AMO - Advanced Modeling and Optimization, Volume 12, Number 2, 2010[*]

Stochastic Based on Multi-objective Transportation Problems Involving Normal Randomness

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Abstract: This paper is concerned with the use of fuzzy programming technique to the objective function, and the stochastic method have been applied for the randomness of sources and destination parameters in inequality type of constraints of multi-objective stochastic unbalanced transportation problem. In this paper, we focus on the solution procedure of the specified problems where the objective functions are minimization type (i.e. non-commensurable and conflicting in nature) and the supplies and demands are replaced by the random variables. By using the chance constrained technique we first converted the multi-objective stochastic transportation problem multi-objective into equivalent deterministic transportation problem. By introducing the concept of linear membership function of fuzzy programming, multi-objective deterministic transportation problem is to convert into single objective deterministic transportation problem and then solved it we obtained the optimal compromise solution. Lastly a numerical example is provided for illustrating the methodology.

***Keywords*:** Transportation Problem, Stochastic Programming, Fuzzy Programming, Multi-objective Decision Making.

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[*] **AMO - Advanced Modeling and Optimization. ISSN: 1841-4311**

1. Introduction:

The probability theory has been the only avenue to analyze the uncertain situation. The probability distribution represents the information that each of possible events occur, and this distribution becomes the evidence for testing the hypotheses. Many problems of the transportation are dealt with by probability and that in which the fuzzy set theory is an appropriate measure. Probability density function and the membership function of fuzzy set are fundamentally different. The probability density function represents a summary of the information from the random experiments, and it is used to measure the truth of a proposition in probability. The basic transportation problem involves the following situation such as follows:

i) the transportation or physical distribution of products from supply points to demand points ,

ii) minimization of cost for transportation of products,

iii) minimization of transportation time,

iv) consideration of requirement of goods at each demand points,

v) variety of shipping routs,

vi) mode of transportation.

The transportation problem is an earliest application of linear programming problem. Hitchcock [9] was first developed the basic concept of transportation problem and later discussed in details by Koopmans [11]. In 1973, Appa [1] considered variants of the transportation in which all constraints involving the supply and demand are of inequality type. However, he has not considered the supply and demand constraints are of mixed type. Brigden [4] extended the concept of Appa [1] and considered the mixed type constraints. Then the original problem is converted into a related transportation problem with equality type of constraints by augmenting the original problem with the addition of two sources and two destinations. He obtained the optimal solution of the original problem from the optimal solution of the transformed transportation problem.

The existent method are available to get an initial basic feasible solution for transportation problem and they are North West Corner Rule, Row Minima Method, Column Minima Method, Vogel Approximation Method and gives a better feasible solution. The latest version of TORA, LINDO, LINGO packages are also helpful to solve the transportation problem.

The stochastic programming was first formulate as mathematical programming by Dantzig [6] and noted that the most promising fields for future research which has single objective or multi-objectives, the parameters have involved by the probability distribution which has treated as random variables. As a result, the objective and the constraints of such a problem turn into stochastic ones. Charnes and Cooper [5] introduced Chance Constrained Programming technique to solve the problems involving probabilistic constraint. For treatment of this technique, the basic approach is E-model which optimizes the expected values of the linear objective function.

Sahoo et al. [18] have studied in 2003, the equivalent deterministic mathematical programming technique of probabilistic linear programming for normal random variable. Kambo [10] discussed the Chance Constrained method for solving the stochastic linear programming technique. Mohon et al. [13] have developed the idea on the fuzzifying approach to multi-objective stochastic programming problem. Rao [14] gives the conception about the technique on stochastic multi-objective linear programming problems. Bit et al. [2] applied the fuzzy programming technique with the linear membership function to solve the chance constrained multi-objective transportation problem where the parameters are random variables on standard normal distribution. Roubens et al. [16] presented the comparison between the methodology for multi-objective fuzzy linear programming & multi-objective stochastic programming. Lee. et al, [12] have studied in 1993, the optimization of multi-objective transportation problem. An alternative procedure to generate all non-dominated solution to the multi-objective programming problems presented by Diaz [7] , whose approach depends upon apriori measure to the ideal solution. Ringuest et al. [15] have developed two interactive algorithms for the linear multi-objective stochastic transportation problem. Zimmermann [22] presented an application of fuzzy linear programming technique to solve these type of problem are always gives an efficient solution and also gives an optimal compromise solution.

These kinds of vagueness can be treated as the basic approaches of fuzzy programming called flexible programming which has proved so far to be one of the most successful applications of the fuzzy set theory initiated by Zadeh [21], Zimmermann [22] and Sakawa [17]. In some researches the fuzzy set theory has been used in dealing with stochastic linear programming problem. Again, Bit et al. [3] have developed the multi-objective transportation problem with probabilistic constraint using the fuzzy programming technique on probabilistic constraint.

The protrude method of Goicoechea et al. [8], the strange method of Teghem et al. [20] are used to solve multi-objective stochastic transportation problem. The multi-objective stochastic transportation problem has been presented by Stancu- Minasion [19]. They are mainly presented the stochastic programming which recognized in the literature as follows,

(i) Risk programming in linear programming models which includes some stochastic linear programming models,

(ii) Decision theoretic models,

(iii) Stochastic sensitivity analysis.

Most of the researchers have followed the conventional fuzzy approach for the solution of multi-objective stochastic transportation problems with two constraints having only one deterministic and other probabilistic without reviewing the multi-objective stochastic transportation problem with two probabilistic constraints to the practical sense of real life problems. In this paper, we have studied the multi-objective stochastic transportation problem with two probabilistic constraints which are of inequality type, and the parameters of supply and demand are followed by normal random variables.

1.1 **Multi-objective Transportation Problem :**

Consider m origin (or supply) O_i ($i = 1, 2, 3, ..., m$) and n destination (or demand) D_j ($j = 1, 2, 3, ..., n$). The sources may be production facilities and they are characterized by available supplies $a_1, a_2, a_3, ..., a_m$. The destination may be public destination center and they are characterized by demand levels $b_1, b_2, b_3, ..., b_n$. A penalty c_{ij} is the transportation cost or time cost, associated from origin i to the destination j and the variables x_{ij} are represented the unknown quantity goods to be transported from origin O_i to destination D_j.

The single objective transportation problem can be extended to multi-objective transportation problem by considering the k-th ($k = 1, 2, 3, ..., K$) cost coefficient c_{ij}^k ($k = 1, 2, 3, ..., K$) in the objective functions. Then the mathematical model of multi-objective transportation problem can be represented as follows:

Model 1:
$$\min : z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij}, \; k = 1, 2, 3, ..., K \tag{1}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \quad i = 1, 2, 3, ..., m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \; j = 1, 2, 3, ..., n \tag{3}$$

$$x_{ij} \geq 0, \quad i = 1, 2, 3, ..., m \;\&\; j = 1, 2, 3, ..., n \tag{4}$$

The balanced transportation problem is define when the total availability at supply point is equal to the total requirement at demand point with an equilibrium condition ($\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$) for the existence of a feasible solution.

2 Mathematical Model Involving Stochastic Multi-objective Transportation Problem :

Here, we have presented the mathematical model of multi-objective stochastic transportation problem with probabilistic constraints as follows:

Model 2:
$$\min : z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k x_{ij}, \; k = 1, 2, 3, ..., K \tag{5}$$

subject to

$$\text{Prob } \left(\sum_{j=1}^{n} x_{ij} \leq a_i\right) \geq 1 - \alpha_i, \quad i = 1, 2, 3, ..., m \tag{6}$$

$$\text{Prob } \left(\sum_{i=1}^{m} x_{ij} \geq b_j\right) \geq 1 - \beta_j, \quad j = 1, 2, 3, ..., n \tag{7}$$

$$x_{ij} \geq 0, \quad i = 1, 2, 3, ..., m \;\&\; j = 1, 2, 3, ..., n \tag{8}$$

where $0<\alpha_i<1, i=1,2,3,...,m$ and $0<\beta_j<1$, $j=1,2,3,...,n$. The above problem is a multi-objective stochastic problem where $a_i\,(i=1,2,3,...,m)$ and $b_j\,(j=1,2,3,...,n)$ are random variables with known distribution and c_{ij}^k $(k=1,2,3,...,K)$ is defined as deterministic cost coefficients for $i=1,2,3,...,m$ & $j=1,2,3,...,n$.

Depending upon the situation, let us consider the following three cases:

Case 1: Only a_i considered as random variables, $i=1,2,3,...,m$

Case 2: Only b_j considered as random variables, $j=1,2,3,...,n$

Case 3: Only a_i and b_j are considered as random variables, $i=1,2,3,...,m$ & $j=1,2,3,...,n$.

Now we have formulated the mathematical model of the above cases:

Case 1. Only a_i is random variable:

Model 3: $$\min: z_k=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^k x_{ij}, \qquad k=1,2,3,...,K \tag{9}$$

subject to

$$\text{Prob }\left(\sum_{j=1}^{n}x_{ij}\le a_i\right)\ge 1-\alpha_i,\ i=1,2,3,...,m \tag{10}$$

$$\sum_{i=1}^{m}x_{ij}\ge b_j, \qquad j=1,2,3,...,n \tag{11}$$

$$x_{ij}\ge 0, \quad i=1,2,3,...,m \text{ \& } j=1,2,3,...,n \tag{12}$$

The above problem is multi-objective stochastic transportation problem where the availability a_i is random variable. More precisely a_i follows normal distribution with mean of a_i i.e., $\overline{a_i}=E\left(a_i\right)$ and variance of a_i i.e., var (a_i) are known. Then our problem is to convert the above problem i.e., probabilistic multi-objective transportation problem into deterministic model. To do this, we have to change only the probabilistic constraint i.e., Prob $\left[\sum_{j=1}^{n}x_{ij}\le a_i\right]\ge 1-\alpha_i$,

$i=1,2,3,...,m$, (6) into deterministic constraint and the mean and variance for the random variables a_i are $\overline{a_i}$ and var (a_i), $i=1,2,3,...,m$. Let 1-$\alpha_i = p_i$, then equation (6) becomes:

$$\Pr ob\left[\sum_{j=1}^{n} x_{ij} \leq a_i\right] \geq p_i \tag{13}$$

This expression can be further stated as:

$$\Pr ob\left[\frac{\sum_{j=1}^{n} x_{ij} - \overline{a_i}}{\sqrt{\text{var}(a_i)}} \leq \frac{a_i - \overline{a_i}}{\sqrt{\text{var}(a_i)}}\right] \geq p_i \tag{14}$$

$$\Pr ob\left[\frac{a_i - \overline{a_i}}{\sqrt{\text{var}(a_i)}} \geq \frac{\sum_{j=1}^{n} x_{ij} - \overline{a_i}}{\sqrt{\text{var}(a_i)}}\right] \geq p_i \tag{15}$$

where $\frac{a_i - \overline{a_i}}{\sqrt{\text{var}(a_i)}}$ is a standard normal variate with mean zero and unit variance. Let $\Phi(z)$ denotes the cumulative density function of the standard normal variate evaluated at z , then k_{α_i} ($i=1,2,3,...,m$) represents the value of the standard normal variables. Then the equation (15) can be stated as

$$1-\Phi\left[\frac{a_i - \overline{a_i}}{\sqrt{\text{var}(a_i)}} \leq \frac{\sum_{j=1}^{n} x_{ij} - \overline{a_i}}{\sqrt{\text{var}(a_i)}}\right] \geq p_i \tag{16}$$

$$\Rightarrow \Phi\left[\frac{a_i - \overline{a_i}}{\sqrt{\text{var}(a_i)}} \leq \frac{\sum_{j=1}^{n} x_{ij} - \overline{a_i}}{\sqrt{\text{var}(a_i)}}\right] \leq 1 - p_i \tag{17}$$

Then $\Phi\left(-k_{\alpha_i}\right) = 1 - p_i = \alpha_i$; Using the cumulative density function of the standard normal variate, it can be simplified as:

$$\Phi\left(\frac{\sum_{j=1}^{n} x_{ij} - \overline{a_i}}{\sqrt{\operatorname{var}(a_i)}}\right) \le \Phi\left(-k_{\alpha_i}\right) \tag{18}$$

Then
$$\frac{\sum_{j=1}^{n} x_{ij} - \overline{a_i}}{\sqrt{\operatorname{var}(a_i)}} \le -k_{\alpha_i}, i = 1, 2, 3...m \tag{19}$$

Finally, the probabilistic constraint can be transformed into deterministic constraint as:

$$\sum_{j=1}^{n} x_{ij} - \overline{a_i} + k_{\alpha_i}\sqrt{\operatorname{var}(a_i)} \le 0, \tag{20}$$

Using (20) the equivalent deterministic multi-objective linear transportation problem of **Model 3** can be restated as.

$$\min : z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, k = 1, 2, 3, ..., K \tag{21}$$

$$\sum_{j=1}^{n} x_{ij} - \overline{a_i} + k_{\alpha_i}\sqrt{\operatorname{var}(a_i)} \le 0, \tag{22}$$

$$\sum_{i=1}^{n} x_{ij} \ge b_j \tag{23}$$

$$\sum_{i=1}^{m} x_{ij} \ge 0 \ , i = 1, 2, 3, ..., m \,\&\, j = 1, 2, 3, ..., n \tag{24}$$

Case 2. When b_j is random variable:

Then the mathematical model can be represented as follows:

Model 4:
$$\min : z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, \quad k = 1, 2, 3, ..., K \tag{25}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1, 2, 3, ..., m \tag{26}$$

$$\text{Prob}\left(\sum_{i=1}^{m} x_{ij} \ge b_j\right) \ge 1 - \beta_j, \ j = 1, 2, 3, ..., n \tag{27}$$

$$x_{ij} \geq 0, \quad i = 1,2,3,...,m \; \& \; j = 1,2,3,...,n \tag{28}$$

To do this, we have change only the probabilistic constraint i.e., Prob$\left(\sum_{i=1}^{m} x_{ij} \geq b_j\right) \geq 1 - \beta_j$, $j = 1,2,3,....,n$ (27) into deterministic constraint and the mean and variance for the random variable b_j are $\overline{b_j}$ and var(b_j) , $j = 1,2,3,....,n$. Let 1- β_j $= q_j$, then equation (27) becomes,

$$\Pr ob\left[\sum_{i=1}^{m} x_{ij} \geq b_j\right] \geq q_j \tag{29}$$

This can be further simplified as:

$$\Pr ob\left[\frac{\sum_{i=1}^{m} x_{ij} - \overline{b_j}}{\sqrt{\operatorname{var}(b_j)}} \geq \frac{b_j - \overline{b_j}}{\sqrt{\operatorname{var}(b_j)}}\right] \geq q_j \tag{30}$$

where $\frac{b_j - \overline{b_j}}{\sqrt{\operatorname{var}(b_j)}}$ is a standard normal variate with zero mean and unit variance. Let $\Phi(z)$ denotes the C.D.F. of the standard normal variate evaluated as z, then k_{β_j} ($j = 1,2,3,...,n$) represents the value of the standard normal variate.

$$\text{Then, } \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2} dt . \tag{31}$$

The cumulative density function (C.D.F) of the standard normal random variable is defined as,

$$\text{Hence} \qquad \Phi\left(\frac{\sum_{i=1}^{m} x_{ij} - \overline{b_j}}{\sqrt{\operatorname{var}(b_j)}}\right) \geq \Phi\left(k_{\beta_j}\right) \tag{32}$$

Therefore $$\frac{\sum_{i=1}^{m} x_{ij} - \overline{b_j}}{\sqrt{\mathrm{var}(b_j)}} \geq k_{\beta_j} \tag{33}$$

Finally, the probabilistic constraint can be transformed into deterministic constraint as:

$$\sum_{i=1}^{m} x_{ij} \geq b_j + k_{\beta_j}\sqrt{\mathrm{var}(b_j)},\ j = 1, 2, 3, ..., n \tag{34}$$

This is an equivalent deterministic constraint for equation (27). Finally, we obtained the mathematical model of multi-objective stochastic transportation problem for the above case 2, using (34) can be represented as:

$$\min : z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, k = 1, 2, 3, ..., K \tag{35}$$

subject to

$$\sum_{j=1}^{m} x_{ij} \leq a_i, \quad i = 1, 2, 3, ..., m \tag{36}$$

$$\sum_{i=1}^{m} x_{ij} - \overline{b_j} - k_{\beta_j}\sqrt{\mathrm{var}(b_j)} \geq 0, \quad j = 1, 2, 3, ..., n \tag{37}$$

$$x_{ij} \geq 0, \quad i = 1, 2, 3, ..., m \ \&\ j = 1, 2, 3, ..., n \tag{38}$$

Case 3. When $a_i \& b_j$ are normal random variables:

The mean and variance of $a_i \& b_j$ are known and defined earlier. In this case, the equivalent deterministic model for the chance constrained of the linear multi-objective stochastic Transportation problem of Model 2 can be represented as:

$$\min : z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}, \quad k = 1, 2, 3, ..., K \tag{39}$$

subject to

$$\sum_{j=1}^{m} x_{ij} - \overline{a_i} + k_{\alpha_j}\sqrt{\text{var}\left(a_i\right)} \le 0\,, \quad i = 1,2,3,...,m \tag{40}$$

$$\sum_{i=1}^{m} x_{ij} - \overline{b_j} - k_{\beta_j}\sqrt{\text{var}\left(b_j\right)} \ge 0\,, \quad j = 1,2,3,...,n \tag{41}$$

$$x_{ij} \ge 0\,, \quad i = 1,2,3,...,m \,\&\, j = 1,2,3,...,n \tag{42}$$

3. Solution Procedure :

To obtain an ideal solution to the linear multi-objective stochastic transportation problem, we should first consider in each objective function separately and other objective functions is to be ignored and solved with the constraints .

We have already converted equations (5), (6), (7) from multi-objective stochastic transportation problem into multi-objective deterministic transportation problem i,e. eqn. (39) to (42). To solve multi-objective deterministic transportation problem, we apply the fuzzy programming technique on consideration of multi-objective vector minimum problem. At first , we find the lower bound L_r (best) and upper bound U_r (worst) for corresponding objective function z_r where $r = 1,2,3,...,K.$

Let, $L_r =$ aspiration level of achievement for objective r ,

$U_r =$ highest acceptable level of achievement for objective r ,

$d_r = U_r - L_r =$ the degradation allowance for objective r ,

when the aspiration level and degradation allowance for each objective are specified.

Algorithm:

Step-1 : Solve multi-objective deterministic transportation problem as a single objective used each time and all other ignored.

Step-2 : Determine the corresponding values for every objective at each solution derived.

Step-3 : We construct a pay-off matrix, according to every objective w. r. to each solution The pay-off matrix in the main program gives the set of non dominated solution which shown in the following table.

$$\begin{array}{c} \\ x^{(1)} \\ x^{(2)} \\ x^{(3)} \\ \vdots \\ x^{(k)} \end{array} \begin{array}{c} \begin{array}{ccccc} z_1 & z_2 & z_3 & \cdots & z_k \end{array} \\ \begin{bmatrix} z_{11} & z_{12} & z_{13} & \cdots & z_{1k} \\ z_{21} & z_{22} & z_{23} & \cdots & z_{2k} \\ z_{31} & z_{32} & z_{33} & \cdots & z_{3k} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ z_{k1} & z_{k2} & z_{k3} & \cdots & z_{kk} \end{bmatrix} \end{array}$$

where $x^{(1)}, x^{(2)}, x^{(3)}, ..., x^{(k)}$ is the ideal solution for the objective $z_1, z_2, z_3, ..., z_k$ respectively.

Let $z_{ij} = z_j\left(x^i\right),\ i = 1,2,3,...,k$ & $j = 1,2,3,...,k$, are the minimum value (best) for each objective z_r, $r = 1,2,3,...,k$.

Step-4 : To find the best (L_r) & worst (U_r) for each objectives corresponding to the set of solution i.e., $L_r = z_{rr}$ and $U_r = \max\limits_{r\geq 1}\{z_{1r}, z_{2r}, z_{3r}........z_{kr}\}$. For satisfy, $z_r \leq L_r$, $r = 1,2,3,...,k$ and constraints (8) , (20), (34).

Step-5: To construct a membership function as,

$$\mu_{z_r}\left(x_{ij}\right) = \begin{cases} 1 & \text{if } z_r \leq L_r, \\ 1 - \dfrac{z_r - L_r}{U_r - L_r} & \text{if } L_r < z_r < U_r, \\ 0 & \text{if } z_r \geq U_r, \end{cases}$$

If $\mu_{z_r}\left(x_{ij}\right) = 1$; then z_r is perfectly achieved,

$= 0$; then z_r is nothing achieved,

If $0 < \mu_{z_r}\left(x_{ij}\right) < 1$; then z_r is partially achieved,

Step-6: Let $\lambda_r = \dfrac{U_r - z_r}{U_r - L_r}$, $r = 1, 2, 3, ..., k$.

Using max-min / min-max operator, we have

$$\max\left[\min\left(\lambda_1, \lambda_2, \lambda_3, ..., \lambda_k\right)\right],$$

then we have, max : λ

$$\lambda_1 \geq \lambda$$
$$\lambda_2 \geq \lambda$$
$$\lambda_3 \geq \lambda$$
$$\cdots\cdots\cdots$$
$$\cdots\cdots\cdots$$
$$\lambda_k \geq \lambda$$

where $\lambda = \min\limits_{r}\left\{\mu_{z_r}\left(x_{ij}\right)\right\}$, $i = 1, 2, 3, ..., m \;\&\; j = 1, 2, 3, ..., n$.

Finally we obtained the mathematical model for all cases through fuzzy programming technique as follows :

$$\max \;:\; \lambda \tag{43}$$

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} + \lambda(U_K - L_K) - U_K \leq 0 , \tag{44}$$

$$\sum_{j=1}^{n} x_{ij} - \overline{a_i} + k_{\alpha_i}\sqrt{\operatorname{var}\left(a_i\right)} \leq 0 , \tag{45}$$

$$\sum_{i=1}^{m} x_{ij} - \overline{b_j} - k_{\beta_i}\sqrt{\operatorname{var}\left(b_j\right)} \geq 0 , \tag{46}$$

$$x_{ij} \geq 0 ,\ \forall i = 1, 2, 3, ..., m \;\&\; j = 1, 2, 3, ..., n \tag{47}$$

$$\lambda \geq 0, \tag{48}$$

4. Numerical Example :

A renounce company collect or procure the baby food products from the three production sources and supply to four destination centre in which all availability and demand (parameters) are random in nature and they follow the normal distribution with known means and variances. The decision maker lays emphasis on criteria, such as minimization of transportation cost, transportation time or (delivery time) and loss during transportation through a given route (i, j) where $i = 1,2,3$ & $j = 1,2,3,4$. Here z_1, z_2, z_3 represents the total transportation cost with Rs. one lack per unit, transportation time with month per unit, loss during transportation with by Rs. thousand respectively from each production sources to each destination center along with availability and demand are represented by the matrix in C^1, C^2 and C^3 as mentioned below:

$$C^1 = \begin{bmatrix} 8 & 9 & 7 & 2 \\ 5 & 6 & 4 & 7 \\ 3 & 7 & 7 & 5 \end{bmatrix}, \quad C^2 = \begin{bmatrix} 2 & 9 & 8 & 1 \\ 4 & 3 & 6 & 7 \\ 5 & 2 & 8 & 2 \end{bmatrix} \text{ and } C^3 = \begin{bmatrix} 2 & 4 & 7 & 3 \\ 6 & 4 & 8 & 4 \\ 8 & 2 & 5 & 1 \end{bmatrix}$$

The decision maker is also interested to transport baby food products by tons from the i- source to the j-th destination so as to satisfy all the requirements.

$$\min : z_1 = 8x_{11} + 9x_{12} + 7x_{13} + 2x_{14} + 5x_{21} + 6x_{22} + 4x_{23} + 7x_{24} + 3x_{31} + 7x_{32} + 7x_{33} + 5x_{34}, \tag{49}$$

$$\min : z_2 = 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{21} + 3x_{22} + 6x_{23} + 7x_{24} + 5x_{31} + 2x_{32} + 8x_{33} + 2x_{34}, \tag{50}$$

$$\min : z_3 = 2x_{11} + 4x_{12} + 7x_{13} + 3x_{14} + 6x_{21} + 4x_{22} + 8x_{23} + 4x_{24} + 8x_{31} + 2x_{32} + 5x_{33} + 3x_{34}, \tag{51}$$

subject to

$$\text{Prob}\left[\sum_{j=1}^{4} x_{1j} \le a_1\right] \ge 1 - \alpha_1, \tag{52}$$

$$\text{Prob}\left[\sum_{j=1}^{4} x_{2j} \le a_2\right] \ge 1 - \alpha_2, \tag{53}$$

$$\text{Prob}\left[\sum_{j=1}^{4} x_{3j} \le a_3\right] \ge 1 - \alpha_3, \tag{54}$$

$$\text{Prob}\left[\sum_{i=1}^{3} x_{i1} \ge b_1\right] \ge 1 - \beta_1, \tag{55}$$

$$\text{Prob}\left[\sum_{i=1}^{3} x_{i2} \geq b_2\right] \geq 1 - \beta_2, \tag{56}$$

$$\text{Prob}\left[\sum_{i=1}^{3} x_{i3} \geq b_3\right] \geq 1 - \beta_3, \tag{57}$$

$$\text{Prob}\left[\sum_{i=1}^{3} x_{i4} \geq b_4\right] \geq 1 - \beta_4, \tag{58}$$

$$x_{ij} \geq 0, \forall i = 1,2,3 \; \& \; j = 1,2,3,4. \tag{59}$$

mean of $a_1 = \overline{a_1}$ =13 variance of a_1 = var (a_1)=3 predetermined confidence level, α_1 =0.01

mean of $a_2 = \overline{a_2}$ =15 variance of a_2 = var(a_2) =2 predetermined confidence level, α_2 =0.02

mean of $a_3 = \overline{a_3}$ =19 variance of a_3 = var(a_3)=7 predetermined confidence level, α_3 =0.03

mean of $b_1 = \overline{b_1}$ =7 variance of b_1 = var(b_1) =5 predetermined confidence level, β_1 =0.04

mean of $b_2 = \overline{b_2}$ =5 variance of b_2 = var(b_2)=3 predetermined confidence level, β_2 = 0.05

mean of $b_3 = \overline{b_3}$ =6 variance of b_3 = var(b_3)=2 predetermined confidence level, β_3 = 0.06

mean of $b_4 = \overline{b_4}$ =4 variance of b_4 = var (b_4)=1 predetermined confidence level, β_4 = 0.07

As described in Section 3, we can converted into the deterministic multi-objective unbalanced transportation problem as follows.

$$\min: z_1 = 8x_{11} + 9x_{12} + 7x_{13} + 2x_{14} + 5x_{21} + 6x_{22} + 4x_{23} + 7x_{24} + 3x_{31} + 7x_{32} + 7x_{33} + 5x_{34}, \tag{60}$$

$$\min: z_2 = 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{21} + 3x_{22} + 6x_{23} + 7x_{24} + 5x_{31} + 2x_{32} + 8x_{33} + 2x_{34}, \tag{61}$$

$$\min: z_3 = 2x_{11} + 4x_{12} + 7x_{13} + 3x_{14} + 6x_{21} + 4x_{22} + 8x_{23} + 4x_{24} + 8x_{31} + 2x_{32} + 5x_{33} + 3x_{34}, \tag{62}$$

subject to

$$\sum_{j=1}^{4} x_{1j} \leq 8.84 \;, \tag{63}$$

$$\sum_{j=1}^{4} x_{2j} \le 12.03 \ , \quad (64)$$

$$\sum_{j=1}^{4} x_{3j} \le 15.2 \ , \quad (65)$$

$$\sum_{i=1}^{3} x_{i1} \ge 11.02 \ , \quad (64)$$

$$\sum_{i=1}^{3} x_{i2} \ge 7.94 \ , \quad (66)$$

$$\sum_{i=1}^{3} x_{i3} \ge 8.26 \ , \quad (67)$$

$$\sum_{i=1}^{3} x_{i4} \ge 5.5 \ , \quad (68)$$

$$x_{ij} \ge 0, \forall i = 1,2,3 \,\&\, j = 1,2,3,4. \quad (69)$$

Using LINGO 9.0 software, we have obtained the lower bounds of the above deterministic problem as (L_1, L_2, L_3) = (128.91, 102.84 , 119.94), and for the same problem the upper bounds as, I^- = (U_1, U_2, U_3) = (232.52, 148.86 , 192.56).

Using case 3, we can formulated the following models

$$\max : \lambda \quad (70)$$

subject to

$$z_1 + 103.61\lambda \le 232.52 \ , \quad (71)$$

$$z_2 + 46.02\lambda \le 148.86 \ , \quad (72)$$

$$z_3 + 80.62\lambda \le 192.56 \ , \quad (73)$$

$$\sum_{i=1}^{4} x_{1j} \le 8.84 \ , \quad (74)$$

$$\sum_{i=1}^{4} x_{2j} \le 12.03 \ , \quad (75)$$

$$\sum_{i=1}^{4} x_{3j} \leq 15.2 \quad , \tag{76}$$

$$\sum_{i=1}^{3} x_{j1} \geq 11.02 \ , \tag{78}$$

$$\sum_{i=1}^{3} x_{i2} \geq 7.94 \ , \tag{79}$$

$$\sum_{i=1}^{3} x_{i3} \geq 8.26 \ , \tag{80}$$

$$\sum_{i=1}^{3} x_{i4} \geq 5.5 \ , \tag{81}$$

$$x_{ij} \geq 0, \forall i = 1,2,3 \,\&\, j = 1,2,3,4. \tag{82}$$

$$\lambda \geq 0 \, . \tag{83}$$

The above problem is solved by the LINGO 0.8 package for obtaining the optimal compromise solution of the deterministic problem. We get λ = 0.5923031 and optimal compromise solution as x_{11} = 3.340000, x_{14} = 5.500000, x_{21} = 6.616295, x_{23} = 5.413705, x_{31}= 1.063705, x_{32} = 7.940000, x_{33}=2.846295, and rest all are zero. The optimal value of each objective functions i.e., z_1, z_2, z_3 are 171.1515, 115.0963, 144.8085 respectively. Also we obtained the non dominated solution for each objective functions z_1, z_2, z_3 i.e., (128.91, 197.74, 232.52), (129.81, 102.84, 148.86), (192.56, 144.70, 111.94) respectively.

5. Conclusion:

The main objective of this paper is to present a solution procedure for multi-objective stochastic unbalanced transportation problem with normal random variables. The transportation problem is an efficient tool to cope with many real life problems of practical importance. Multi-objective transportation problems involve the design, modeling, and planning of many complex resource allocation systems, transportation in which demand and supply are random in nature.

After converting probabilistic constraints into an equivalent deterministic constraints using chance constraint programming, the fuzzy programming is applied to obtain a compromise solution from the set of non-dominated solution. In our approach, we have presented two types of probabilistic constraints of practical importance instead of two constraints having one deterministic, another probabilistic. So our technique is highly fruitful in the sense of real life problems of practical importance. A practical numerical example is provided to demonstrate the feasibility of all decision variables of the proposed method.

6. References:

[1] Appa, G.M. (1973) "The Transportation Problem and Its Variants", *Operations Research Quarterly* , Vol. 24, pp. 79-99.

[2] Bit, A.K., Biswal, M.P. and Alam, S.S. (1992) "Fuzzy Programming Approach to Multi-criteria Decision Making Transportation Problem", *Fuzzy Sets and Systems*, Vol. 50, pp. 135-141.

[3] Bit, A.K., Biswal, M.P. and Alam, S.S. (1994) "Fuzzy Programming Approach to Chance Constrained Multi-objective Transportation Problem", *The Journal of Fuzzy Mathematics*, Vol. 2, pp. 117-130.

[4] Brigden, M.E.B. (1974) " A Variant of the Transportation Problem in which the Constraints are of Mixed Type", *Operations Research Quarterly*, Vol. 25, pp. 437-445.

[5] Charnes, A., and Cooper, W.W. (1963) "Deterministic Equivalents for Optimizing the Satisficing Under Chance Constraints", *Operations Research* , Vol. 11, pp. 18-39.

[6] Dantzig, G.B. (1963) *"Linear Programming and Extensions"* Princeton University Press, Princeton, New Jersey.

[7] Diaz, J.A. (1979) "Finding a Complete Description of all Efficient Solutions to a Multi-Objective Transportation Problem" , *Ekonomicky - matematicky, Obzor* , Vol. 15, pp. 62-73.

[8] Goicoechea , A., Hansen , D.R., and Duckstein, L. (1982) *"Multi - objective Decision Analysis with Engineering and Business Application"*, John Wiley

[9] Hitchcock, F.L. (1941) " The Distribution of a Product from Several Sources to

Numerous Localities", *J. Math* , *Phys* , Vol. 20 , pp. 224 -230.

[10] Kambo, N. S. (1984) *Mathematical Programming Techniques*, New Delhi, Affilated East-West Press , India.

[11] Koopmans, T. C. (1949) "Optimum Utilization of the Transportation System", *Econometrica*, Vol. 17, pp. 3-4.

[12] Lee, S. M. and Moore, C .J. (1973) "Optimizing Transportation Problems with Multiple Objectives", *A I I E Transactions* , Vol. 5, pp. 333-338.

[13] Mohon , C. and Nguyen , H.T. (1997) "A Fuzzifying Approach to Stochastic Programming", *Opsearch* , Vol. 34, pp. 73 – 96.

[14] Rao, S.S. (1984) "*Optimization Theory and Applications*", Wiley Eastern Limited, New Delhi, 2nd Edn.

[15] Ringuest, J.L. and Rinks, D.B (1987) "Interactive Solutions for the Linear Multi-objective Transportation Problem", *European Journal of Operational Research*, Vol. 32, pp. 96-106.

[16] Roubens, M. and Teghem, J. Jr. (1991) "Comparison of Methodologies for Fuzzy and Stochastic Multi-objective Programming", *Fuzzy Sets and Systems*, Vol. 42 , pp. 119-132 .

[17] Sakawa , M. (1983) "Fuzzy Sets and Interactive Multi-objective Optimization", *Plenum Press*, New York.

[18] Sahoo, S.N. and Biswal, M.P. (2003) "Some Stochastic Multi-objective Linear Programming Problems", Proceedings of the APORS, pp. 168-180.

[19] Stancu-Minasian , I.M. (1984) "Stochastic Programming with Multiple Objective Functions", *Reidel*, Dordrecht.

[20] Teghem, J. Jr., Dufrane, D., Thauvoye, M. and Kunsch, P. (1986) " Strange an Interactive Method for Multi-objective Linear Programming under Uncertainty", European J. Oper. Res. Vol. 26, pp. 65-82.

Approach to Probabilistic Multi-objective Transportation Problems with Pareto Optimum Solutions", *The Journal of Fuzzy Mathematics*, Vol. 4, pp. 301-314

[21] Zadeh, L. A. (1965) "Fuzzy Sets" , *Information and Control* , Vol. 8, pp. 338-353.

[22] Zimmermann, H.-J. (1978) "Fuzzy Programming and Linear Programming with Several Objective Functions", *Fuzzy Sets and Systems*, Vol. 1, pp. 45-55.