IMPROVED ROUTH-PADÉ APPROXIMANTS USING VECTOR EVALUATED GENETIC ALGORITHM TO CONTROLLER DESIGN

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Abstract: This note describes a novel approach to Routh-Padé approximation problem relating to the construction of a stable reduced-order approximants for continuous-time Controller. In this method, stability and the first *r* time-moments/Markov-parameters are preserved as well as the errors between a set of subsequent time-moments/Markov-parameters of the system and those of the model are minimized. For the solution of this problem a method using the concept of Pareto-optimality is proposed. Pareto-optimal curve is the solution of Multi-objective Optimization problem. Evolutionary Algorithm such as real parameter Genetic Algorithm is used to get Pareto-optimal curve. The search area for GA is very wide and it usually converges to a point near global optima.

Index Terms: Model reduction, Padé approximation, Routh criterion, Pareto-optimal solutions, Controller design, VEGA.

1 Introduction

Controllers of low complexity are often desirable in practice. Unfortunately, modern controller design techniques frequently lead to high complexity controllers, so there is a real need for reliable model reduction methods which allow a low-order controller to be extracted from high-order controller without incurring too much error. Simple linear controllers are normally preferred over complex linear controllers for linear time-in-variant plants. Such methods can be classified in to two classes: *direct*, in which the parameter defining a low order controller are computed by some optimization or other procedure and indirect, in which a high-order controller is first found, and a procedure used to simplify it. One well established way of obtaining a low order plant is the so called balanced truncation method reported by Moore B.C. [29]. The method has some appealing properties: it generally leads to a stable reduced order model and an error bound exists in terms of the truncated Hankel singular values. As suggested by Anderson and Liu [2], that approximations early in the design process may lead to the undesirable propagation of errors as the design progresses. LQG and H_{∞} controllers are of high order and are impractical to implement. Open-loop balanced truncation suffers from these criticisms and if the plant happens to be unstable, compensation may be needed any way. One closed loop model reduction procedure was introduced to obtain reduced-order model by Jonkheere and Silverman [7]. The open loop system (which may be unstable) is first compensated with standard linear quadratic Gaussian controller (the so called normalized LQG controller). Mustafa and Glover [11] reported that H_{∞} balanced truncation may be used to obtain reduced order plants or controllers. The plant (possibly unstable) is compensated using a particular robustly stabilizing controller. Computationally the H_{∞} balanced truncation method is very simple, since H_{∞} characteristic values are easily calculated from the solutions to the design Ricatti equations in advance of doing any model reduction. The strategy to determine a set of algebraic equations which constitute necessary (but not sufficient) condition on the controller parameter to achieve a minimum value for the performance index was assessed by Bernstein and Hyland [3]. Model reduction by balanced realization proposed by Moore B.C. [29] eliminates a small part of the system and requires transformation of the system in to a special form. Yousuff and Skelton [26] reported methods based on q-variance equivalent

realization. In usual form, these realizations replace on stable high-order model by a second order stable low order model that usually is not an optimal. Reduced-order controllers have been developed using optimal projection in the literature by Bernstein and Hyland [3] and co-prime factorization by Anderson and Liu [2]. A new conceptual approach to controller

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reduction is proposed, that is based upon closed loop considerations by Villemange and Skelton [22]. This method assesses the interactions between the plant and high order controller as described by canonical correlation coefficients. Kreisselmeier and Mevenkamp [8] proposed reduced-order controller for single-input single-output (SISO) plants by first calculating a controller for some, not necessary accurate, reduced order-model of the plant, and then defining the controller so as to account for the full plant.

In the following, the indirect strategy is used to design a low order controller. The plant is first approximated by low order model using the concept of Pareto-optimality. Pareto-optimal curve is the solution of Multi-objective Optimization problem. Evolutionary Algorithm such as Vector Evaluated Genetic Algorithm [5] is used to get Pareto-optimal curve and a controller is designed for this low-order plant. The controller is then attached to the original plant. The applicability of proposed method is shown by means of numerical example. The search area for GA is very wide and it usually converges to a point near global optima [5]. Though Pareto-optimality, which is a key step in the present technique, is well known to the best of author's knowledge, this is the first instance of explicitly showing its usefulness for obtaining reduced-order models.

This paper is organized as follows. In Sec. 2 we briefly review the results of [26,27,28]. The improvement is presented in Sec. 3 and numerical example is given in Sec. 4. Finally paper is concluded in Sec. 5.

2 Brief Review of Existing Results

Consider a single-input-single-output system described by the transfer function:

$$G_n(s) = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}{s^n + b_1 s^{n-1} + \dots + b_n}$$
(1)

$$= t_1 + t_2 s + \dots + t_n s^{n-1} + \dots$$
(2)
(expansion around $s = 0$)

$$= M_{1}s^{-1} + M_{2}s^{-2} + \dots + M_{n}s^{-n} + \dots$$
(3)
(expansion around $s = \infty$)

The problem is to determine its stable reduced-order (rth-order) approximant

$$G_{r}(s) = \frac{\hat{a}_{1}s^{r-1} + \hat{a}_{2}s^{r-2} + \dots + \hat{a}_{r}}{s^{r} + \hat{b}_{1}s^{r-1} + \dots + \hat{b}_{r}}$$
(4)

$$= \hat{t}_1 + \hat{t}_2 s + \dots + \hat{t}_r s^{r-1} + \dots$$
(5)

$$= \hat{M}_1 s^{-1} + \hat{M}_2 s^{-2} + \dots + \hat{M}_r s^{-r} + \dots .$$
(6)

A. Formulation of the objective function

The formulation of the multiobjective optimization problem will be explained for r being even. Formulation for r being odd can be done in a similar way. It is easy to verify that for r even, the following equations hold true:

$$\hat{a}_{r+1-i} = \sum_{j=1}^{i} \hat{t}_{j} \hat{b}_{r-i+j}$$

$$\hat{a}_{i} = \sum_{j=1}^{i} \hat{M}_{j} \hat{b}_{i-j}$$

$$i = 1, ..., \frac{r}{2}$$

$$(7)$$

$$\hat{t}_{i} = \left(-\sum_{j=1}^{i-1} \hat{t}_{j} \hat{b}_{r-i+j} + \sum_{j=1}^{r+1-i} \hat{M}_{j} \hat{b}_{r+1-i-j}\right) \hat{b}_{r}^{-1} \\
\hat{M}_{i} = \sum_{j=1}^{r+1-i} \hat{t}_{j} \hat{b}_{i-1+j} - \sum_{j=1}^{i-1} \hat{M}_{j} \hat{b}_{i-j}$$
(8)

 $(\hat{b}_0 = 1; \hat{b}_i = 0 \text{ for } i \notin \{0, ..., r\}; \hat{t}_i, \hat{M}_i = 0 \text{ for } i < 1)$. We seek a stable model for which *r* equations given b

$$\hat{t}_i - t_i = 0
\hat{M}_i - M_i = 0$$
 $i = 1, ..., \frac{r}{2}$
(9)

are satisfied ,which implies, from (7),

$$\hat{a}_{r+1-i} = \sum_{j=1}^{i} t_j \hat{b}_{r-i+j}$$

$$\hat{a}_i = \sum_{j=1}^{i} M_j \hat{b}_{i-j}$$

$$i = 1, \dots, \frac{r}{2}.$$

$$(10)$$

There exist an infinite number of stable models for which (10) is satisfied [18]. This arbitrariness in stability preservation is exploited in [33] by minimizing the sum of the weighted squares of errors.

To find the improved model, VEGA [16] is used to generate Pareto-optimal solutions by minimizing objective functions Z_n^t , Z_n^M given by

Using (8) subject to (9), (11) can be expressed as

$$z_{\frac{r}{2}+i}^{t} = (1 - \frac{\hat{t}_{\frac{r}{2}+i}}{t_{\frac{r}{2}+i}})^{2} = f(\hat{b}_{1}, \hat{b}_{2}, ..., \hat{b}_{r})$$

$$i = 1, ..., \frac{r}{2}$$

$$(12)$$

$$z_{\frac{r}{2}+i}^{M} = (1 - \frac{\hat{M}_{\frac{r}{2}+i}}{M_{\frac{r}{2}+i}})^{2} = f(\hat{b}_{1}, \hat{b}_{2}, ..., \hat{b}_{r})$$

B. Formulation of the stability constraints

Now following [29], the denominator polynomial of (4) can be expressed as

$$s^{r} + \dot{d}_{1}s^{r-1} + (\dot{d}_{2} + \dot{d}_{3} + \dots + \dot{d}_{r})s^{r-2} + \dot{d}_{1}(\dot{d}_{3} + \dot{d}_{4} + \dots + \dot{d}_{r})s^{r-3} + [\dot{d}_{2}(\dot{d}_{4} + \dot{d}_{5} + \dots + \dot{d}_{r}) + \dot{d}_{3}(\dot{d}_{5} + \dot{d}_{6} + \dots + \dot{d}_{r}) + \dot{d}_{4}(\dot{d}_{6} + \dot{d}_{7} + \dots + \dot{d}_{r}) + \dots + \dot{d}_{r-2}\dot{d}_{r}]s^{r-4} + \dots + \dot{d}_{1+q}\dot{d}_{3+q}\dots\dot{d}_{r-2}\dot{d}_{r}$$
(13)

which is constructed by taking the coefficients of the first two rows of the Routh array with the elements of its first column given by

$$1, \hat{d}_1, \hat{d}_2, \hat{d}_1 \hat{d}_3, \hat{d}_2 \hat{d}_4, \hat{d}_1 \hat{d}_3 \hat{d}_5, \dots, \hat{d}_{1+q} \hat{d}_{3+q} \dots \hat{d}_{r-2} \hat{d}_r$$
(14)

where
$$q = 1$$
 for r even and $q = 0$ for r odd. By setting
 $\hat{d}_1 \hat{b}_1^{-1} = 1, (\hat{d}_2 + \hat{d}_3 + ... + \hat{d}_r) \hat{b}_2^{-1} = 1,$
 $\hat{d}_1 (\hat{d}_3 + \hat{d}_4 + ... + \hat{d}_r) \hat{b}_3^{-1} = 1, ..., (\hat{d}_{1+q} \hat{d}_{3+q} ... \hat{d}_{r-2} \hat{d}_r) \hat{b}_r^{-1} = 1$

(13) is matched with the denominator polynomial of the model in (4), namely, with

$$\hat{b}_r + \hat{b}_{r-1}s + \dots + \hat{b}_1s^{r-1} + s^r$$
(16)

(15)

and the necessary and the sufficient condition that all the roots of (14) be strictly in the left half plane is:

$$d_1 > 0, d_2 > 0, \dots, d_r > 0 \tag{17}$$

which, of course, implies

$$\hat{b}_1 > 0, \hat{b}_2 > 0, ..., \hat{b}_r > 0$$
 (18)

Note that, for a given r, \hat{b}_i , i = 1,...,r, can easily be expressed in terms of \hat{d}_i , i = 1,...,r, by constructing an inverse Routh array (i.e., with the element of its first column given by (14)). Thus, pertaining to r = 4, (15) becomes

$$\hat{b}_1 = \hat{d}_1, \ \hat{b}_2 = \hat{d}_3 + \hat{d}_3 + \hat{d}_4, \ \hat{b}_3 = \hat{d}_1(\hat{d}_3 + \hat{d}_4), \ \hat{b}_4 = \hat{d}_2\hat{d}_4.$$
 (19)

C. Background

The problem is to derive the transfer function of the controller C(s) for control system proposed by Aguirre [1], which yields the desired response of the closed loop system. A classical approach to the design of the controller C(s) is to specify the desired (also called reference) closed loop transfer function T(s), equate it to the closed loop transfer function, equate to the closed loop transfer function and solve for the controller proposed by Aguirre [1]. Thus,



Fig. 1 : Control Configuration

T(z)

$$T(s) = \frac{G(s)C(s)}{1 + G(s)C(s)H(s)}$$
(20)

On simplification for controller, results:

$$C(s) = \frac{T(s)}{G(s)[1 - T(s)H(s)]} = \frac{TN(s)GD(s)HD(s)}{GN(s)[TD(s)HD(s) - TN(s)HN(s)]}$$
(21)

 $XN(\cdot)$ and $XD(\cdot)$ indicate numerator and denominator respectively.

Consider the closed loop system shown in Fig. 1. By approximating $G_1(s)$ by a reduced-order transfer function R(s) Fig. 2 is obtained. In other words, the system of Fig. 2 is approximated by that of Fig. 3, where H(s) is assumed to be same in both these figures. Pertaining to these two systems, the following result was previously arrived at the results reported by Shamash [30].



Fig.2 : A Closed-loop System



Fig. 3: A reduced-order approximant of the system

D. Theorem

Shamash [30] addressed that the first α of the Markov-parameters and the first β time-moments are identical for $G_1(s)$ and R(s) Then, under any stable dynamic feedback law initial Markov-parameters and initial time-moments of the corresponding closed loop transfer functions in Fig. 2 and 3 will be identical.

An illustration of the above theorem, consider the following high order transfer function reported by Shamash [30]:

$$G(s) = \frac{s^3 + 12s^2 + 54s + 72}{s^4 + 18s^3 + 97s^2 + 180s + 100}$$
(22)

The reduced-order model R(s) which fits the first two time-moments and the first two Markov-parameters of (20) is given by:

$$R(s) = \frac{s + 10.060606060}{s^2 + 16.0606066s + 13.973064}$$
(23)

Let
$$(24)$$

$$H(s) = \frac{s+10}{s+1}$$

Thus, the overall closed-loop transfer function in Fig. 2 is

$$\frac{s^4 + 13s^3 + 66s^2 + 126s + 72}{s^5 + 20s^4 + 137s^3 + 451s^2 + 892s + 820}$$
(25)

The time-moments and the Markov-parameters of (24) are given by

Overall closed-loop transfer function in Fig 3 is:

$$\frac{s^2 + 11.06060606s + 10.06060606}{s^3 + 18.0606066s^2 + 50.09427612s + 114.5791246}$$
(26)

The time-moments and the Markov-parameters of (25) are given by

$$t_1 = 0.087804878$$
 $t_2 = -58143962$ $M_1 = 1$ $M_2 = -7$

This therefore, illustrates the above theorem.

The procedure to obtain a reduced order controller C(s), for the system shown in Fig. 1 is explained in the following example. The results of Theorem which relate to matching of the initial time-moments and the Markov-parameters of the corresponding closed-loop transfer function in Fig. 2 and 3, is exploited in this procedure.

3. Application of VEGA

Now, the problem is to minimize (12), satisfying (17a). The vector evaluated genetic algorithm (VEGA) [5] is proposed herein for solving the above stated problem. VEGA is the simplest possible multi-objective GA [5] and is straightforward extension of a single-objective extension of multi-objective optimization. Since a number of objectives (say Q) have to be handled, GA population is divided at every generation into Q equal subpopulations randomly. Each subpopulation is assigned a fitness value based on different objective function.

After each solution is assigned a fitness value, the selection operator restricted among solutions of each subpopulation, is applied until the complete subpopulation is filled [5]. The following VEGA procedure is used [5].

Step 1: Set, for population size N, an objective function counter i = 1 and define x = N/Q

Step 2: For all solution, j = 1 + (i - 1) * x to j = i * x, assign fitness as: $Z(\hat{\mathbf{b}}^{(j)}) = z_i(\hat{\mathbf{b}}^{(j)})$.

Step 3: Perform proportionate selection on all x solutions to create a mating pool P_i .

Step 4: If i = Q, go to Step 5. Otherwise, increment *i* by one and go to Step 2.

Step 5: Combine all mating pools together: $P = \bigcup_{i=1}^{Q} P_i$. Perform crossover and mutation on P to create a new population [5].

In this VEGA, linear crossover operator is used. It creates three solutions, $0.5(\hat{b}_i^{(1,t)} + \hat{b}_i^{(2,t)})$, $(1.5\hat{b}_i^{(1,t)} - 0.5\hat{b}_i^{(2,t)})$, $(-0.5\hat{b}_i^{(1,t)} + 1.5\hat{b}_i^{(2,t)})$ from two parent solutions $\hat{b}_i^{(1,t)}$ and $\hat{b}_i^{(2,t)}$ at generation *t*, with the best two solutions being chosen as offspring. For performing mutation, random mutation is used. Instead of creating a solution from the entire search space, a

solution in the vicinity of parent solution with a uniform probability distribution is chosen: $y_i^{(1,t+1)} = \hat{b}_i^{(1,t)} + (r_i - 0.5)\Delta_i$ where r_i is a random number in [0,1].

4 Reduction Procedure Example

Suppose G(s) and H(s) considered by Aguirre L.A. [1]:

$$G(s) = \frac{18.439s^3 + 14.446s^2 + 11.454s + 13.765}{35.83s^4 + 40.388s^3 + 32.541s^2 + 13.94s + 1}$$
(27)

$$= \frac{0.5164s^3 + 0.4018s^2 + 0.31967s + 0.03842}{s^4 + 1.12722s^3 + 0.9082s^2 + 0.389s + 0.02791}$$
and
$$H(s) = 1$$
(28)

and

The problem is to find a reduced-order controller

System function G(s) = 1 has following Time-moments and Markov-parameters:

Time-moments:

$$t_1 = 1.376567$$
, $t_2 = -7.7324$, $t_3 = 77.3739$ $M_2 = -7$
Markov-parameters:
 $M_1 = 0.5146$, $M_2 = -0.17826$ $M_3 = 0.053248$

Step 1:

Select a reference model which satisfies the control specifications. In this example, a standard second order transfer function is selected with damping ratio:

 $\xi = 0.7$ and natural frequency $\omega_n = 1.5$ rad/s. Thus,

$$T(s) = \frac{2.25}{s^2 + 2.1s + 2.25}$$
(29)

Step 2:

Derive a reduced-order model using Pareto-Optimality and V.E.G.A.

Equations of Time-moments and Markov-parameters of the model are the following:

$$\hat{t}_{2} = (1.376567\hat{b}_{1} + 0.5146)\hat{b}_{2}^{-1}, \quad \hat{t}_{3} = -(1.376567 + \hat{t}_{2}\hat{b}_{1})\hat{b}_{2}^{-1}$$

$$\hat{M}_{2} = 1.376567\hat{b}_{2} - 0.5146\hat{b}_{1}, \qquad \hat{M}_{3} = -(\hat{b}_{2} + \hat{M}_{2}\hat{b}_{1})$$
(30)

Applying Pareto-Optimality and V.E.G.A., algorithm converges to the following optimal solution (numerator and denominator polynomial coefficients):

$$\hat{a}_1 = M_1 = 0.5146, \qquad \hat{a}_2 = t_1 \cdot \hat{b}_2 = 0.629862,$$
(31)

$$\hat{b}_1 = 0.628256, \qquad \hat{b}_2 = 0.045756$$
(32)

For the following population of initial conditions, the population after crossover and mutation operators are shown in following table:

Sl. No.	Initial Population		Population after Selection Operator		Population after Crossover & Mutation Operator		Assigned Fitness Value
	b 1	b ₂	b ₁	b ₂	b 1	b ₂	
1.	0.6815	0.0999	0.6815	0.0999	0.642556	0.050556	0.002885
2.	0.6915	0.0995	0.6915	0.0995	0.642556	0.050556	0.002885
3.	0.6885	0.0998	0.6885	0.0998	0.628056	0.045756	0.000118
4.	0.7115	0.1100	0.7115	0.1100	0.662556	0.061556	0.194634
5.	0.7250	0.1110	0.7250	0.1110	0.690306	0.068556	0.238191
6.	0.7300	0.1115	0.7300	0.1115	0.681056	0.066056	0.194634

Table 1

Therefore, model takes the form after applying Pareto-optimality and VEGA:

$$G_{2}(s) = \frac{0.5146s + 0.629852}{s^{2} + 0.628056s + 0.045756}$$
(33)
Step 3:
Derive the reduced-order controller. From (20):

$$C_{2}(s) = \frac{C_{2}N(s)}{C_{2}D(s)} = \frac{TN(s)G_{2}D(s)HD(s)}{G_{2}N(s)[TD(s)HD(s) - TN(s)HN(s)]}$$
Here:

$$TN(s) = 2.25$$

$$TD(s) = s^{2} + 2.1s + 2.25$$

$$G_{2}N(s) = 0.5146s + 0.629852$$

$$G_{2}D(s) = s^{2} + 0.628256s + 0.045756$$
and $HN(s) = HD(s) = 1$ Therefore, $C_{2}(s)$ turns out to be;

$$C_{2}(s) = \frac{2.25s^{2} + 1.413126s + 0.102951}{0.5146s^{2} + 0.120247}$$
(34)
(35)
(36)

$$0.5146s^3 + 1.14273s^2 + 0.130347s$$

Note by contrast, the controller transfer function without reducing G(s) turns out to be:

$$C(s) = \frac{T(s)}{G(s)[1 - T(s)H(s)]}$$
(37)

$$=\frac{806175s^{4} + 90873s^{3} + 73.2172s^{2} + 31.365s + 2.25}{18.439s^{5} + 53.1679s^{4} + 41.7906s^{3} + 25.4294s^{2} + 2.8896s}$$
(38)
which is of fifth-order.
The transfer function $O(s)$ of the system with reduced order controller $O(s)$ takes the form:

The transfer function, O(s) of the system with reduced-order controller $C_2(s)$ takes the form:

And if the original plant G(s) along with full order controller C(s) then overall closed loop transfer function takes the form

$$F(s) = \frac{+492.5416s^{2} + 68.9105s + 3.097125}{660.669s^{9} + 2649.7201s^{8} + 5731.229s^{7}}$$
(41)

$$F(s) = \frac{+492.5416s^{2} + 68.9105s + 3.097125}{660.669s^{9} + 2649.7201s^{8} + 5731.229s^{7}}$$

The step responses of (27), (32) and (39), (40) are shown in Fig. 4 and Fig. 5. It can be observed that the response of the system with reduced-order controller is satisfactory.



Fig. 4 Step responses of original system (27) and its reduced order model



Fig. 5 Step responses of system O(s) and F(s) reduced order models

5 Conclusions

The emphasis of this note has been to present, in the context of reduced-order control, the idea of controller design as a model approximation problem in the feedback loop. It is clear that proposed reduced order modelling technique is more sophisticated, optimization based, computer-aided controller design. In this note, the problem of finding Routh-Padé approximants has been viewed as a multi-objective optimization problem. It is shown that, using Pareto-Optimality and V.E.G.A., the denominator of the model can be chosen so as to minimize errors between the (r+1)th and the subsequent time-moments and Markov-parameters of the model and the corresponding time-moments and Markov-parameters of the system while preserving stability. Having obtained the denominator in this manner, the numerator parameters can be determined in the usual manner, namely, by fully retaining the first r time-moments/Markov-parameters of the system. The present approach, therefore, leads to an improved approximant for continuous-time systems controller design and result is then successfully applied to discrete-time systems, 2-D systems and many electrical systems. The effectiveness and superiority of proposed method has been illustrated with the help of a numerical example. The method guarantees that a stable system is reduced to stable model only.

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