Theorem 3.2.

Proof. See [6].

Theorem 3.9. If the objective function of Problem (12) is changed to " max ", then the best optimum value of the objective function obtains by solving the following problem:

$$\begin{aligned} \max \quad & \overline{Z} = \sum_{j=1}^{n} c''_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a'_{ij} x_j \geq \underline{b}_i \qquad i = 1, 2, ..., m \\ & \sum_{j=1}^{n} a''_{ij} x_j \leq \overline{b}_i \qquad i = 1, 2, ..., m \\ & x_j \in X^{si} \end{aligned}$$

and the worst value of objective function obtains by solving one of the following two linear programming problems:

$$\begin{aligned} \max \quad & \underline{Z}_1 = \sum_{j=1}^{n} c'_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a'_{ij} x_j = \underline{b}_i \qquad i = 1, 2, ..., m \\ & x_j \in X^{si} \end{aligned}$$

$$\begin{aligned} \max \quad & \underline{Z}_2 = \sum_{j=1}^{n} c''_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a''_{ij} x_j = \overline{b}_i \qquad i = 1, 2, ..., m \\ & x_j \in X^{si} \end{aligned}$$

where $a'_{ij}, a''_{ij}, c'_j, c''_j$ are as defined in Theorem 3.2.

Proof. See [6].

4 Solving fuzzy linear programming with interval linear programming approach

In this section we present a method for solving fuzzy linear programming. Firstly, we convert fuzzy coefficients in problem to interval coefficient by $\alpha-$cut. So we solve determined interval linear programming.

Definition 4.1. We define a fuzzy linear programming problem as follows:

$$\begin{aligned} \min \quad & \widetilde{Z} = \sum_{j=1}^{n} \widetilde{c}_j x_j \\ s.t. \quad & \sum_{j=1}^{n} \widetilde{a}_{ij} x_j \geq \widetilde{b}_i \qquad i = 1, 2, ..., m \\ & x_j \geq 0 \end{aligned} \tag{16}$$

where $\widetilde{c}_j$, $\widetilde{a}_{ij}$, $\widetilde{b}_i$ are trapezoidal fuzzy numbers.

Now, we can convert Problem (16) to interval linear programming by $\alpha-$cuts. Let $\alpha \in [0, 1]$, $\widetilde{c}_j = (c_j^1, c_j^2, c_j^3, c_j^4)$, $\widetilde{a}_{ij} = (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4)$ and $\widetilde{b}_i = (b_i^1, b_i^2, b_i^3, b_i^4)$ be trapezoidal fuzzy numbers. then

$$\sum_{j=1}^{n} \widetilde{c}_j x_j = (\sum_{j=1}^{n} c_j^1 x_j, \sum_{j=1}^{n} c_j^2 x_j, \sum_{j=1}^{n} c_j^3 x_j, \sum_{j=1}^{n} c_j^4 x_j)$$

According to the Definition 2.3, we have

$$\widetilde{Z}_\alpha = [\sum_{j=1}^{n}(c_j^1 + \alpha(c_j^2 - c_j^1))x_j, \sum_{j=1}^{n}(c_j^4 - \alpha(c_j^4 - c_j^3))x_j]$$

Also, the constraints are converted to

$$[\sum_{j=1}^{n}(a_{ij}^1 + \alpha(a_{ij}^2 - a_{ij}^1))x_j, \sum_{j=1}^{n}(a_{ij}^4 - \alpha(a_{ij}^4 - a_{ij}^3))x_j] \geq [b_i^1 + \alpha(b_i^2 - b_i^1), b_i^4 - \alpha(b_i^4 - b_i^3)]$$

Therefore, FLP Problem (16) is converted to ILP problem as

$$\begin{aligned} \min \quad & Z = \sum_{j=1}^{n}[\underline{c}_j, \overline{c}_j]x_j \\ s.t. \quad & \sum_{j=1}^{n}[\underline{a}_{ij}, \overline{a}_{ij}]x_j \geq [\underline{b}_i, \overline{b}_i] \qquad i = 1, 2, ..., m \\ & x_j \geq 0 \end{aligned} \tag{17}$$

where

$$\begin{aligned} &\underline{c}_j = c_j^1 + \alpha(c_j^2 - c_j^1)\ ,\ \overline{c}_j = c_j^4 - \alpha(c_j^4 - c_j^3) \\ &\underline{a}_{ij} = a_{ij}^1 + \alpha(a_{ij}^2 - a_{ij}^1)\ ,\ \overline{a}_{ij} = a_{ij}^4 - \alpha(a_{ij}^4 - a_{ij}^3) \\ &\underline{b}_i = b_i^1 + \alpha(b_i^2 - b_i^1)\ ,\ \overline{b}_i = b_i^4 - \alpha(b_i^4 - b_i^3) \end{aligned}$$

5 Numerical examples

In this section we will explain previous methods with presenting several examples. Also we will show infirmity of Tong-Shaocheng method for finding the worst value of objective function in comparison with the proposed method in this paper.

Example 1.

$$\begin{aligned} \min \quad & z = (0, 2, 4, 6)x_1 + (2.5, 3.5, 3.7, 4.3)x_2 \\ s.t. \quad & (0.5, 1.5, 2, 4)x_1 + (4.5, 5.5, 5.8, 6.2)x_2 \geq (2.8, 3.2, 4.5, 5.5) \\ & x_1 + (-2.5, -1.5, -1.3, -0.7)x_2 \leq (0, 2, 3, 5) \\ & (1.5, 2.5, 2.7, 3.3)x_1 + x_2 = (2.5, 3.5, 3.8, 4.2) \\ & x_1, x_2 \geq 0 \end{aligned}$$

With cut $\alpha = 0.5$, the following interval linear programming will be obtained:

$$
\begin{aligned}
\min \quad & z = [1,5]x_1 + [3,4]x_2 \\
s.t. \quad & [1,3]x_1 + [5,6]x_2 \geq [3,5] \\
& x_1 + [-2,-1]x_2 \leq [1,4] \\
& [2,3]x_1 + x_2 = [3,4] \\
& x_1, x_2 \geq 0
\end{aligned}
$$

The best value is $\underline{z}^* = 1$ which by considering Sections 3, 4 and 5 obtain by solving the following problem

$$
\begin{aligned}
\min \quad & \underline{z} = x_1 + 3x_2 \\
s.t. \quad & 3x_1 + 6x_2 \geq 3 \\
& x_1 - 2x_2 \leq 4 \\
& 3x_1 + x_2 \geq 3 \\
& 2x_1 + x_2 \leq 4 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

and the worst value is $\overline{z}^* = 11$ which obtain by solving the following problems:

$$
\begin{aligned}
\min \quad & \overline{z}_1 = 5x_1 + 4x_2 \\
s.t. \quad & x_1 + 5x_2 \geq 5 \\
& x_1 - x_2 \leq 1 \\
& 3x_1 + x_2 = 3 \\
& x_1, x_2 \geq 0
\end{aligned}
\qquad\qquad
\begin{aligned}
\min \quad & \overline{z}_2 = 5x_1 + 4x_2 \\
s.t. \quad & x_1 + 5x_2 \geq 5 \\
& x_1 - x_2 \leq 1 \\
& 2x_1 + x_2 = 4 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

where $\overline{z}_1^* = 7, \overline{z}_2^* = 11$. By use of Tong-Shaocheng method the best value is $\underline{z}^* = 1$, but problem related to the worst value is infeasible.

Example 2.

$$
\begin{aligned}
\min \quad & z = (0,2,3,7)x_1 + (1,3,5,7)x_2 \\
s.t. \quad & (1,3,4,6)x_1 + x_2 = (1,5,6,8) \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Let $\alpha = 0.5$, then the Problem is converted to interval linear programming as follows

$$
\begin{aligned}
\min \quad & z = [1,5]x_1 + [2,6]x_2 \\
s.t. \quad & [2,5]x_1 + x_2 = [3,7] \\
& x_1, x_2 \geq 0
\end{aligned}
$$

The best and worst values of objective function are $\frac{3}{5}$ and $\frac{35}{2}$, with optimum solutions $\begin{pmatrix} \frac{3}{5} \\ 0 \end{pmatrix}$. and $\begin{pmatrix} \frac{7}{2} \\ 0 \end{pmatrix}$. respectively. But by use of Tong-Shaocheng method the worst problem will be infeasible.

6 Conclusion

In this paper a method has been presented for solving fuzzy and so interval linear programming (ILP) and it has been compared with Shaocheng method. For solving ILP problems with inequality constraints we use Theorems 3.2 and 4.2 and for solving ILP problems with equality constraints we use Theorem 3.8. Solutions obtained by the proposed method and Shaocheng method for the best values of the objective function of ILP are the same. However, this is not true for the worst value. These statements have been shown by several examples.

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