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COLOR CLASS DOMINATION NUMBER OF MIDDLE GRAPH AND CENTER GRAPH OF $K_{1,n}$, C_n AND P_n

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ABSTRACT

We give the color class domination number of the center graph and middle graph of $K_{1,n}$, C_n and P_n.

1 INTRODUCTION

By a graph we mean a finite, undirected graph without loops or multiple edges. Terms not defined here are used in the sense of Harary [2].

For a given graph G = (V,E) we do an operation on G, by subdividing each edge exactly once and joining all the non adjacent vertices of G. The graph obtained by this process is called central graph [4] of G, denoted by C(G).

The middle graph [1] of G, denoted by M(G) is defined as follows: The vertex set of M(G) is V(G) ∪ E(G). Two vertices x, y in the vertex set of M(G) are adjacent in M(G) in case one of the following holds: (i) x, y are in E(G) and x, y are adjacent in G. (ii) x is in V(G), y is in E(G), and x, y are incident in G.

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Domination and coloring are well studied concepts in graph theory. In [3] the concept of color class domination partition of a graph G was introduced. Let $\Pi = \{ V_1 , V_2 , V_3 , \ldots ,V_k \}$ be a proper color partition of G. Π is called a color class domination partition of G if for every i, $1 \le i \le k$, there exists a vertex $u \in V(G)$ such that V_i is dominated by u. The minimum cardinality of a color class domination partition of G is denoted by $\chi_{cd}(G)$ and is called color class domination number of G.

Theorem 1.1. [3] $\chi_{cd}(C_n) = \lceil n/2 \rceil$, if $n \equiv 0, 1, 3 \pmod 4$ and $n/2+1$ if $n \equiv 2 \pmod 4$.

Theorem 1.2. [3] $\chi_{cd}(P_n) = \lceil n/2 \rceil$, if $n \equiv 0, 1, 3 \pmod 4$ and $n/2+1$ if $n \equiv 2 \pmod 4$.

2 Color class domination number of middle graphs

Theorem 2.1: For any star graph $K_{1,n}$, $\chi_{cd}(M(K_{1,n}))=n+1$.

Proof: Let $V(K_{1,n})=\{v,v_1,v_2,v_3,\ldots ,v_n\}$ and $E(G)=\{e_1,e_2,e_3,\ldots,e_n\}$. By the definition of Middle graph, we have $V[M(K_{1,n})]=\{v\} \cup \{e_i \,/\, 1\le i \le n\} \cup \{v_i \,/\, 1 \le i \le n\}$, in which the vertices $e_1,e_2,e_3,\ldots,e_n,v$ induces a clique of order $n+1$. Hence, $\chi_{cd}(M(K_{1,n}))\ge n+1$.

$\Pi=\{\{e_1,v_2\}, \{e_2,v_3\},\{e_3,v_4\},\ldots,\{e_{n-1},v_n\},\{e_n,v_1\},\{v\}\}$ forms a color class domination partition of $M(K_{1,n})$. Hence, $\chi_{cd}(M(K_{1,n}))\le n+1$. Therefore, $\chi_{cd}(M(K_{1,n}))=n+1$.

Theorem 2.2: For any cycle C_n, $\chi_{cd}(M(C_n))=n$.

Proof: Let $V(C_n)=\{ v_1,v_2,v_3,\ldots ,v_n\}$ and $E(C_n)=\{ e_1,e_2,e_3,\ldots,e_n\}$ where $e_i=v_iv_{i+1}$ ($1 \le i \le n-1$), $e_n=v_nv_1$. By the definition of middle graph, $M(C_n)$ has the vertex set

$V(C_n) \cup E(C_n)$ in which e_i is adjacent with e_{i+1} ($i=1,2,3,\ldots,n-1$) and e_n is adjacent with v_1.

In $M(C_n)$, $v_1,e_1,v_2,e_2,v_3,e_3,\ldots,e_{n-1},v_1$ induces a cycle of length 2n. Therefore, $\chi_{cd}(M(C_n))\geq n$. $\Pi=\{\{e_2,v_1\}, \{e_3,v_2\},\{e_4,v_3\},\ldots,\{e_n,v_{n-1}\},\{e_n,v_1\},\{v_n,e_1\}\}$ forms a color class domination partition of $M(C_n)$. Hence, $\chi_{cd}(M(C_n))\leq n$. Therefore, $\chi_{cd}(M(C_n))=n$.

Theorem 2.3: For any path P_n, $\chi_{cd}(M(P_n))=n$.

Proof: Let P_n: $v_1,v_2,v_3,\ldots,v_{n+1}$ be a path of length n and let $v_iv_{i+1}=e_i$. By the definition of middle graph, $M(P_n)$ has the vertex set $V(P_n)\cup E(P_n) = \{ v_i / 1 \leq i \leq n+1\} \cup \{ e_i / 1 \leq i \leq n\}$ in which each v_i is adjacent to e_i and e_i is adjacent to v_{i+1}. Also e_i is adjacent to e_{i+1}.

Case (i): $|V(P_n)| = 2k$.

$|V(M(P_n))| = 4k-1$. The vertices $v_1, e_1,v_2,e_2,\ldots, e_{2k-1},v_k$ induces a path of length 4k. Therefore, $\chi_{cd}(M(P_n))\geq 2k$.

$\Pi=\{ \{ v_1, v_2 \}, \{ v_3, v_4 \}, \ldots , \{v_{2k-1},v_{2k}\},\{e_1,e_2\},\{e_3,e_4\},\ldots,\{e_{2k-3},e_{2k-1}\},\{e_{2k}\}\}$ is a color class domination partition. Hence, $\chi_{cd}(M(P_n))\leq 2k$.

Case (ii): $|V(P_n)| = 2k+1$.

$|V(M(P_n))| = 4k+1$. The vertices $v_1, e_1,v_2,e_2,\ldots, e_{2k},v_{2k+1}$ induces a path of length 4k. Therefore, $\chi_{cd}(M(P_n))\geq 2k$.

$\Pi=\{\{v_1, v_2\}, \{v_3, v_4\}, \ldots , \{v_{2k-1},v_{2k}\},\{v_{2k+1}\},\{e_1,e_2\},\{e_3,e_4\},\ldots,\{e_{2k-3},e_{2k-2}\},\{e_{2k-1},e_{2k}\}\}$ is a color class domination partition. Hence, $\chi_{cd}(M(P_n))\leq 2k$. Therefore, $\chi_{cd}(M(P_n))=n$.

3 Color class domination number of center graphs

Theorem 3.1: For any star graph $K_{1,n}$, χ_{cd} $(C(K_{1,n}))$=n+1.

Proof: Let $V(C(K_{1,n}))=\{v,v_1,v_2,...,v_n,e_1,e_2,...,e_n\}$. All e_i's are adjacent to v. e_i is adjacent to v_i, for every i ($1\leq i \leq n$).

The set of vertices $v,v_1,v_2,v_3,...,v_n$ forms a clique of order n+1. Therefore, χ_{cd} $(C(K_{1,n}))\geq n+1$. $\Pi=\{\{e_1,e_2,e_3,...,e_n\},\{v,v_1\},\{v_2\},\{v_3\},...,\{v_n\}\}$ forms a color class domination partition of $C(K_{1,n})$. Hence, χ_{cd} $(C(K_{1,n}))\leq n+1$. Therefore, χ_{cd} $(C(K_{1,n}))=n+1$.

Theorem 3.2: For any cycle C_n, $\chi_{cd}(C(C_n))$= n/2 if n is even and (n+1)/2 if n is odd.

Proof: Let $V(C_n)=\{v_1,v_2,v_3,...,v_n\}$ and $E(C_n)=\{e_1,e_2,e_3,...,e_n\}$. By the definition of $C(C_n)$, we have (i) v_i is adjacent to e_{i-1} and e_i, i = 2 to n-1. (ii) v_1 is adjacent to e_1 and e_n (iii) v_n is adjacent to e_n and e_{n-1} (iv) v_i is adjacent to v_j, i = 2 to n-1, $j\neq i-1,i+1$ (v) v_1 is adjacent to v_j $3 \leq j\leq n-1$. (vi) v_n is adjacent to v_j $2\leq j\leq n-2$.

Case (i): n even.

The vertices $v_1,e_1,v_2,e_2,v_3,e_3,...,v_n$ induces a subgraph C_{2n} and $\chi_{cd}(C(C_n))\geq n/2$.

Subcase (i): n/2 is even.

The following is a color class domination partition $\Pi=\{\{e_1,e_2,v_{1+n/2},v_{2+n/2}\}, \{e_3,e_4,v_{3+n/2},v_{4+n/2}\},...,\{e_{1+n/2},e_{2+n/2},v_1,v_2\},..., \{e_{n-1},e_n,v_{(n/2)-1},v_{n/2}\}\}$. Therefore, $\chi_{cd}(C(C_n))\leq n/2$. Hence, $\chi_{cd}(C(C_n))= n/2$.

Subcase (ii): n/2 is odd.

$\Pi=\{\{v_1,v_2,e_{n-2},e_{n-1}\},\{v_3,v_4,e_{n-4},e_{n-3}\},...,\{v_{n/2},v_{1+n/2},e_n,e_1\},...,\{v_{n-1},v_n,e_{(n/2)-1},e_{n/2}\}\}$ forms a color class domination partition. Therefore, $\chi_{cd}(C(C_n))\leq n/2$. Hence, $\chi_{cd}(C(C_n))= n/2$.

Case (ii): n is odd.

The vertices $v_1,e_1,v_2,e_2,v_3,e_3,\ldots,e_n,v_n$ induces a subgraph C_{2n} of length 4n+2. Therefore, $\chi_{cd}(C(C_{2n}))= (n+1)/2$.
$\Pi=\{\{v_{(n+1)/2},v_{(n+3)/2},e_1,e_2\},\ldots,\{v_n,v_1,e_{(n+1)/2},e_{(n+3)/2}\},\ldots,\{v_{(n-5)/2},v_{(n-3)/2},e_{n-2},e_{n-1}\},\ldots,$ $\{v_{n-1},v_n,e_{(n/2)-1},e_{n/2}\}\}$ forms a color class domination partition. Therefore, $\chi_{cd}(C(C_n))\leq (n+1)/2$. Hence, $\chi_{cd}(C(C_n))=(n+1)/2$.

Theorem 3.3: For any path P_n, $\chi_{cd}(C(P_n)) =\lceil n/2\rceil$.

References

[1] Danuta Michalak, On Middle and total graphs with coarseness number equal 1; Graph theory, Proc.Conf;Lagow/pol.1981, Lect.Notes Math.1018 Springer, Berlin 1983, 139-150.

[2] F.Harary, Graph theory, Addison Wesley Reading Mass, 1969.

[3] R.Sundareswaran, V. Swaminathan, Color class domination and anti Domination in graphs. (accepted for publication)

[4] Vernold Vivin.J, Harmonious coloring of total graph, n-leaf, central graphs and circumdetic graphs; Ph.D thesis, Bharathiar University (2007), India.