

A Generalized Approach to Comparing the Systems of Different Order

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Abstract

Comparing systems of different order is a very important problem in reliability engineering. From a number of systems of same or different order choosing the best one is of great concern to the reliability practitioners. Here we propose a criterion based on stochastic ordering using which we can select the system whose chance of being best is the most. In our study we take recourse to the concept of minimal cut sets of the systems under comparison. In this paper we consider both situations-the component lives to be i.i.d. and to be independent but not identical (i.n.i.d.). Various component life distributions are considered and numerical examples are given to illustrate the method.

Keywords and phrases: minimal cut set, reliability, stochastic ordering, system life.

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1 Introduction

Comparison of similar systems via system life for determining the best system is an important issue in reliability engineering. Various methods and criteria for the said purpose are available in the literature. Mention may be made of the work of Barlow and Proschan(1981), Kochar et al.(1999). All these works have considered the comparison between two similar systems of same order and consisting of i.i.d. life distribution of components. Comparison becomes difficult for complex systems with a large number of components. Analytical difficulties in computing the values of a particular measure of system performance (for example, the systems' expected lifetimes) make the comparison almost impossible. In a recent paper of Roychowdhury and Bhattacharya (2009) the performance of systems consisting of the components with independently and identically distributed lives and also with independently but not identically distributed lives have been compared using a direct method of comparison of system lives and an indirect method of comparison via signatures of the systems. It has been observed that the comparison among systems of different orders is possible by direct method only when the systems are simple and do not involve too many components. But it is almost impossible to use the direct method for comparing two complex systems of same or different orders. In that situation the indirect method of comparison via signatures of the systems looks promising. But there also the way of comparing the two systems of same order via signature vectors fails if the tail probability vector of one system does not have a clear dominance over the tail probability vector of the other. Moreover if the systems are of different order, the associated signature vectors belong to the different vector spaces, and hence they are not directly comparable. In this paper an attempt has been made to resolve this long-standing issue of comparing the systems of same or different orders using the minimal cut set (MCS) representation of a system. The paper introduces the definition of the most likely minimal cut set (MLMCS) of a system. We illustrate our idea with some examples. The novelty of this paper is that we have used a very general comparison criterion which is independent of the number of components of the systems whether or not they are identically distributed and has potential applications in practice.

2 Identification of Most Likely Minimal Cut Set of a System

For determining the life of a system based on the lives of the individual components that the system is comprised of, the system can be represented as a combination of a number of sets of components, called minimal cut sets. A cut set of a system is defined as a set of components in the system whose failure will cause the system to fail.

A cut set is said to be minimal if the set cannot be reduced without losing its status as a cut set. A minimal cut set (MCS) of a system is a minimal set of components whose failing causes the system to fail (Barlow and Proschan, 1981). The number of different basic components in an MCS is called the order of the MCS.

Let X_1, X_2, \dots, X_N be the independently and identically distributed random lives of the components of an N -component system which has r minimal cut sets, K_1, K_2, \dots, K_r , of orders N_1, N_2, \dots, N_r , respectively. $\sum_{i=1}^r N_i \geq N$, which means, different MCS can share same components. The life of a system is, then, given by

$$T = \min_{i=1(1)r} \max_{j \in K_i} X_j = \min_{i=1(1)r} Y_i, \quad (1)$$

where $Y_i = \max_{j \in K_i} X_j$, the maximum of the lives of the components of the i^{th} MCS. Let us call it maximum component life of the i^{th} MCS, or *life of the i^{th} MCS*, since the component having maximum life among the lives of all components in the i^{th} MCS, by failing, will cause the system to fail. We can think of reliability block diagram (RBD) representation of a system producing full MCS representation for the system, in which all minimal cut sets can be considered to be connected in a series configuration, and hence failure of any one MCS will cause the system to fail. Thus the system life will be the minimum of the lives of all minimal cut sets, i.e., minimum of Y_1, Y_2, \dots, Y_r . If Y_i is the minimum of Y_1, Y_2, \dots, Y_r , then $T = Y_i$.

The maximum component life, Y_i , of the i^{th} MCS will be the most likely value of the system life if its chance of being minimum among all such Y -values is maximum, i.e., if

$$P(Y_i \leq a) \geq P(Y_h \leq a), \quad \forall a \in R \text{ and for all } h \neq i, \quad i, h = 1(1)r, \quad (2)$$

which indicates that Y_i is stochastically smaller than any other Y_h ,

$$\text{i.e., } Y_i \preceq^{st} Y_h, \text{ for all } h \neq i, \quad i, h = 1(1)r,$$

where the symbol ‘ \preceq^{st} ’ stands for ‘stochastically smaller’.

Let us now define the *most likely MCS (MLMCS)* of a system. The i^{th} MCS, K_i , will be called the *most likely MCS* of the system if its life is same as the system life.

If X_1, X_2, \dots, X_N are i.i.d. as $F(\cdot)$, then

$$P(Y_i \leq a) = P(\max_{j \in K_i} X_j \leq a) = \{F(a)\}^{N_i}, \quad (3)$$

since i^{th} MCS, K_i , has N_i components.

Similarly,

$$P(Y_h \leq a) = P(\max_{j \in K_h} X_j \leq a) = \{F(a)\}^{N_h}, \quad (4)$$

since h^{th} MCS, K_h , has N_h components, $h \neq i$, $i, h = 1(1)r$.

Hence, using (3) and (4), from (2) we can say that the i^{th} MCS will be the *MLMCS* if

$$\{F(a)\}^{N_i} \geq \{F(a)\}^{N_h},$$

$\forall a \in R$ and for all $h \neq i$, $i, h = 1(1)r$,

or

$$N_i \leq N_h, \quad \text{for all } h \neq i, \quad i, h = 1(1)r,$$

since $0 \leq F(a) \leq 1$ always.

Hence we have the following lemma:

Lemma 1. Let X_1, X_2, \dots, X_N be the independently and identically distributed random lives of a N -component system having r minimal cut sets, K_1, K_2, \dots, K_r , of orders N_1, N_2, \dots, N_r , respectively ($\sum_{i=1}^r N_i \geq N$). Then the i^{th} minimal cut set, K_i , will be the most likely minimal cut set (*MLMCS*), if its order is minimum of the orders of all minimal cut sets of the system, i.e.,

$$N_i \leq N_h, \quad \text{for all } h \neq i, \quad i, h = 1(1)r.$$

In case of equality of N_i and N_h , i^{th} and h^{th} , both minimal cut sets will be treated as the most likely MCS.

3 Main Theorem

Suppose there are m systems (of same or different order) comprising of i.i.d. component lives. We want to choose the best system, or arrange the systems in order from best to worst. Let T_l be the life of the l^{th} system, which is comprising of N_l components, $l = 1(1)m$. The l^{th} system has r_l minimal cut sets, viz., $K_{l1}, K_{l2}, \dots, K_{lr_l}$, having orders $N_{l1}, N_{l2}, \dots, N_{lr_l}$, respectively, $\sum_{i=1}^{r_l} N_{li} \geq N_l$. The life of the l^{th} system is given by

$$T_l = \min_{i=1(1)r_l} \max_{j \in K_{li}} X_{lj} = \min_{i=1(1)r_l} Y_{li},$$

where $X_{l1}, X_{l2}, \dots, X_{lN_l}$ are the independently distributed random lives of the N_l components of the l^{th} system, and Y_{li} is the life of i^{th} MCS (K_{li}), of l^{th} system.

For comparing the m systems and choosing the best one, we compare their chances of being best, and for this their chances of meeting some reliability target are compared. The p^{th} system is the best system if its chance of meeting any preset reliability target is more than all other systems, or in other words, its life is stochastically larger than the life of any other system. If K_{pt} , t^{th} MCS of p^{th} system, is the *MLMCS* of p^{th} system, and K_{qs} , the s^{th} MCS of q^{th} system, is the *MLMCS* of q^{th} system, then p^{th} system will be the best system if

$$Y_{pt} \succ^{st} Y_{qs}, \text{ for all } q \neq p, \quad p, q = 1(1)m, \quad (5)$$

where ' \succ^{st} ' means 'stochastically larger'. Here (5) indicates that Y_{pt} is stochastically larger than Y_{qs} .

Let the lives of the components of all m systems be i.i.d. as $F(\cdot)$. Then (5) can be rewritten as

$$P(Y_{pt} \leq c) \leq P(Y_{qs} \leq c), \quad \forall c \in R \text{ and for all } q \neq p, \quad p, q = 1(1)m, \quad (6)$$

which means that the probability of failing to meet any reliability target c is less for the p^{th} system than any other system, or, in other words, the probability of meeting any reliability target c is more for the p^{th} system than any other system.

Here (6) is equivalent to

$$P(\max_{j \in K_{pt}} X_{pj} \leq c) \leq P(\max_{j \in K_{qs}} X_{qj} \leq c), \quad (7)$$

$\forall c \in R$ and for all $q \neq p$, $p, q = 1(1)m$,
 where the order of MCS K_{pt} is N_{pt} and the order of the MCS K_{qs} is N_{qs} .

Then (7) reduces to

$$\{F(c)\}^{N_{pt}} \leq \{F(c)\}^{N_{qs}},$$

$\forall c \in R$ and for all $q \neq p$, $p, q = 1(1)m$,
 or

$$N_{pt} \geq N_{qs},$$

for all $q \neq p$, $p, q = 1(1)m$.

Hence for comparing m systems having i.i.d. component lives and choosing the best one from them we have the following theorem:

Theorem 1. The p^{th} system is the best system among m systems having i.i.d. component lives if

$$N_{pt} \geq N_{qs}, \text{ for all } q \neq p, \quad p, q = 1(1)m,$$

where N_{pt} is the order of the t^{th} minimal cut set of p^{th} system, which is the most likely minimal cut set of the p^{th} system, and N_{qs} is the order of the s^{th} minimal cut set of q^{th} system, which is the most likely minimal cut set of the q^{th} system.

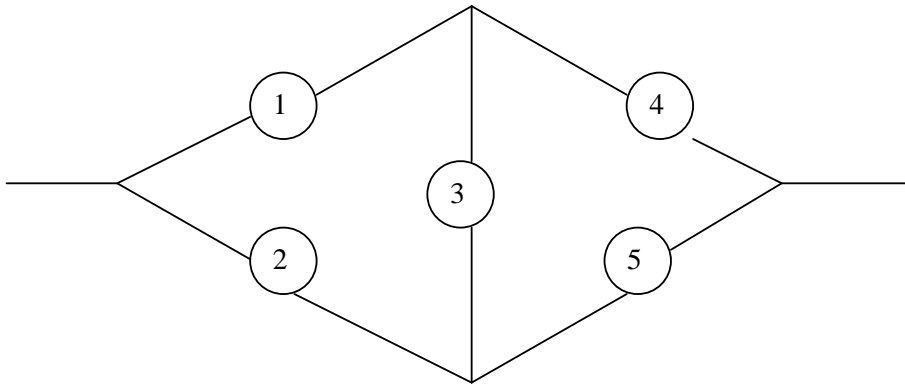
3.1 An Example

Let us consider the following systems, as figured.

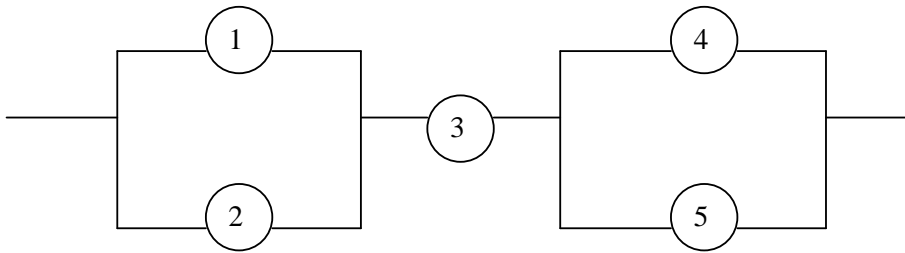
Suppose we want to compare the bridge system and the hi-fi system. The minimal cut sets of bridge system are: $K_{11} = \{1,2\}$, $K_{12} = \{4,5\}$, $K_{13} = \{1,3,5\}$, $K_{14} = \{2,3,4\}$, and the minimal cut sets of hi-fi system are: $K_{21} = \{1,2\}$, $K_{22} = \{3\}$, $K_{23} = \{4,5\}$. The order of the *MLMCS* of the bridge system is 2, whereas it is 1 for hi-fi system. Thus, by the above theorem, the bridge system is better in this case.

Now let us consider the component lives of both the systems to be i.i.d. as $\exp(\lambda = 0.5)$. We draw a random sample of size 5 from an $\exp(\lambda = 0.5)$ distribution for the lives of the components of a bridge system, and another random sample of size 5 for the lives of the components of a hi-fi system, from an $\exp(\lambda = 0.5)$ distribution.

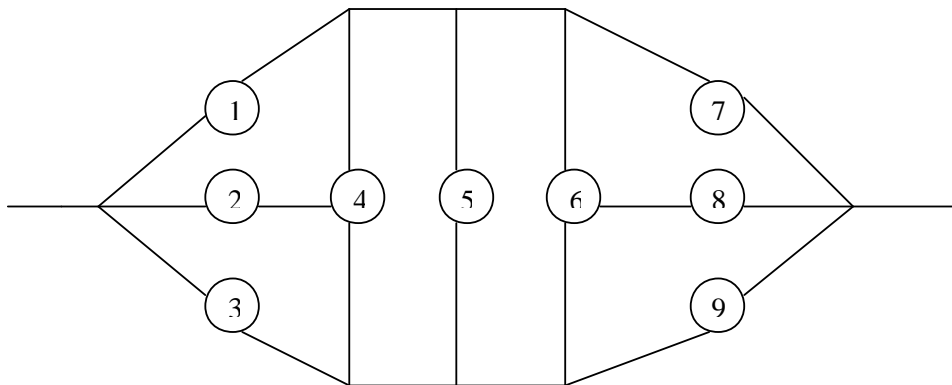
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System 1: Bridge system



System 2: Hi-fi system



System 3

Let the component lives of a bridge system be (0.83490839, 1.66356002, 0.05264756, 6.40622553, 0.79965116) yielding the system life as 0.83490839 (in appropriate unit), by (1), and the component lives of a hi-fi system be (0.386725, 0.1166548, 0.1266139, 4.4228664, 2.007046) yielding the system life to be 0.1266139 which shows that the bridge system is better here.

If we considered the first set of random component lives to be the lives of the hi-fi system and the second set of random lives to be the lives of the bridge system, then also we would get the system life as 0.386725 for the bridge system and 0.05264756 for the hi-fi system, which indicates that the bridge system is still better.

Let us consider the component lives to be lognormally distributed with a mean of 2 and a standard deviation of 1. Let two random samples of size 5 be drawn from this population for the component lives of a bridge system and a hi-fi system. The component lives for the bridge system are (5.224905, 9.508514, 33.776723, 10.348908, 9.983604) with a system life 9.508514, and the component lives for the hi-fi system are (2.756115, 1.834089, 5.762983, 23.840486, 3.840642) which gives the system life as 2.756115. In this case also the bridge system comes out to be better.

The minimal cut sets of System 3 are $K_{31} = \{1,2,3\}$, $K_{32} = \{1,3,4\}$, $K_{33} = \{6,7,9\}$, $K_{34} = \{7,8,9\}$, $K_{35} = \{1,4,5,6,9\}$, $K_{36} = \{3,4,5,6,7\}$. If we compare System 1 (a five-component system), System 2 (a five-component system) and System 3 (a nine-component system), by Theorem 1, System 3 comes out to be the best, since the orders of their most likely minimal cut sets are, respectively, 2, 1 and 3.

4 Comparison of Systems having Independent but not Identical Component Lives

In this paper we developed a way to determine the best system from a number of systems with i.i.d. component lives. If the systems under comparison have the components with independent lives, but not i.i.d., then also we can proceed in the similar manner to get the best solution, though the result will not be as precise as the i.i.d. case, as it will no longer be a distribution-independent result. In our study we consider two situations- the component lives to be i.i.d. within the systems and to be independent but not identical within the systems.

4.1 Independent and Identical Component Lives within Systems

If the component lives of the l^{th} system are i.i.d. among themselves as $F_l(\cdot)$, $l = 1(1)m$, and the lives of the components of different systems are independent (not i.i.d.), then (7) reduces to

$$\{F_p(c)\}^{N_{pt}} \leq \{F_q(c)\}^{N_{qs}}, \quad (8)$$

$\forall c \in R$ (c being the preset reliability target) and for all $q \neq p$, $p, q = 1(1)m$, and the p^{th} system will be the best system.

Arranging similar quantities in order of magnitude we can order the systems according to the order of preference (from best to worst). Note that the relation is distribution-specific. In particular, if the component lives of l^{th} system are i.i.d. as exponential with parameter λ_l , $l = 1(1)m$, and the lives of different systems are independent, then (8) will reduce to

$$(1 - e^{-\lambda_p c})^{N_{pt}} \leq (1 - e^{-\lambda_q c})^{N_{qs}}, \quad (9)$$

$\forall c \in R$ and for all $q \neq p$, $p, q = 1(1)m$, and the p^{th} system will be the best system then. Comparing the similar quantities we can decide on the order of preference of different systems, as before.

For the complex systems with a large number of components in the minimal cut sets, (8) reduces to

$$e^{-N_{pt}\overline{F}_p(c)} \leq e^{-N_{qs}\overline{F}_q(c)}, \quad (10)$$

since $F(c)^n \rightarrow e^{-n\overline{F}(c)}$, as $n \rightarrow \infty$ (van der Vaart, 1998).

Here (10) reduces to

$$e^{-N_{pt}e^{-\lambda_p c}} \leq e^{-N_{qs}e^{-\lambda_q c}},$$

or

$$N_{pt}e^{-\lambda_p c} \geq N_{qs}e^{-\lambda_q c},$$

or

$$\frac{N_{pt}}{N_{qs}} \geq e^{-(\lambda_q - \lambda_p)c},$$

$\forall c \in R$ and for all $q \neq p$, $p, q = 1(1)m$.

This way we can compare different systems and find the best one, which has the maximum chance of meeting any preset reliability target.

4.1.1 An Example

Suppose we want to compare the bridge system and the hi-fi system. The order of the *MLMCS* of the bridge system is 2, whereas it is 1 for hi-fi system. Let the component lives of the bridge system be i.i.d. as $\exp(\lambda_1 = 0.5)$ and the component lives of the hi-fi system be i.i.d. as $\exp(\lambda_2 = 0.16)$. Then considering $c = 1$, we have $(1 - e^{-\lambda_1 c})^{N_1} = 0.154818$ and $(1 - e^{-\lambda_2 c})^{N_2} = 0.147856211$, where N_1 and N_2 are the orders of the *MLMCS* of bridge system and hi-fi system, respectively. By (9), hi-fi system is better here.

Now let us draw a random sample of size 5 (lives of the components of a bridge system) from an $\exp(\lambda_1 = 0.5)$ distribution, another random sample of size 5 (lives of the components of a hi-fi system) from an $\exp(\lambda_2 = 0.16)$ distribution. The random component lives of a bridge system are (3.8263308, 2.3208434, 0.4905767, 2.8401476, 0.3338295), yielding the system life as 2.8401476, by (1). The random component lives of a hi-fi system are (1.128532, 9.458008, 7.025433, 3.113249, 8.428159), yielding the system life as 7.025433 (in appropriate unit), which shows that the hi-fi system is better here.

Here we note that the hi-fi system will be better than the bridge system according to our criterion if (9) holds, or equivalently, if $\lambda_2 < -\log_e(1 - (1 - e^{-\lambda_1})^2)$, when $c = 1$.

4.2 Independent, but not Identical, Component Lives within Systems

Now consider the case where the component lives for each system are independent, but not identically distributed (*i.n.i.d.*). For this situation also we can apply the same logic to reach a meaningful decision as to which system to choose among m systems as the best one and how to order them from best to worst. Here also we cannot get a precise expression, but we can propose a rule that can give us the best solution.

Suppose that $X_{l1}, X_{l2}, \dots, X_{lN_l}$, the component lives of an N_l -component system, are independently distributed as $F_{l1}(\cdot), F_{l2}(\cdot), \dots, F_{lN_l}(\cdot)$, respectively, $l = 1(1)m$. Let the system have r_l minimal cut sets, $K_{l1}, K_{l2}, \dots, K_{lr_l}$. The life of the system is, then, given by

$$T_l = \min_{i=1(1)r_l} \max_{j \in K_{li}} X_{lj} = \min_{i=1(1)r_l} Y_{li},$$

where $Y_{li} = \max_{j \in K_{li}} X_{lj}$.

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The i^{th} MCS, K_{li} , is the most likely MCS of the l^{th} system if

$$Y_{li} \preceq^{st} Y_{lh}, \text{ for all } h \neq i, \quad i, h = 1(1)r_l,$$

i.e.,

$$P(Y_{li} \leq a) \geq P(Y_{lh} \leq a),$$

$\forall a \in R$ and for all $h \neq i, \quad i, h = 1(1)r_l$.

Here

$$P(Y_{li} \leq a) = P(\max_{j \in K_{li}} X_{lj} \leq a) = \prod_{j \in K_{li}} \{F_{lj}(a)\},$$

and

$$P(Y_{lh} \leq a) = P(\max_{j \in K_{lh}} X_{lj} \leq a) = \prod_{j \in K_{lh}} \{F_{lj}(a)\},$$

Hence the i^{th} MCS, K_{li} , is the *MLMCS* of the l^{th} system if

$$\prod_{j \in K_{li}} \{F_{lj}(a)\} \geq \prod_{j \in K_{lh}} \{F_{lj}(a)\}, \quad (11)$$

$\forall a \in R$ and for all $h \neq i, \quad i, h = 1(1)r_l$.

Now for comparing the m systems and choosing the best one, we compare their chances of meeting any preset reliability target. The p^{th} system is the best system if its chance of meeting the preset reliability target is more than all other systems, or in other words, if K_{pt} , the t^{th} MCS of p^{th} system, is the *MLMCS* of p^{th} system, and K_{qs} , the s^{th} MCS of q^{th} system, is the *MLMCS* of q^{th} system, then p^{th} system will be the best system if

$$Y_{pt} \succ^{st} Y_{qs}, \text{ for all } q \neq p, \quad p, q = 1(1)m,$$

or

$$P(Y_{pt} \leq c) \leq P(Y_{qs} \leq c), \quad \forall c \in R \text{ and for all } q \neq p, \quad p, q = 1(1)m,$$

which reduces to

$$\prod_{j \in K_{pt}} \{F_{pj}(c)\} \leq \prod_{j \in K_{qs}} \{F_{qj}(c)\}, \quad (12)$$

$\forall c \in R$ and for all $q \neq p, \quad p, q = 1(1)m$.

Using (11) and (12) we can compare m systems.

In particular if the component lives, $X_{l1}, X_{l2}, \dots, X_{lN_l}$, follow exponential distribution with parameters $\lambda_{l1}, \lambda_{l2}, \dots, \lambda_{lN_l}$, respectively, independently of each other, then by (11), the i^{th} MCS, K_{li} , will be the *MLMCS* of the l^{th} system ($l = 1(1)m$) if

$$\prod_{j \in K_{li}} (1 - e^{-\lambda_{lj}a}) \geq \prod_{j \in K_{lh}} (1 - e^{-\lambda_{lj}a}),$$

$\forall a \in R$ and for all $h \neq i$, $i, h = 1(1)r_l$.

And, by (12), the p^{th} system will be the best system if

$$\prod_{j \in K_{pt}} (1 - e^{-\lambda_{pj}c}) \leq \prod_{j \in K_{qs}} (1 - e^{-\lambda_{qj}c}),$$

$\forall c \in R$ and for all $q \neq p$, $p, q = 1(1)m$.

4.2.1 An Example

Suppose we want to compare the bridge system and the hi-fi system. Let the component lives of the bridge system be independently distributed exponential with parameters $\lambda_{11} = 0.5, \lambda_{12} = 0.3, \lambda_{13} = 0.4, \lambda_{14} = 0.1, \lambda_{15} = 0.3$, and the component lives of the hi-fi system be independent as exponential with parameters $\lambda_{21} = 0.2, \lambda_{22} = 0.4, \lambda_{23} = 0.5, \lambda_{24} = 0.2, \lambda_{25} = 0.2$. For bridge system the minimal cut sets are $K_{11} = \{1, 2\}, K_{12} = \{4, 5\}, K_{13} = \{1, 3, 5\}, K_{14} = \{2, 3, 4\}$, and for hi-fi system, $K_{21} = \{1, 2\}, K_{22} = \{3\}, K_{23} = \{4, 5\}$. Considering $a = 1$, for bridge system,

$$\prod_{j \in K_{11}} (1 - e^{-\lambda_{1j}a}) = (1 - e^{-0.5})(1 - e^{-0.3}) = 0.10198,$$

$$\prod_{j \in K_{12}} (1 - e^{-\lambda_{1j}a}) = (1 - e^{-0.1})(1 - e^{-0.3}) = 0.024664,$$

$$\prod_{j \in K_{13}} (1 - e^{-\lambda_{1j}a}) = (1 - e^{-0.5})(1 - e^{-0.4})(1 - e^{-0.3}) = 0.033621,$$

$$\prod_{j \in K_{14}} (1 - e^{-\lambda_{1j}a}) = (1 - e^{-0.3})(1 - e^{-0.4})(1 - e^{-0.1}) = 0.008131.$$

Hence the MCS K_{11} is the *MLMCS* of bridge system here.

Similarly, for hi-fi system,

$$\prod_{j \in K_{21}} (1 - e^{-\lambda_{2j}a}) = (1 - e^{-0.2})(1 - e^{-0.4}) = 0.059761,$$

$$\prod_{j \in K_{22}} (1 - e^{-\lambda_{2j}a}) = (1 - e^{-0.5}) = 0.393469,$$

$$\prod_{j \in K_{23}} (1 - e^{-\lambda_{2j}a}) = (1 - e^{-0.2})^2 = 0.032859.$$

Hence the *MLMCS* for hi-fi system is K_{22} .

For $c = 1$, $\prod_{j \in K_{11}} (1 - e^{-\lambda_{1j}c}) = (1 - e^{-0.5})(1 - e^{-0.3}) = 0.10198$, $\prod_{j \in K_{22}} (1 - e^{-\lambda_{2j}c}) = (1 - e^{-0.5}) = 0.393469$. Thus, according to our rule, the bridge system is better here.

Now for each component life of the bridge system, draw a random sample observation from each of the exponential distributions having parameters $\lambda_{11} = 0.5, \lambda_{12} = 0.3, \lambda_{13} = 0.4, \lambda_{14} = 0.1, \lambda_{15} = 0.3$. Let the sample lives be 0.2228708, 0.3790935, 0.8697001, 2.161026, 0.4887504. Then the life of the bridge system will be 0.3790935. Next, for the hi-fi system, draw a random sample observation from each of the exponential distributions having parameters $\lambda_{21} = 0.2, \lambda_{22} = 0.4, \lambda_{23} = 0.5, \lambda_{24} = 0.2, \lambda_{25} = 0.2$. Let the sample lives be 1.496667, 0.006879814, 0.0873931, 0.1214046, 0.9813434. The system life of this hi-fi system will then be 0.0873931. Hence the bridge system comes out to be better here.

5 Conclusion and Discussion

In this paper we developed a generalized methodology to determine the best system which has the maximum chance of meeting all preset reliability targets, from a number of systems of same or different order with i.i.d. component lives. Even if the systems under comparison have i.n.i.d. component lives, then also we can proceed in the similar manner to get the best solution. The results have been derived under a general framework enabling us to compare the systems and find the best one, under a less restrictive environment.

References

- [1] Barlow, R.E. and Proschan, F.(1981), *Statistical Theory of Reliability and Life Testing: Probability Models*, Silver Spring, MD.
- [2] Kochar,S., Mukherjee, H. and Samaniego, F.J.(1999), The Signature of a Coherent System and its Application to Comparison among Systems, *Naval Research Logistics*, 46, 507-523.
- [3] Roychowdhury, S. and Bhattacharya, D. (2009), A Note on Comparison of Performances of Systems of Different Order, *Journal of Applied Statistical Science*,16(3), 293-301.
- [4] van der Vaart, A.W.(1998), *Asymptotic Statistics*, Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge Univ. Press.