# Special Issue on Algorithmic Approaches to Solve Complementarity and Variational Inequality Problems

## Guest Editorial

The subject of complementarity and variational inequality problems , their diverse applications in linear/quadratic programming, operations research, engineering (dynamic rigid-body model, nonlinear obstacle problems), economics equilibrium models, and science, have become a well-established and fruitful discipline within mathematical programming.

This special issue focus on algorithmic approaches to solve complementarity and variational inequality problems. It includes 12 papers selected after a peer revision. We hope that the readers of Advanced Modeling and Optimization will find in these papers stimulating ideas and useful results. The summaries of the 12 papers in this issue are listed below:

The first paper by M. J. Alves and J. J. Júdice develops a pivoting heuristic based on Tabu search and its integration into an enumerative framework to solve the Linear Complementarity Problem (LCP). The heuristic uses Tabu search principle by imposing Tabu restrictions on pivot operations in order to avoid inversions or repetitions of recent moves. Computational experiments with this Branch and Tabu Pivoting (BTP) algorithm have shown that for some types of NP-hard LCPs the algorithm is able to find a solution in a few iterations. Computational experience on test problems is re- ported to show the efficiency of the proposed method.

The second paper by Q. Li and D. Li proposes a smoothing function to the nonlinear complementarity problem. The function possesses the Jacobian consistency property. Based on this function, they develop a smoothing Newton method for solving the nonlinear complementarity problem. Under appropriate conditions, the global and superlinear convergence of the method is established. they also show that when applied to solve a linear complementarity problem, the method terminates at a solution after infinitely many iterations.

The third paper by C. Bidard presents a general equilibrium technique to show the existence of a solution for a family of linear complementarity problems or complementarity problems which involve a function admitting a copositive extension.

It is well-known that the second order cone complementarity problems (SOCCP), as a special case of symmetric cone complementarity problems, have widely applications and have been extensively studied both in theoretical analysis and in methods design. Most existing results for SOCCPs are investigated for the monotone case which to some extent restrict their application scope. In the fourth paper by G.Q. Wang et al. focuses on a special non-monotone linear SOCCP and proposes a new interior point algorithm for such a class of problems. This shows a certain research value of this paper. Though the proposed algorithm is exactly an extension from linear optimization to the Cartesian $P_*(\kappa)$ second-order cone linear complementarity problem, it is not straightforward resulting from the non-polyhedral property of the second-order cone. By employing the algebraic and geometrical structures of the second order cone, together with the feature of a finite kernel function, the authors derived the iteration bounds of the proposed algorithm with large-update and small-update strategies, which are as good as the ones for standard $P_*(\kappa)$-LCP over the simple nonnegative orthants.

Based on the unconstrained implicit Lagrangian reformulation of the symmetric cone complementarity problem (SCCP), the fifth paper by S. Pan and J.-S. Chen suggests a derivative-free method, with a nonmonotone line search, for solving the SCCP. In particular, the authors have shown that the proposed method is globally convergent under mild assumptions; and if the SCCP is strongly monotone then the method is R-linearly convergent for all small values of $\mu$ (a parameter involved in the method). Numerical results demonstrate that the method is effective.

The Sixth paper by A. Moudafi proposes a generalized version of an algorithm introduced by Bermudez and Moren 1981 to solve general variational inequalities. The results in this paper extend, improve and develop some known results in this field.

The seventh paper by M.A. Noor et al. defines a new general class of variational problems and shows by well-known results from monotone operator theory using resolvent technique the equivalence of such problems to a fixed point problem. This suggests an iterative solution of these variational problems by a fixed iteration. As their main result, the authors show norm convergence of this iterative method to a solution of the problem.

The eighth paper by R. U. Verma introduces a new approach based the relatively maximal $(m)$-relaxed monotonicity and applies the approach to the linear convergence analysis to the context of the approximation solvability of a general class of inclusion problems. Further, based on the new model, the author considers convergence analysis, turns out to be more general than that of the celebrated technique of Rockafellar that is limited to the Lipschitz continuity at 0 of the inverse of the set-valued mapping. The results presented in this paper generalize, improve and unify the known results of some recent works.

The ninth paper by B. He presents some alternating directions like methods (ADM)for linearly constrained structured variational inequalities. And the convergence of the different versions of the proposed methods follows from the general framework of the contraction methods. The main feature of this paper is that the sub-problems of the new methods are simple and applicable.

The tenth paper by H. He et al. uses the projection type methods to solve monotone variational inequalities. These kinds of methods require less storage and are easy to implement. By introducing a new descent direction, the authors have studied the contraction and convergence properties of their proposed method. Some preliminary numerical experiments are tested to illustrate the performance of the proposed method

The eleventh paper by Y. Qing et al. considers the problem of finding a common element of the fixed point set of a strict pseudocontraction and the solution set of variational inequalities. Also as applications, the authors have considered the problem of approximating a common fixed point of three strict pseudocontractions in a real Hilbert space. The main results improve the recent corresponding results of several authors in literature.

The last paper by M. Li proposes some improved relaxed CQ methods to solve the split feasibility problem. Convergence of the proposed methods is discussed under some suitable conditions. Some preliminary computational results are given to illustrate the efficiency and implementation of the proposed methods.

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