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Genetic Algorithm Approach to Entropy Matrix Game via Fuzzy Programming

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Abstract

In this paper the entropy function of the matrix game has been considered as an objective function to the matrix game and formulate a new game model namely Entropy Matrix Game Model. For each player we have generated multi-objective non-linear programming models. To determine the solutions of the formulated models, we applied genetic algorithm and fuzzy programming. Numerical examples are presented to illustrate the methodology.

Keywords: Matrix Game, Entropy, Multi-objective Decision Making, Genetic Algorithm, Fuzzy Programming.

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1 Introduction

Game Theory has a remarkable importance in both Operations Research and Systems Engineering due to its great applicability. Many real conflict problems can be modeled as games. However, the encountered conflict problems in economical, military and political fields become more and more complex and uncertain due to the existence of diversified factors. This situation will bring some difficulties in application of classical game theory. To remove this difficulties, we have employed the entropy on two-person zero-sum game. Most of the real conflict problem are not enough to express in linear model of classical game theory.

Every probability distribution has some "uncertainty" associated with it. The concept of "entropy" is introduced to provide a quantitative measure of uncertainty. According to the maximum-entropy principle, given some partial information about a random variate, scalar or vector, we should choose that probability distribution for it, which is consistent with given information, but has otherwise maximum uncertainty associated with it.

In this paper, few references are presented including their work. [Fernandez, Puerto and Monroy (1998A)] considered to solve the two-person multicriteria zero-sum games. As they have considered a multicriteria game, the solution concept is based on Pareto optimality and finally they obtained the Pareto efficient solution for their proposed games. [Fernandez and Puerto (1996)], developed a methodology to get the whole set of Pareto-optimal security strategies based on solving a multiple criteria linear program. This approach shows the parallelism between these strategies in multicriteria games and minimax strategies in scalar zero-sum matrix games. This notion of security is based on expected payoffs. For this reason, only when the game is played many times, these strategies can provide us with a real sense of security. On the contrary, if the game is played only once; as in one shot game, a better analysis should consider not only the payoffs but also the probability to get them. [Roy, Biswal and Tiwari (2000)], presented a new solution procedure to solve fuzzy matrix game and the elements of the pay-off matrix are trapezoidal fuzzy number. And then the linear programming models using the pay-off matrix by introducing the imprecise tolerances for the soft inequalities have been formulated. [Ghose and Prasad (1989)] have proposed a solution concept based on Pareto-optimal security strategies for these games. They also introduced the concept based on the similarity with security levels determined by the saddle points in scalar matrix games. This concept is independent of the notion of equilibrium so that the opponent is only taken into account to establish the security levels for one's own payoffs. When it is used to select strategies, the concept of security levels has important property that the payoff obtained by these strategies cannot be diminished by the opponent's deviation in strategy. [Das and Roy (2008A)] have proposed a new solution concept by considering the entropy function to the objectives of the players. This model is known as entropy optimization model on two-person zero-sum game. Solution concept is based on the Kuhn-Tucker conditions, Maximum Entropy Principle, and Minimum Cross-Entropy Principle. Without considering the pay-offs of the players, we have shown that the optimal strategy and the value of the game for each player are equivalent to the results of classical game model. [Das and Roy] have proposed a new solution concept by considering the entropy function as an objectives of the players to the multicriteria game and formulated some models, Known as multicriteria entropy game model. Solutions are obtained by determining Pareto-optimal Security Strategies(POSS) and it is shown that said models may have risk factor in payoffs for each players with their measure of uncertainties in strategies. [Das and Roy] have proposed a game model by considering entropy functions into the objectives of the players to the multicriteria goal game and named as multicriteria entropy goal game model. Solutions are obtained by determining G-goal security strategies(GGSS) which includes as a part of solution with the probabilities of obtaining presanctified values of the pay-off functions when the players are wanted to maximize the information about their strategies. Several methodologies have been proposed to solve two-person matrix (zero-sum) game. Most of these methods are based on concept of Pareto-optimal security strategies and equilibrium solution. Here, we mainly concentrated on matrix game under entropy environment.

2 Mathematical Model

A payoff matrix of the player I and II are defined as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \tag{1}$$

Let the mixed strategies for player I and II are

$$y = \{y_1, y_2, \dots y_m\}^T \tag{2}$$

and

$$z = \{z_1, z_2, \dots z_n\}^T \tag{3}$$

Then from our classical game theory, we can determine an optimal strategy y of player I which is the solution of the following linear programming model.

Model 1

$$\max : \quad v$$

subject to

$$\sum_{i=1}^{m} a_{ij} y_i \geq v, \quad j = 1, 2, \dots, n \tag{4}$$

$$\sum_{i=1}^{m} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, \dots, m \tag{5}$$

Similarly, an optimal strategy z for the player II is the solution of the following programming model.

Model 2

$$\min : \quad w$$

subject to

$$\sum_{j=1}^{n} a_{ij} z_j \leq w, \quad i = 1, 2, \ldots, m \tag{6}$$

$$\sum_{j=1}^{n} z_j = 1; \quad z_j \geq 0, \quad j = 1, 2, \ldots, n \tag{7}$$

By the duality theorem of the linear programming, the maximum value of v will be equal to the minimum value of w. This value represents the value of the matrix game.
Again [15] each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the two players are involved in maximizing their entropies. The mathematical form of entropies are as follows:

$$H_1 = -\sum_{i=1}^{m} y_i \ ln(y_i) \tag{8}$$

$$H_2 = -\sum_{j=1}^{n} z_j \ ln(z_j) \tag{9}$$

i.e. they are interested in making their strategies as spread out as possible. However they are primarily interested in maximizing their expected payoffs.

2.1 Entropy Matrix Game Models

Every probability distribution has some "uncertainty" associated with it. The concept of "entropy" is introduced to provide a quantitative measure of uncertainty. According to the maximum-entropy principle, given some partial information about a random variate, scalar or vector, we should choose that probability distribution for it, which is consistent with given information, but has maximum uncertainty associated with it.
We first established the entropy optimization model for maximization type by considering following principle.
" Out of all possible distributions that are consistent with moment constraint, choose one that has the maximum entropy". This principle was proposed by [Janes(1957)] and has been known as Maximum Entropy principle or Janes' Maximum Entropy principle. From

this point of view, we formulated a new mathematical model namely Entropy Optimization Model on matrix game in which the entropy function of the matrix game has been considered to an objective function. Therefore we have defined the entropy optimization model for player I named as **Model 3**, as follows.

Model 3

$$\max: \quad H_1$$

subject to

$$\sum_{i=1}^{m} a_{ij} y_i \geq v, \quad j = 1, 2, \ldots, n \tag{10}$$

$$H_1 = -\sum_{i=1}^{m} y_i \; ln(y_i) \tag{11}$$

$$\sum_{i=1}^{m} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, \ldots, m \tag{12}$$

With out loss of generality, let us combine the **Model 1** and **Model 3**, we formulated a new mathematical model namely Entropy Matrix Game Model which is a multi-objective non-linear programming model. This model is defined for player I as follows:

Model 4

$$\max: \quad v$$

$$\max: \quad H_1$$

subject to

$$\sum_{i=1}^{m} a_{ij} y_i \geq v, \quad j = 1, 2, \ldots, n \tag{13}$$

$$H_1 = -\sum_{i=1}^{m} y_i \; ln(y_i) \tag{14}$$

$$\sum_{i=1}^{m} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, \ldots, m \tag{15}$$

Similarly the Entropy Matrix Game Model for player II is as follows:
Model 5

$$\min : \quad w$$
$$\max : \quad H_2$$

subject to

$$\sum_{j=1}^{n} a_{ij} z_j \leq w, \quad i = 1, 2, \ldots, m \tag{16}$$

$$H_2 = -\sum_{j=1}^{n} z_j \; ln(z_j) \tag{17}$$

$$\sum_{j=1}^{n} z_j = 1; \quad z_j \geq 0, \quad j = 1, 2, \ldots, n \tag{18}$$

3 Solution Procedures

In previous section, we have seen that, **Model 3** and **Model 4** are both multi-objective non-linear programming (MONLP) problem. To get a satisfactory solution of the above models we have introduced the fuzzy programming which is defined as follows.

3.1 Basic concepts of Fuzzy Set and Membership Function

Fuzzy sets first introduced by [Zadeh(1965)] in 1965 as a mathematical way to representing impreciseness or vagueness in everyday life.
Fuzzy set: A fuzzy set A in a discourse X is defined as the following set of pairs $A = (x, \mu_A) : x \in X$, where $\mu_A : X \longrightarrow [0, 1]$ is a mapping, called membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set A. The larger $\mu_A(x)$ is the stronger grade of membership form in A.
Fuzzy number: A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. A fuzzy number A is a fuzzy set of real line R whose membership

function $\mu_A(x)$ has following characteristic with $a < b$.

$$\mu_A(x) = \begin{cases} 0 & \text{if } \ x \leq a \\ \frac{x-a}{b-a} & \text{if } \ \ a \leq x \leq b \\ 1 & \text{if } \ x \geq b \end{cases} \tag{19}$$

In fuzzy programming technique, first we construct the membership function for each objective function in **Model 4**. Let $\mu_{11}(v)$, $\mu_{12}(H_1)$ be the membership function for both objectives respectively and they are defined as follows:

$$\mu_{11}(v) = \begin{cases} 0 & \text{if } \ v \leq v^- \\ \frac{v-v^-}{v^+-v^-} & \text{if } \ \ v^- \leq v \leq v^+ \\ 1 & \text{if } \ v \geq v^+ \end{cases} \tag{20}$$

and

$$\mu_{12}(H_1) = \begin{cases} 0 & \text{if } \ H_1 \leq H_1^- \\ \frac{H_1-H_1^-}{H_1^+-H_1^-} & \text{if } \ \ H_1^- \leq H_1 \leq H_1^+ \\ 1 & \text{if } \ H_1 \geq H_1^+ \end{cases} \tag{21}$$

where v^+, v^- represents maximum and minimum value of v and H_1^+, H_1^- represents maximum and minimum value of H_1 for player I.

Similarly we can construct the membership function for each objective function in **Model 5**. Let $\mu_{21}(w)$, $\mu_{22}(H_2)$ be the membership function for both objectives respectively and they are defined as follows:

$$\mu_{21}(w) = \begin{cases} 1 & \text{if } \ w \leq w^- \\ \frac{w^+-w}{w^+-w^-} & \text{if } \ \ w^- \leq w \leq w^+ \\ 0 & \text{if } \ w \geq w^+ \end{cases} \tag{22}$$

and

$$\mu_{22}(H_2) = \begin{cases} 0 & \text{if } \ H_2 \leq H_2^- \\ \frac{H_2-H_2^-}{H_2^+-H_2^-} & \text{if } \ \ H_2^- \leq H_2 \leq H_2^+ \\ 1 & \text{if } \ H_2 \geq H_2^+ \end{cases} \tag{23}$$

where w^+, w^- represents maximum and minimum value of w and H_2^+, H_2^- represents maximum and minimum value of H_2 for player II.

3.2 Fuzzy Programming

To conversion in a single objective non-linear model from multi-objective non-linear model, we have introduced the concept of fuzzy programming with the help of (20), (21) and the **Model 4**, then we formulated the following single objective non-linear model and this model is denoted by **Model 6**.

Model 6

$$\max: \quad \lambda$$

subject to

$$\lambda \leq \frac{v - v^-}{v^+ - v^-} \tag{24}$$

$$\lambda \leq \frac{H_1 - H_1^-}{H_1^+ - H_1^-} \tag{25}$$

$$\sum_{i=1}^{m} a_{ij} y_i \geq v, \quad j = 1, 2, \ldots, n \tag{26}$$

$$H_1 = -\sum_{i=1}^{m} y_i \; ln(y_i) \tag{27}$$

$$\sum_{i=1}^{m} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, \ldots, m \tag{28}$$

and for player II, the similar model may be formulated by the help of **Model 5** and (22) and (23) and this model is denoted by **Model 7**.

Model 7

$$\max: \quad \delta$$

subject to

$$\delta \leq \frac{w^+ - w}{w^+ - w^-} \tag{29}$$

$$\delta \leq \frac{H_2 - H_2^-}{H_2^+ - H_2^-} \tag{30}$$

$$\sum_{j=1}^{n} a_{ij} z_j \leq w, \quad i = 1, 2, \ldots, m \tag{31}$$

$$H_2 = -\sum_{j=1}^{n} z_j \ ln(z_j) \tag{32}$$

$$\sum_{j=1}^{n} z_j = 1; \quad z_j \geq 0, \quad j = 1, 2, \ldots, n \tag{33}$$

Now to solve the above two models, **Model 6** and **Model 7**, we apply the Genetic Algorithm which is depicted in the next section.

3.3 Genetic Algorithm

My revised genetic algorithm is illustrated as follows:

Gene Type

The gene is defined as $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ in this study: y_1^a, $y_2^a, \ldots, y_{m-1}^a$ are randomly given values. Please notice a gene must satisfy that $y_1^a + y_2^a + y_3^a + \ldots + y_m^a = 1$.

Generating genes

This process is randomly generating each element in $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ and $y_1^a + y_2^a + y_3^a + \ldots + y_m^a = 1$; Moreover the number of gene is limited to 25 when each new run begins.

Crossover

Since it is not easy to design a crossover between genes for satisfying that $y_1^a + y_2^a + y_3^a + \ldots + y_m^a = 1$, there fore no cross over is applied in this study.

Mutation

Mutation is designed as a order of elements in $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ by randomly determined cut-point. Consider an example: if the original gene is $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ and cut-point is randomly determined between the string: y_1^a and y_2^a, $y_3^a, \ldots, y_m^a$, then moreover newly mutated gene $(y_1^{'}, y_2^{'}, y_3^{'}, \ldots, y_m^{'})$ is $(y_2^a, y_3^a, y_m^a, \ldots, y_1^a)$.

Reproduction

The reproduction is also omitted to prevent the early- matured solution, which will limit the variety of solution.

Evaluation

Once $(y_1^a, y_2^a, y_3^a, \ldots, y_m^a)$ is determined, the corresponding v^a and H_1^a can be computed by (4) and (8).

Iteration

The number of iteration is set to 30 runs, each of which begins with the different random seed.

Similar technique apply for player II.

4 Numerical Examples

Example 1:

Let us consider a matrix game as follows:

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 2 & 1 & 1 \end{bmatrix} \tag{34}$$

The following results are summarized in Table-1 which computed Genetic Algorithm.

	maximum value	*minimum value*
v	$v^+ = 1.7158$	$v^- = 1.1373$
w	$w^+ = 1.9336$	$w^- = 1.7158$
H_1	$H_1^+ = 1.386295$	$H_1^- = 0.0$
H_2	$H_2^+ = 1.386295$	$H_2^- = 0.0$

Table - 1

With the help of above values in Table-1 the mathematical **Model 5** and **Model 6** for the player I and II respectively are redefined as follows:

Model 7

$$\max : \quad \lambda$$

subject to

$$\lambda \leq \frac{v - 1.1373}{1.7158 - 1.1373} \tag{35}$$

$$\lambda \leq \frac{H_1 - 0.0}{1.386295 - 0.0} \tag{36}$$

$$\sum_{i=1}^{m} a_{ij} y_i \geq v, \quad j = 1, 2, \ldots, n \tag{37}$$

$$H_1 = -\sum_{i=1}^{m} y_i \ ln(y_i) \tag{38}$$

$$\sum_{i=1}^{m} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, \ldots, m \tag{39}$$

and

Model 8

$$\max: \quad \delta$$

subject to

$$\delta \leq \frac{1.9336 - w}{1.9336 - 1.7158} \tag{40}$$

$$\delta \leq \frac{H_2 - 0.0}{1.386295 - 0.0} \tag{41}$$

$$\sum_{j=1}^{n} a_{ij} z_j \leq w, \quad i = 1, 2, \ldots, m \tag{42}$$

$$H_2 = -\sum_{j=1}^{n} z_j \ ln(z_j) \tag{43}$$

$$\sum_{j=1}^{n} z_j = 1; \quad z_j \geq 0, \quad j = 1, 2, \ldots, n \tag{44}$$

Example 2:

Let us consider a matrix game as follows:

$$A = \begin{bmatrix} 4 & 5 & 7 & 3 \\ 2 & 7 & 6 & 2 \\ 4 & 3 & 2 & 5 \\ 3 & 2 & 1 & 4 \end{bmatrix} \tag{45}$$

The following results are summarized in Table-2 which computed Genetic Algorithm.

	maximum value	*minimum value*
v	$v^+ = 3.428572$	$v^- = 2.152600$
w	$w^+ = 4.851533$	$w^- = 3.428572$
H_1	$H_1^+ = 1.374611$	$H_1^- = 0.234374$
H_2	$H_2^+ = 1.374611$	$H_2^- = 0.234374$

Table - 2

With the help of above values in Table-2 the mathematical **Model 5** and **Model 6** for the player I and II respectively are redefined as follows:

Model 9

$$\max : \quad \lambda$$

subject to

$$\lambda \leq \frac{v - 2.152600}{3.428572 - 2.152600} \tag{46}$$

$$\lambda \leq \frac{H_1 - 0.234374}{1.374611 - 0.234374} \tag{47}$$

$$\sum_{i=1}^{m} a_{ij} y_i \geq v, \quad j = 1, 2, \ldots, n \tag{48}$$

$$H_1 = -\sum_{i=1}^{m} y_i \; ln(y_i) \tag{49}$$

$$\sum_{i=1}^{m} y_i = 1; \quad y_i \geq 0, \quad i = 1, 2, \ldots, m \tag{50}$$

and

Model 10

$$\max : \quad \delta$$

subject to

$$\delta \leq \frac{4.851533 - w}{4.851533 - 3.428572} \tag{51}$$

$$\delta \le \frac{H_2 - 0.234374}{1.374611 - 0.234374} \tag{52}$$

$$\sum_{j=1}^{n} a_{ij} z_j \le w, \quad i = 1, 2, \ldots, m \tag{53}$$

$$H_2 = -\sum_{j=1}^{n} z_j \; ln(z_j) \tag{54}$$

$$\sum_{j=1}^{n} z_j = 1; \quad z_j \ge 0, \quad j = 1, 2, \ldots, n \tag{55}$$

4.1 Results

Results of Example 1:

The aspiration level with two objective for a given solution, λ_a and δ_a are obtained from above models (**Model 7** and **Model 8**) by the help of Lingo package. The optimal solutions for player I and player II are represented in the following Table-3.

aspiration level	*optimal value*	*entropy*	*optimal strategy*
$\lambda^* = 0.6269663$	$v^* = 1.5$	$H_1^* = 1.386294$	$y^* = (0.25, 0.25, 0.25, 0.25)$
$\delta^* = 1.0$	$w^* = 1.7158$	$H_2^* = 1.386295$	$z^* = (0.25, 0.25, 0.25, 0.25)$

Table - 3

Results of Example 2:

The optimal solutions for player I and player II are represented in the following Table-4.

aspiration level	*optimal value*	*entropy*	*optimal strategy*
$\lambda^* = 0.72852$	$v^* = 3.3980$	$H_1^* = 0.7605657$	$y^* = (0.035, 0.275, 0.005, 0.685)$
$\delta^* = 0.72852$	$w^* = 3.497184$	$H_2^* = 1.065064$	$z^* = (0.0, 0.285, 0.0, 0.715)$

Table - 4

5 Conclusions

Two-person matrix (zero-sum) game is analyzed in entropy environment with the help of fuzzy programming and genetic algorithm approach. In fuzzy programming technique we first fixed the bounds of the objective values and uncertainties of players. Then define a triangular membership function for the both objectives and entropies. Using fuzzy programming, we convert multi-objective non-linear model to single objective non-linear model and then we determine the value of the game and optimal strategies of the players. To study the numerical results, we have seen that each player may be looser(in pay-offs) than their classical game model due to the diversity factor 'entropy', in the entropy game model i.e., if the players are more interested for surprising movement of their strategies then their expected pay-off may be decrease. In some other words, if it is seen that expected pay-off be less than their classical game model, then it may be happen that they were interested for movement of opponent's strategies. The uncertainty function 'entropy' may be a cause for not achieving the value of the game.

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