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Solving Airline Crew-Scheduling Problem with imprecise service time using Genetic Algorithm

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Abstract

This paper deals with a special case of the well known airline crew-scheduling problem which has been formulated considering the day-to-day assignment of the technical crew members to their legal round-trip rotations for all the scheduled flights connecting only two cities that will minimize the overall service times (including rest times) of all the crews. In this problem, the service times of crews from their starting city to another city are imprecise in nature. This impreciseness is represented by intervals. For solving this problem, two different methods are proposed: (a) an elitist genetic algorithm (EGA) with interval valued fitness function and (b) EGA approach after converting it into a multi-objective assignment problem with crisp objectives considering the centre and width values of the corresponding intervals. However, for the second method, at first, the multi-objective assignment problem is transformed into a single objective optimization problem with the help of Global Criterion Method (GCM) and then the reformulated problem is solved by EGA. The experimental results of the proposed methods to a realistic airline crew-scheduling problem are compared. Finally, the effect of changes of different genetic parameters on success rate of both the methods, computation times and function evaluations is observed by sensitivity analysis taking one at a time.

Keywords: Crew-scheduling, airline, combinatorial optimization, interval order relation, Global Criterion Method, genetic algorithm

1 Introduction

Airline crew-scheduling problem is a world wide NP-hard combinatorial optimization problem with considerable economic significance. The basic airline crew-scheduling problem concerns the daily assignment of the crew members to round-trips for all the scheduled flights so that the total service time is minimized.

In the past, a variety of approaches using exact methods and efficient heuristics have already been proposed for solving airline crew-scheduling problems. Also there were a large number of contributions on various extensions to the basic problem.

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Marsten and Shepardson (1981), Gershkoff (1989) and Barutt and Hull (1990) solved small size problems using LP relaxation method. Hoffman and Padberg (1993) proposed exact method like branch and cut to solve airline crew-scheduling problems. Levine (1996) developed a heuristic method like hybrid genetic algorithm consisting of a steady-state genetic algorithm and a local search heuristic to solve the same problem. Ozdemir and Mohan (2001) also proposed genetic algorithm for crew scheduling in airlines. A bi-criterion approach for the airline crew rostering problem was proposed by Moudani et al. (2001). In this approach, the solution is associated with acceptable satisfaction levels for the crew staff. Klabjan et al. (2001) developed an airline crew scheduling model that maximizes the repetition or regularity of crew itineraries over a weekly horizon in addition to minimizing cost. Cordeau et al. (2001) proposed simultaneous aircraft routing and crew scheduling based on Benders decomposition method for finding a minimum cost set of aircraft routes as well as crew pairings with some side constraints. Recently, Zeghal and Minoux (2005) studied a new approach to the crew assignment problem in airlines that formulated and solved the two sub-problems, viz. Crew Pairing Problem followed by the Working Schedules Construction Problem. Again Schaefer et al. (2005) developed better approximate solution method for airline crew scheduling under uncertainty due to disruptions where they provided a lower bound on the cost of an optimal crew schedule in operations.

To the best of our knowledge, among all the aforesaid works the coefficients or cost parameters have been specified precisely by fixed (deterministic) real numbers. However, in real-life, there may be many diverse situations arising due to rainy/foggy/cloudy weather, etc. for which the time taken by a flight for a trip from one place to another will not be fixed and so the service times of crews from their starting city to another city will be imprecise in nature. To represent such imprecise numbers, stochastic, fuzzy and fuzzy-stochastic approaches may be used. In stochastic approach, the coefficients/parameters are viewed as random variables with known probability distributions. On the other hand, in fuzzy approach, the parameters, constraints and goals are viewed as fuzzy sets/fuzzy numbers. It is also assumed that their membership functions are known. Again, in fuzzy-stochastic approach, some parameters are viewed as fuzzy sets and others, as random variables. However, it is not always easy for a decision maker to specify the appropriate membership function for fuzzy approach, exact probability distribution of a parameter for stochastic approach and both for fuzzy-stochastic approach. For these reasons, in this approach, impreciseness has been represented by intervals (wherein the actual service times are expected to lie). To solve this type of interval valued crew-scheduling problem, order relations between interval numbers are essential. To the best of our knowledge, very few researchers defined the order relations between interval valued numbers. Among them, one may refer to the works of Moore (1979), Ishibuchi and Tanaka (1990) and Chanas and Kuchta (1996). However, their definitions are not complete. Sengupta and Pal (2000) proposed two different approaches (one is deterministic and another is fuzzy) to compare any two interval valued numbers with respect to the optimistic as well as pessimistic decision makers' point of view. However, in some cases, both of their approaches fail to find out the order relation between two interval valued numbers.

Genetic Algorithm (GA) is a powerful computerized heuristic search and optimization method based on the well-known Darwin's principle of evolution, viz. "Survival of the fittest". Holland (1976) developed the primary concept of GA. After that, a number of researchers have contributed much to the development of GA. At present, there are several text books on GA. Among them, the books of Goldberg (1989), Mitchell (1996), Gen and Cheng (1997) and Michalewicz (1999) are worth mentioning.

In the present paper, an airline crew-scheduling problem with interval valued time parameters has been proposed considering the service time (including rest time) of each crew as interval. Here, the problem (with interval objective) has been formulated as an assignment problem using interval arithmetic and existing recently developed complete definitions due to Mahato and Bhunia (2006) of interval order relations with respect to the pessimistic decision makers' preference. To solve this interval valued crew-scheduling problem, two different methods have been proposed:

(i) an elitist genetic algorithm (EGA) with interval valued fitness function and

(ii) EGA approach after converting it into a multi-objective assignment problem with crisp objectives considering both the centre and width values of the corresponding intervals.

For the method (ii), at first, the multi-objective assignment problem is transformed into a single objective optimization problem with the help of Global Criterion Method (GCM) and then the reformulated problem is solved by EGA. Finally, the results of the proposed methods have been compared with the help of an example and to study the effect of changes of various genetic parameters on the performances of both the methods, sensitivity analyses have been done.

2 Order relations between interval valued numbers

An interval valued number is defined either by its lower and upper limits or by its centre and width as

$$A = [a_L, a_R] = \{x : a_L \le x \le a_R, x \in R\}$$
$$= \langle a_c, a_w \rangle = \{x : a_c - a_w \le x \le a_c + a_w, x \in R\}$$

where a_L and a_R are the lower and upper limits respectively, $a_c = (a_L + a_R)/2$, $a_w = (a_R - a_L)/2$ are the centre and width of A and R, the set of all real numbers.

Next, we shall discuss the order relations for finding the decision maker's preference between interval valued times of minimization problems. We shall restrict only to pessimistic decision making for our crew scheduling problem as this will be very much beneficial for airline company. Let the uncertain times from two alternatives be represented by two closed intervals $A = [a_L, a_R] = \langle a_c, a_w \rangle$ and $B = [b_L, b_R] = \langle b_c, b_w \rangle$ respectively. It is also assumed that the time of each alternative lies in the corresponding interval. These two intervals A and B may be of the following three types:

Type–I: Both the intervals are disjoint.

Type–II: Intervals are partially overlapping.

Type–III: One interval is contained in the other.

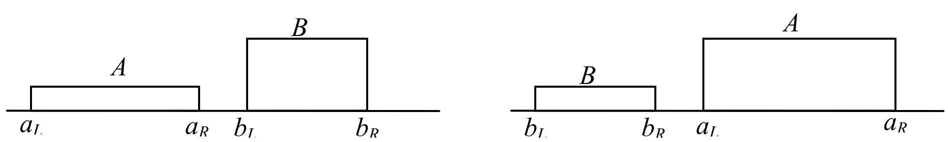

Fig. 1(a) Type I intervals

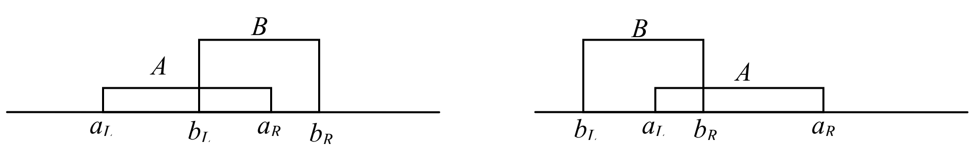

Fig. 1(b) Type II intervals

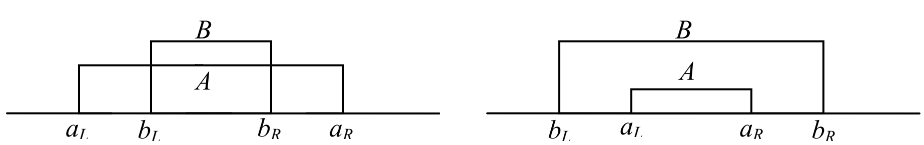

Fig. 1(c) Type III intervals

The three types of intervals are shown in Fig. 1(a), 1(b) and 1(c) for different situations.

For pessimistic decision making, the decision maker expects the minimum cost/time for minimization problems according to the principle "Less uncertainty is better than more uncertainty".

According to Mahato and Bhunia (2006) the order relations of interval numbers for minimization problems in case of pessimistic decision making are as follows:

Definition 1. Let us define the order relation $\leq_{p\min}$ between $A=\left[a_L,a_R\right]=\left\langle a_c,a_w\right\rangle$ and $B=\left[b_L,b_R\right]=\left\langle b_c,b_w\right\rangle$ as

$$A<_{p\min} B \Leftrightarrow a_c<b_c \text{ for Type-I and Type-II intervals}$$

$$A<_{p\min} B \Leftrightarrow \left(a_c \leq b_c\right) \wedge \left(a_w<b_w\right) \text{ for Type-III intervals.}$$

However, for Type–III intervals with $\left(a_c<b_c\right) \wedge \left(a_w>b_w\right)$, the pessimistic decision cannot be taken. Here, the optimistic decision is to be considered.

3 Formulation of the crisp problem

Let us consider the following assignment problem with interval objective:

$$\text{Minimize} \quad F\left(x\right)=\sum_{i=1}^{n}\sum_{j=1}^{n}[a_{L_{ij}},a_{R_{ij}}]x_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{i=1}^{n} x_{ij}=1, \quad j=1,2,\ldots,n \tag{2}$$

$$\text{and} \quad \sum_{j=1}^{n} x_{ij}=1, \quad i=1,2,\ldots,n \tag{3}$$

$$\text{where} \quad x_{ij}\in\{0,1\}, \quad i,j=1,2,\ldots,n \tag{4}$$

and $[a_{L_{ij}},a_{R_{ij}}]$ being an interval representing the uncertain time for the assignment problem.

Now we formulate the interval valued objective function in (1) of the earlier mentioned problem as a crisp multi-objective one using **Definition 1**.

Definition 2. $x^{/} \in S$ is an optimal solution of (1) subject to the constraints (2), (3) and restrictions (4) if and only if there is no other solution $x \in S$ which satisfies $F(x) <_{p\min} F(x^{/})$, S being the set of all feasible solutions of the problem.

As the order relation of two interval valued numbers depends upon both the centre and width values of the corresponding intervals, the optimization of both of them is to be considered here for optimization of the interval objective.

The centre $F_c(x)$ and the width $F_w(x)$ of the interval objective function $F(x)$ in (1) is

$$F_c(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{c_{ij}} x_{ij} \tag{5}$$

$$F_w(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{w_{ij}} x_{ij} \tag{6}$$

where $a_{c_{ij}}$ and $a_{w_{ij}}$ are the centre and width respectively of the interval $[a_{L_{ij}}, a_{R_{ij}}]$.

The solution set of (1) subject to the constraints (2), (3) and restrictions (4) defined by **Definition 2** can be obtained from the following crisp multi-objective problem:

$$\text{Minimize}\{F_c, F_w\} \tag{7}$$

subject to the constraints (2), (3) and restrictions (4).

4 Assumptions and notations

The following assumptions and notations are used in developing the proposed crew-scheduling problem.

Assumptions

(1) The flight service between two cities C_1 and C_2 of an airline company is considered.

(2) Running time of a flight from a city to another city lies in an interval.

(3) Rest time of the crew in a city away from his starting city as well as his service time lie within intervals.

(4) Every crew should be provided with more than t_1 hours of rest before the return trip again and should not rest for more than $t_2\,(> t_1)$ hours for the return trip.

(5) The airline company has residential facilities for the crews at both cities C_1 and C_2.

Notations

$[t_{i_1}^1, t_{i_1}^2]$: time (hours) taken by the i_1-th flight for a trip from city C_1 to another city C_2,

$[t_{i_2}^2, t_{i_2}^1]$: time (hours) taken by the i_2-th flight for return trip via a different route,

$t_{C_1D_{i_1}}$: departure time of i_1-th flight from C_1,

$t_{C_2D_{i_2}}$: departure time of i_2-th flight from C_2,

$t_{C_2A_{i_1}}$: arrival time of i_1-th flight at C_2,

$t_{C_1A_{i_2}}$: arrival time of i_2-th flight at C_1,

i_1, i_2: flight index i.e., 1, 2, ..., n,

n: number of flights/crews,

x_{ij}: decision variables.

The task is to find the optimal assignment schedule of the crews for a single day which minimizes the total service time (including rest time).

5 Formulation of the problem

To formulate the problem, the following two cases may arise:

Case – 1:

If all the crew is asked to reside at city C_1 (so that they start from C_1 and come back to C_1 with minimum rest time at C_2), then the total service time (including the rest time at C_2) for different flights (i.e., i_1 corresponds to i_2) are given by the following $n \times n$ matrix:

$$T_1^S = \left([a_{S_{ij}}^{(1)}, b_{S_{ij}}^{(1)}]\right)$$

where

$[a_{S_{ij}}^{(1)}, b_{S_{ij}}^{(1)}]$ = interval representing total service time (including rest time at C_2) for crew starting from C_1 with transport vehicle i_1 in the up direction and transport vehicle i_2 in the down direction

$= t_{C_1A_{i_2}} - t_{C_1D_{i_1}}$

Case – 2:

If all the crew is asked to reside at city C_2 (so that they start from C_2 and come back to C_2 with minimum rest time at C_1), then the total service time (including the rest time at C_1) for different flights (i.e., i_2 corresponds to i_1) are similarly (as in **Case – 1**) given by the following $n \times n$ matrix:

$$T_2^S = \left([a_{S_{ij}}^{(2)}, b_{S_{ij}}^{(2)}]\right)$$

where

$$[a_{S_{ij}}^{(2)}, b_{S_{ij}}^{(2)}] = t_{C_2 A_{i_1}} - t_{C_2 D_{i_2}}$$

As a crew can reside either at city C_1 or at C_2, the minimum total service times (including rest times) can be obtained for different flights by choosing minimum value out of two interval times from T_1^S and T_2^S, using interval order definition, viz. **Definition 1**.

Thus we get the following $n \times n$ matrix:

$$T^S = \left([t_{L_{ij}}^S, t_{R_{ij}}^S]\right)$$

where

$[t_{L_{ij}}^S, t_{R_{ij}}^S]$= minimum of $[a_{S_{ij}}^{(1)}, b_{S_{ij}}^{(1)}]$ and $[a_{S_{ij}}^{(2)}, b_{S_{ij}}^{(2)}]$, if both the rest times (at C_1 and at C_2) $\in \left[t_1, t_2\right]$

= $[a_{S_{ij}}^{(1)}, b_{S_{ij}}^{(1)}]$, if the rest time (at C_1) $\notin \left[t_1, t_2\right]$

= $[a_{S_{ij}}^{(2)}, b_{S_{ij}}^{(2)}]$, if the rest time (at C_2) $\notin \left[t_1, t_2\right]$

Then the crew-scheduling problem will be as follows:

$$\text{Minimize } Z^S = \sum_{i=1}^{n}\sum_{j=1}^{n} [t_{L_{ij}}^S, t_{R_{ij}}^S] x_{ij} \tag{8}$$

$$\text{subject to } \sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n \tag{9}$$

and $$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1,2,\ldots,n \tag{10}$$

where $x_{ij} \in \{0,1\}, \quad \forall\, i,j=1,2,\ldots,n$ (11)

Our objective is to find the optimal assignment of crews by solving the above minimization problem.

6 Solution Procedure

Now, to solve the above mentioned constrained minimization problem with interval objective we shall develop two different methods **M-1** and **M-2** with the help of Elitist Genetic Algorithm (EGA). These methods are as follows:

M-1: Elitist Genetic Algorithm (EGA) with interval valued fitness function and

M-2: Elitist Genetic Algorithms (EGA) after converting it into the following crisp multi-objective problem using equation (7):

Crisp Problem:

$$\text{Minimize } \{ Z_c^S, Z_w^S \} \tag{12}$$

subject to the constraints (9) and (10) together with the restrictions (11),

where $Z_c^S = \sum_{i=1}^{n}\sum_{j=1}^{n} t_{c_{ij}}^S x_{ij}$ and $Z_w^S = \sum_{i=1}^{n}\sum_{j=1}^{n} t_{w_{ij}}^S x_{ij}$

$t_{c_{ij}}^S$, $t_{w_{ij}}^S$ being centre and width of the interval valued coefficient in (8) respectively.

In the first method **M-1**, we shall solve the problem (8) – (11) using interval valued fitness function and interval order relations with respect to pessimistic decision maker's point of view. On the other hand, for the second method **M-2**, we shall, at first, transform the above multi-objective optimization problem (12) into a single objective optimization problem with the help of Global Criterion Method (GCM) as follows:

Global Criterion Method (GCM)

In this method, the ideal objective vector is used as a reference point. An objective vector minimizing each of the objective functions is called an ideal objective vector. To transform the problem (12) into the single objective optimization problem, the following steps are followed:

Step-1. Solve the problem:

Minimize $Z_c^S = \sum_{i=1}^{n}\sum_{j=1}^{n} t_{c_{ij}}^S \, x_{ij}$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \cdots, n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \cdots, n$$

where $x_{ij} \in \{0,1\}$, $\forall\, i, j = 1, 2, \cdots, n$.

and obtain the optimum value, say, $Z_c^{S'}$. Similarly, minimize the other objective function Z_w^S separately subject to the same constraints and restrictions and obtain the optimum value, say, $Z_w^{S'}$. Thus, the ideal objective vector is $\left(Z_c^{S'}, Z_w^{S'}\right)$.

Step-2. Now, using the above reference point, formulate the normalized distance function Z as

$$Z = \left[\left(\frac{Z_c^S - Z_c^{S'}}{Z_c^{S'}}\right)^p + \left(\frac{Z_w^S - Z_w^{S'}}{Z_w^{S'}}\right)^p\right]^{\frac{1}{p}}$$

Step-3. Thus the problem is to solve the following auxiliary problem:

Minimize Z (of Step-2) (13)

subject to the same constraints and restrictions as in Step-1.

The exponent $\frac{1}{p}$ may be dropped. Problem with or without the exponent $\frac{1}{p}$ are equivalent for $1 \le p < \infty$, since problem in Step-3 is an increasing function of the corresponding problem without the exponent. Generally, p is taken as 2. The solution say, $\left(Z_c^{S^*}, Z_w^{S^*}\right)$ of the problem (13) in Step-3 is Pareto optimal (Miettinen, 1999).

In each of the constrained minimization problems in Step-1 and Step-3, EGA has been developed for n^2 integer variables x_{ij} (whose values are either 0 or 1).

Implementation of Elitist Genetic Algorithm (EGA)

The working steps of the elitist GA (EGA) have been given in Majumdar and Bhunia (2006). We shall now discuss the different processes/operators like initialization of chromosomes, genetic operators and elitism in details.

In our developed GA, a chromosome has been represented by a matrix (of order n) containing n^2 genes x_{ij} (i, j =1,2,…,n) whose values are either 0 or 1 (see Majumdar and Bhunia, 2007). This representation ensures that the constraints (9) and (10) are automatically satisfied.

Next, an initial population of GA consisting of p_{size} (population size) chromosomes has been generated using a random initialization scheme (Majumdar and Bhunia, 2007) where '0's have been set to all the n^2 genes of a chromosome and then for a randomly chosen gene of this chromosome, a '1' has been set in each row and in each column.

As in our case, the fitness value of each chromosome is interval valued, usual ranking selection has been used here following the definition (**Definition 1**) for comparing interval valued numbers from the view point of pessimistic decision maker. The probability of the i-th chromosome being selected in this selection method is defined by

$$\text{P(select the } i\text{-th chromosome)} = p(1-p)^{i-1}$$

where 'p' is the probability of selecting the best chromosome and 'i', the rank of the chromosome.

Here matrix binary crossover (MBX) (Majumdar and Bhunia, 2007) has been used which is an extension of the conventional 2-point crossover on strings that deals with column positions rather than bit positions. In this crossover scheme, two crossover sites are selected and marked from two randomly selected chromosomes of the population and all the entries of the selected chromosomes determined by the crossover sites are exchanged.

Due to the above crossover operation, some infeasible chromosomes (solutions) may generate. To avoid this possibility, a repair procedure (Majumdar and Bhunia, 2007) has been embedded after the crossover operation.

In our GA, inversion mutation (Gen and Cheng, 1997) has been used as in (Majumdar and Bhunia, 2007) in which two positions within a randomly chosen chromosome are selected at random and then the sub-matrix specified by these two positions is inverted.

To maintain monotonic non-degradation of the best solution in subsequent generations as well as to add good quality chromosomes for mating, an *elitist* strategy has been proposed in our GA that preserves the best chromosome of the previous generation. If the best found solution of the current generation is worse than that of the previous generation, the latter one would replace the worst result of the current generation.

7 Experimental Results and discussion

In this section, the computational results of our proposed methods separately on a realistic airline crew-scheduling problem have been presented. The developed algorithm has been coded in C programming and implemented on a Pentium IV 3.0 GHz with 1 GB RAM PC under LINUX environment. For all the experiments, we have performed 50 trials with different sets of random numbers.

To illustrate our proposed methods, the following numerical example has been considered. The arrival and departure times of the problem have not been selected from any case study, but the values considered here are all realistic.

Example: A small airline company, owing six planes operates on all the seven days of a week. Flights between the two cities C_1 and C_2 has the typical time table given in Table 1.

Table 1. **Time table**

$C_1 \rightarrow C_2$			$C_2 \rightarrow C_1$		
Flight No.	**Departure**	**Arrival**	**Flight No.**	**Departure**	**Arrival**
A	06-00	11-45 to 12-00	I	05-30	10-30 to 10-45
B	09-45	16-00 to 16-15	II	09-30	15-00 to 15-15
C	14-00	20-00 to 20-15	III	13-45	18-45 to 19-15
D	19-15	01-15 to 01-30	IV	16-45	22-15 to 22-30
E	22-00	03-30 to 04-00	V	21-15	02-30 to 02-45
F	00-30	06-00 to 06-15	VI	23-45	04-30 to 04-45

The cost of providing this service by the airline company partially depends upon the time spent by the crew (pilots and officers) away from their places in addition to service times. There are six crews. Every crew must have a minimum and maximum rest times of 4 hours and 24 hours respectively before the return trip again. The airline company has residential facilities for the crews at city C_1 as well as at C_2. Find the optimal schedule (pairing of flights and base city) of the crews minimizing the overall service time (including rest time).

For this problem, the best found objective function values for $\left(Z_c^S, Z_w^S\right)$ together with the best found solution Z^S obtained using two different methods **M-1** and **M-2** have been presented in Table-2 and also the best found schedule for the crews has been displayed in Table-3.

Table 2. **Best found objective values**

Method	Z_c^S	Z_w^S	**Best found Objective value** **(** Z^S **)**
M-1	102.5	0.75	[101.75,103.25]
M-2	102.5	0.75	[101.75,103.25]

Table 3. **Best found Schedule**

Crew	**Residence at**	**Service Schedule (Flight No.)**		**Minimum Service time (including rest time)**	**Minimum Total Service time (including rest time)**
		Up	**Down**		
1	C_1	A	IV	[16.25,16.5]	
2	C_1	B	V	[16.75, 17]	
3	C_2	VI	C	[20.25,20.5]	[101.75,103.25]
4	C_1	D	I	[15.25,15.5]	
5	C_1	E	II	[17,17.25]	
6	C_2	III	F	[16.25,16.5]	

The earlier mentioned problem is used to study the effect of the changes of the GA parameters p_{size}, m_{gen} (maximum number of generations), p_c (probability of crossover) and p_m (probability of mutation) on the success rate (**SR**) of the trials, **CPU** times (in seconds) and objective function evaluations per trial (**Fn-Count**). In each case, the results are obtained from 50 trials by changing one parameter at a time and keeping the others as their original values. The results of these analyses have been displayed in Table 4-7.

In Table-4 and Table-5, sensitivity analyses of p_{size} and m_{gen} for **M-1** and **M-2** respectively with respect to **SR**, minimum, maximum and average **CPU** times and **Fn-Count** have been reported (taking p_c=0.8 and p_m=0.1).

Table 4. **Sensitivity analyses of p_{size} and m_{gen} on M-1 (where p_c= 0.8 and p_m= 0.1)**

p_{size}	m_{gen}	SR	CPU Time			Fn-Count		
			Min	Max	Avg.	Min	Max	Avg.
200	200	100	0.01	0.04	0.020	400	1200	688
	300	100	0.02	0.04	0.028	600	1200	936
	400	100	0.01	0.02	0.012	400	400	400
	500	100	0.01	0.02	0.011	400	400	400
300	200	100	0.02	0.07	0.035	600	1500	834
	300	100	0.02	0.03	0.023	600	600	600
	400	100	0.02	0.23	0.054	600	3900	1170
	500	100	0.02	0.25	0.051	600	5700	1140
400	200	100	0.04	0.32	0.075	800	5200	1352
	300	100	0.04	0.22	0.059	800	4400	1160
	400	100	0.04	0.05	0.043	800	800	800
	500	100	0.04	0.21	0.064	800	2800	1064
500	200	100	0.06	0.27	0.119	1000	3000	1600
	300	100	0.06	0.37	0.146	1000	4500	1830
	400	100	0.06	0.13	0.071	1000	1500	1060
	500	100	0.05	0.17	0.088	1000	2000	1260

Table 5. **Sensitivity analyses of p_{size} and m_{gen} on M-2 (where p_c= 0.8 and p_m= 0.1)**

p_{size}	m_{gen}	SR	CPU Time			Fn-Count		
			Min	Max	Avg.	Min	Max	Avg.
200	200	80	0	0.27	0.058	400	40200	8360
	300	90	0	0.39	0.045	400	60200	6380
	400	94	0	0.51	0.039	400	80200	5188
	500	98	0	0.01	0.020	400	100200	2396
300	200	88	0.01	0.47	0.069	600	60300	7764
	300	94	0.01	0.68	0.055	600	90300	5982
	400	96	0.01	0.90	0.052	600	120300	5388
	500	100	0.01	0.02	0.016	600	600	600
400	200	96	0.02	0.69	0.051	800	80400	3984
	300	98	0.02	0.03	0.043	800	120400	3192
	400	98	0.02	1.37	0.105	800	160400	10376
	500	100	0.02	0.03	0.025	800	800	800
500	200	94	0.03	0.96	0.090	1000	100500	6970
	300	100	0.03	0.04	0.033	1000	1000	1000
	400	100	0.03	0.04	0.034	1000	1000	1000
	500	100	0.03	2.34	0.082	1000	250500	5990

From the results of Table-4 and Table-5 it is observed that the best solution is found for all combinations using method **M-1** and generally found using method **M-2** as seen from **SR** columns of the corresponding tables. Also the method **M-1** and **M-2** take a minimum CPU time of less than 0.07 seconds and 0.04 seconds, a maximum CPU time of less than 0.4 seconds and 2.4 seconds and an average of less than 0.15 seconds and 0.2 seconds respectively for all the combinations. The minimum and maximum number of objective function evaluations for **M-1** and **M-2** are reported to be 400 & 400 and 5700 & 250500 respectively. Moreover, the average number of objective function evaluation per trial for **M-1** is generally smaller than those for **M-2** excepting for only four cases.

In the interest of fair comparison of the methods on the basis of p_c and p_m, sensitivity analyses are performed keeping p_{size} and m_{gen} fixed at 200 and 500 respectively. The results are presented in Table-6 and Table-7.

Table 6. **Sensitivity analyses of p_c and p_m on M-1 (where p_{size} = 200 and m_{gen} = 500)**

p_c	p_m	SR	CPU Time			Fn-Count		
			Min	Max	Avg.	Min	Max	Avg.
0.8	0.1	100	0.01	0.02	0.111	400	400	400
	0.15	100	0.02	0.08	0.049	800	2000	1320
	0.2	100	0.03	0.07	0.049	600	1600	1364
0.85	0.1	100	0.01	0.05	0.029	400	1400	848
	0.15	100	0.01	0.04	0.017	400	1200	576
	0.2	100	0.01	0.02	0.011	400	400	400
0.9	0.1	100	0.02	0.05	0.029	800	1400	896
	0.15	100	0.02	0.06	0.038	1000	1400	1152
	0.2	100	0.01	0.03	0.015	400	600	468
0.95	0.1	100	0.01	0.04	0.017	400	800	520
	0.15	100	0.01	0.07	0.032	400	1600	952
	0.2	100	0.01	0.02	0.011	400	400	400

It is seen that the **SR** values of **M-1** are 100 while those of **M-2** are 90 or more for all the combinations. Again, **M-1** and **M-2** take an average CPU time of less than 0.05 seconds and 0.8 seconds respectively. Furthermore, the average objective function evaluation for **M-2** is reported to be higher than those for **M-1** excepting for a single case when p_c=0.95 and p_m=0.15.

Further, it is observed from our computational tests that both the methods **M-1** and **M-2** generally found best solution within the first 10 iterations (generations).

Overall, the method **M-1** is proved comparatively better than the method **M-2** in all respects.

Table 7. **Sensitivity analyses of p_c and p_m on M-2 (where p_{size} = 200 and m_{gen} = 500)**

p_c	p_m	SR	CPU Time			Fn-Count		
			Min	Max	Avg.	Min	Max	Avg.
0.8	0.1	98	0	0.01	0.020	400	100200	2396
	0.15	98	0	0.67	0.021	400	100200	2396
	0.2	98	0	0.69	0.022	400	100200	2396
0.85	0.1	98	0	0.64	0.021	400	100200	2396
	0.15	92	0	0.68	0.062	400	100200	8384
	0.2	96	0	0.69	0.035	400	100200	4392
0.9	0.1	96	0	0.64	0.034	400	100200	4392
	0.15	90	0	0.68	0.075	400	100200	10380
	0.2	94	0	0.70	0.049	400	100200	6388
0.95	0.1	96	0	0.65	0.034	400	100200	4392
	0.15	100	0	0.01	0.008	400	400	400
	0.2	96	0	0.71	0.036	400	100200	4392

8 Conclusions

In this paper, for the first time a special realistic day-to-day airline crew-scheduling problem has been formulated assuming imprecise total service time (including rest time) of each crew. This impreciseness has been represented by interval valued numbers that is more general than other representations, like stochastic, fuzzy and fuzzy-stochastic. Here, the problem has been solved by our proposed two different methods based on elitist genetic algorithm (EGA). In these methods, interval ranking for pessimistic decision makers' preference has been considered in order to avoid more uncertainty that will be much more beneficial for the airlines authority. Due to the heuristic nature of the proposed two methods (**M-1** and **M-2**) the performances of those methods have been investigated with the help of sensitivity analyses. These analyses show that our proposed methods perform well and in fact, the performance of the first method **M-1** are better compared with the second method **M-2** in the context of success rate, CPU time and function evaluation.

For future research, one may consider the case where the total service time including rest time is more than 24 hours. In that case, the scheduling can be done for a period (like week/fortnight/month) considering weekly day-off(s) and other extendable benefits for the crews. Further study can be to solve the same problem where there is a constraint like limitation of the residential facilities for the crews at any one city under consideration. Another extension of this work can be to develop a multi-objective crew-scheduling problem minimizing the total rest time and total service time (including rest time) of crews separately.

References

[1] Barutt J., Hull T. Airline crew scheduling: supercomputers and algorithms (1990) 23(6) SIAM News.

[2] Chanas S., Kuchta D. Multiobjective programming in optimization of interval objective functions–A generalized approach. European Journal of Operational Research (1996) 94(3): 594 − 598.

[3] Cordeau J.F., Stojković G., Soumis F., Desrosiers J. Benders Decomposition for Simultaneous Aircraft Routing and Crew Scheduling. Transportation Science (2001) 35(4): 375 – 388.

[4] Gen M., Cheng R. Genetic Algorithms and Engineering Design, New York: Wiley.

[5] Gershkoff, I. (1989), 'Optimizing flight crew schedules', *INTERFACES* 19, 29 – 43.

[6] Goldberg, D.E. (1989), *Genetic Algorithms in Search, Optimization and Machine Learning*, New York: Addison-Wesley.

[7] Hoffman, K., and Padberg, M. (1993), 'Solving airline crew-scheduling problems by branch-and-cut', *Management Science,* 39, 657 – 682.

[8] Holland, J.H. (1976), *Adaptation of Natural and Artificial systems*, Ann Arbor, MI, University of Michigan Press.

[9] Ishibuchi, H., and Tanaka, H. (1990), 'Multiobjective programming in optimization of the interval objective function', *European Journal of Operational Research*, 48, 219 − 225.

[10] Klabjan, D., Johnson, E.L., Nemhauser, G.L., Gelman, E., and Ramaswamy, S. (2001), 'Airline Crew Scheduling with Regularity', *Transportation Science*, 35(4), 359 – 374.

[11] Levine, D. (1996), 'Application of a hybrid genetic algorithm to airline crew-scheduling', *Computers and Operations Research*, 23, 547 − 558.

[12] Majumdar, J. and Bhunia, A.K. (2006), 'Elitist genetic algorithm approach for Assignment Problem', *AMO- Advanced Modeling and Optimization*, 8(2), 135 – 149.

[13] Majumdar, J. and Bhunia, A.K. (2007), 'Elitist genetic algorithm for Assignment Problem with imprecise goal', *European Journal of Operational Research*, 177, 684 – 692.

[14] Marsten, R. and Shepardson, F. (1981), 'Exact solution of crew scheduling problems using the set partitioning model: recent successful applications', *Networks*, 11, 165 – 177.

[15] Michalewicz, Z. (1999), *Genetic Algorithms + Data Structure = Evolution Programs*, Berlin: Springer-Verlag.

[16] Miettinen, K.M. (1999), *Non-linear Multi-objective Optimization*, Kluwer's International series.

[17] Mitchell, M. (1996), *An introduction to genetic algorithms*, Cambridge: MIT Press.

[18] Moore, R.E. (1979), *Method and Applications of Interval Analysis*, Philadelphia: SIAM.

[19] Moudani, W.E., Cosenza, C.A.N., Collgny, M. de and Mora-Camino, F. (2001), 'A Bi-Criterion Approach for Airline Crew Rostering Problem', in *Lecture Notes in Computer Science*, 486.

[20] Ozdemir, H. T. and Mohan, C.K. (2001), 'Flight graph genetic algorithm for crew scheduling in airlines', *Information Sciences*, 133, 165 – 173.

[21] Schaefer, A.J., Johnson, E.L., Kleywegt, A.J., and Nemhauser, G.L. (2005), 'Airline Crew Scheduling Under Uncertainty', *Transportation Science*, 39(3), 340 – 348.

[22] Sengupta, A. and Pal, T.K. (2000), 'Theory and methodology on comparing interval numbers', *European Journal of Operational Research*, 127, 28 − 43.

[23] Zeghal, F.M. and Minoux, M. (2006), 'Modeling and solving a Crew Assignment Problem in air transportation', *European Journal of Operational Research*, 175(1), 187 – 209.