Observations on a certain Comment about Zeta Functions

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Abstract

On a few observations realized by O. Shanker [2010] it brings over of an paper that treats on certain aspects of the Function Zeta [Garrido2009].
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1 Some considerations

First of all, to be grateful for the praises of the author of the Comment [OShanker2010], on the interesting thing that looks like to him the work.

As for the objections of [Shanker2010] it brings over of if Ihara's zeta function treats itself about the function for graphs, or about the so called function zeta for complex networks, we have to add that our objective and principal field of study are precisely the complex networks, and therefore, its applications to these many times ellusive structures.

On the other hand, it is of all known that these are modeled mathematically by means of his representation across Graph's Theory, many times with support in the assignment of probabilities, which does that the models take associate certain distributions of probability.

So, we need frequently the support of directed graphs (digraphs), weighted edges, Hidden Markov Models, and so on.

I recommend the detailed analysis of many articles that I have dedicated to these topics and to related others, inside the 135 published ones up to the date by me.

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1.1 Some features

According to the own expressions by [OShanker2010], "in an intersting study... the bulk of the analysis is valid".

Nevertheless, he says that "properties attributed to the Ihara Zeta functions on pages 261 and 262 are not correct...", because ""these properties... actually hold are now called complex network zeta functions [OShanker2008], to avoid confusion with the Ihara zeta function for graphs".

It is indeed a quasi-semantic problem, also partially related with certain notational terminology, which departs from considering the complex network set as a sophisticated subclass of the family of graphs. And therefore, it must be called "Complex Network Zeta Functions", instead of Ihara Zeta Function, usually assigned for more simple graphs.

Thus, it must be defined as

$$\varsigma_G(\alpha) = \frac{1}{N}\sum_i \sum_{i \neq j} \frac{1}{r_{ij}^{\alpha}}$$

with N the cardinal of the set of nodes of G.

So, introducing the graph surface function, denoted by $S(r)$, it can also be expressed by an equivalent formula

$$\varsigma_G(\alpha) = \sum_r \frac{S(r)}{r^{\alpha}}$$

Hence, being $S(r)$ the cardinality of the set of nodes which are exactly at distance r from a given node, averaging over all nodes of the network.

In addition, since we know, all this depends on the aims that are chased and of the pieces of the puzzle which we have. Because the best definition is going to be depending on which it is the application to which it is destined.

From other aspects, the Note of [OShanker2010] is almost a mere transcription, with many typographical mistakes, of the Wikipedia entry on the "Complex Network Zeta function", page currently visible, and surely due to the same author, according to the Bibliography which is cited.

To show the typology of structures, models and problems to which one tries to apply such mathematical tools as the Function Zeta, let's see the following paragraph.

1.2 On Complex Networks

In Regular Networks, each node is connected to all other nodes. I.e. they are fully connected between them.

As related to *Random Graphs (RGs)*, we can say that each pair of nodes is connected with probability p.

They have a low average path length, according to

$$L \approx (\ln\ n) / \ln \langle k \rangle \simeq \ln\ n,$$
$$for\ n \gg 1$$

It is because the total network may be covered in <k> steps, from which

$$n \propto \langle k \rangle^{L}$$

Moreover, they possess a low clustering coefficient, when the graph is sparse. Thus,

$$C = p = \langle k \rangle / n \ll 1$$

given that the probability of each pair of neighboring nodes to be connected is precisely equal to *p.*

The *Small-World effect* is observed on a network when it has a low average path length.
I.e.

$$L << n,\ for\ n >> 1$$

Recall the now famous "six degrees of separation", also called the

"small-world phenomenon"

Self-similarity on network indicates that it is approximately similar to any part of itself, and therefore, it is *fractal.* In many cases, the real networks possess all these properties, i.e. they are *Fractal, Small-World,* and *Scale-Free.*

Fractal dimensions describe self-similarity of diverse phenomena: images, temporal signals,... Such fractal dimension gives us an indication of how completely a fractal appears to fill the space, as one zooms down to finer and finer scales. It is, so, a statistical measure.

2 Conclusion

So, we hope to have achieved our initial purpose, that of attempting to provide a certain vision on some aspects, and properties, of Complex Networks, from a new Mathematical Analysis point of view.

Our intention is not to argue on the suitability of the definitions for an intention or other one.

For it, we are grateful providing that they are respectful and contribute something, all the contributions that allow to improve the undertaken works. This one is the case of the positive observations of O. Shanker [2010].

References

[1] K. Falconer. *Fractal Geometry: Mathematical Foundations and Applications.* Wiley, second edition, 2003.

[2] A. Garrido (2009) Combinatorial analysis by the Ihara zeta function of graphs, *Advanced Modeling and Optimization,* vol.11, pp. 253-278.

[3] O. Shanker (2007) Zeta Function and Dimension of Complex Network, *Mod. Phys. Lett.* vol. B21, pp. 639-644.

[4] O. Shanker (2008) Sharp dimension transition in a shortcut model, J. Phys. A, vol. 41(28), doi: 10.1088/1751-8113/41/28/285001.

[5] O. Shanker (2009) Defining the Dimension of a Complex Network and Zeta Functions, *International Journal of Computer Research,* vol.16, pp. 293-312.

[6] O. Shanker (2010) Complex network dimension and path counts, *Theoretical Computer Science,* vol. 411, pp. 2454-2458.

[7] R. Albert, H. Jeong, and A.-L. Barabási (1999). Diameter of the Worl-Wide Web. *Nature.* 401: 129-130.

[8] R. Albert, and A.-L. Barabási (2002). Statistical Mechanics of Complex Networks. *Reviews of Modern Physics.* 74: 47-.

[9] A.-L. Barabási, and Bonabeau (2003). Scale-Free Networks. *Scient. Am.* May 2003: 50-59.

[10] A. Barrat et al. (2008). *Dynamical processes in Complex Networks.* Cambridge University Press.

[11] S. Bocaletti et al. (2006). Complex Networks: Structure and Dynamics. *Phys. Rep.* 424: 175-308.

[12] B. Bollobás (2001). *Random Graphs.* Cambridge Studies in Advanced Mathematics 73. Cambridge University Press.

[13] B. Bollobás (1998). *Modern Graph Theory.* Springer Verlag.

[14] B. Bollobás et al. (2009). *Handbook of Large-Scale Random Networks.* (Bólya Society Mathematical). Springer Verlag.

[15] S. Bornholdt, and H. G. Schuster, editors (2003). *Handbook of Graphs and Networks: From the Genome to the Internet.* Wiley, Weinheim.