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An Optimal Ordering Policy for a Stochastic Inventory Model for Deteriorating Items with Time-dependent Selling Price

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Abstract

This paper studies a stochastic inventory model for deteriorating items where the selling price is assumed to be a decreasing function of time. The rate of deterioration of the items are assumed to be constant over time. The selling price decreases monotonically at a constant rate with the deterioration of the items also. The demand and the lead-time both are random. A profit-maximization model has been formulated and solved here for optimum order quantity. Numerical examples are provided to illustrate the model and the results, and sensitivity analyses have been performed to examine how sensitive the solution is to the system parameter values, lead-time distributions, and the form of selling price function.

Keywords and phrases: Deteriorating items, inventory, lead time, order quantity, time-dependent selling price

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1 Introduction

Managing inventory for deteriorating items is of great concern to the retailers, wholesalers, even to the production managers, who are in the business of perishable items, the items that can deteriorate or lose value under normal conditions, such as meat, fish, sea-food, poultry, dairy products, fruit and vegetables, some special type of medicines etc., often transportation of which also needs special care, for example, some items need refrigeration or gel-type ice packing, some needs styrofoam as outer packaging, or thermoplastic bags, and in most of the cases it should be used within a short period of time after delivery, as it may not be possible to preserve them in the same manner after delivery. This type of items, if delivered prior to the time-point when demand arrives, starts deteriorating, and hence loses value.

In this paper we investigate a special type of a stochastic inventory model for deteriorating items where the selling price is assumed to be a decreasing function of time. Price also decreases with the deterioration of the items. The rate of deterioration are assumed to be constant over time. The selling price decreases at a constant rate with the deterioration of the items. The demand and the lead-time both are random. A profit-maximization model has been formulated and solved for optimum order quantity.

Here, in this model the point of time when the demand arrives is assumed to be known in advance. If the supply arrives at that point of time, the items will be sold at the maximum price. Because of the randomness of lead-time, it is not certain exactly when the supply will arrive. If it arrives at some later point, then it has to be sold at some reduced price. On the other hand, if supply reaches prior to the arrival of demand, a carrying cost incurs for holding the items until the demand arrives. Moreover, the items will start deteriorating. Note that here the selling price decreases for two reasons – one, for the delay in supply, and two, for deterioration of the items. Here the items are deteriorating with time, and hence, after delivery of the items, the selling price will decrease at a constant rate with passage of time, due to deterioration of the items, until it is sold. During Christmas season, or other festive seasons, like Easter or Thanksgiving days or so, a large demand comes for various items including perishable items too. Then the inventory policy for such items should be such that the expected profit from them is maximized, keeping in view the facts that the items are perishable, holding costs are there to hold the inventory for early arrival of supply, selling price decreases for late supply, shortages and excesses also lead to a certain amount of loss. Taking all these into account the optimum quantity to be ordered should be obtained in order to maximize the expected profit.

Benhadid et al.(2008) solved a production inventory problem with deteriorating items and dynamic costs, in a deterministic environment. Nahmias(1974) and Fries(1975) considered the problem of determining optimal policies for items with fixed lifetime. Bar-Lev et al.(2005) discussed the control policies for perishable inventory systems with random input, where the shelf-lives of the items are considered to be finite and deterministic. Deng et al.(2007) studied the inventory models for deteriorating items with ramp type demand rate. Dave (1986) developed a probabilistic scheduling period inventory model for continuously decaying items where lead-time was assumed to be deterministic. Wee et al.(2008) studied an inventory model with deteriorating items to develop an optimal replenishment inventory strategy. Hon (2006) derived an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time-discounting over a finite planning horizon, where the solution is obtained by minimizing the total cost function. In this connection mention may be made of the work of Kabak and Weinberg (1972), which is an extension of classical newsboy problem considering supply as a random variable, but newsboy suffers no decrease in expected revenue. In classical newsboy problem the products do not carry any value at all other than the salvage after the time-point at which the demand arrives [Hadley and Whitin (1963), Naddor (1966)]. In these models the items are not considered to be deteriorating. Mukherjee and Roychowdhury (1997) also discussed this type of model where the items were non-perishable, and hence no question of selling price decreasing with deterioration of the items did arise. Hsu et al. (2006) developed and analyzed an inventory model with uncertain demand and with the price depending on the lead-time, and solved the model to determine the optimal stocking quantities. Variable selling price is considered in Hsu et al. (2008). Wang (2008) developed an inventory model with continuous price decrease and variable lead-time.

The main objective of the present study is to determine the optimal order quantity that is expected to be most profitable, based on system parameters of shortage and holding costs, lead-time and demand distributions and other related costs, where the items are deteriorating with time. A closed form solution to maximize the total expected profit is obtained when the demand and lead-time both are random. Numerical examples are provided to illustrate the model and the results. Sensitivity analyses have been done to see how sensitive the optimal solution is to the change in parameter values, lead-time distributions, and the form of selling price function.

2 Assumptions and Notation

A stochastic inventory model is considered here in which the selling price is time-dependent. It decreases with time after the point-of-demand, the time-point when the demand arrives. If the supply arrives late, after the point-of-demand, the selling price will continuously decrease as the time passes. Eventually the items will lose all of its worth and will possess only salvage value or scrap value. The excess items, if any, can be sold at salvage value only. The selling price also depends on the then condition of the item. It decreases with the deterioration of the item. The condition of the item deteriorates with the passage of time. It starts deteriorating at a constant rate from the point-of-supply, the point of time when the supply arrives. The selling price is assumed to be decreasing linearly with time at a constant rate, due to the deterioration of the items, until the items are sold. The items remain in a sellable condition at least up to t_0-time after delivery. The order quantity is our decision variable here. An optimal solution is derived under the following set of assumptions:

1. The demand (X) is a continuous random variable, the distribution of which is characterized by the *c.d.f.* $F(x)$, $f(x)$ being the probability density function.
2. The demand arrives at a particular point of time that is fixed and known in advance.
3. The lead-time (l) is a continuous random variable, the distribution of which is characterized by the *c.d.f.* $G(l)$, $g(l)$ being its probability density function. The lead-time distribution is independent of the demand distribution.
4. The order is placed t_0-time prior to the point-of-demand.
5. C_1 is the holding cost per unit per unit of time. $C_1 > 0$.
6. C_2 is the shortage cost per unit short. $C_2 > 0$.
7. C is the cost per unit. $C > 0$.
8. S is the maximum selling price per unit.
9. R is the salvage value per unit. $R > 0$.
10. β is the rate of decrease in selling price per unit per unit time due to the deterioration of the item.
11. $S(l)$, the selling price per unit, is a non-increasing function of l, the lead-time.

Here $S(l)$ is assumed to be of one of the following forms. The selling price decreases due to deterioration of the items at the rate of β per unit time up to the point-of-demand. The linlin (linear-linear) form of $S(l)$ is as follows, where it is assumed to be a non-increasing linear function of l with a known decrease, b (> 0), in per unit selling price per unit of time when lead-time exceeds t_0 (up to time $t_0 + \frac{S-R}{b}$), where t_0 is the known preponement time, the time-gap between placing the order and the time-point at which the demand arrives. The functional form of a linlin $S(l)$ is given by

$$\begin{aligned} S(l) &= S - \beta\ (t_0 - l) \quad \text{if} \quad 0 \le l \le t_0 \\ &= S - b(l - t_0) \quad \text{if} \quad t_0 < l \le t_0 + \frac{S-R}{b} \\ &= R \quad \text{if} \quad l > t_0 + \frac{S-R}{b}. \end{aligned} \tag{1}$$

The linex (linear-exponential) form of the selling price function is given by

$$\begin{aligned} S(l) &= S - \beta\ (t_0 - l) \quad \text{if} \quad 0 \le l \le t_0 \\ &= S \exp\{-r(l - t_0)\} \quad \text{if} \quad t_0 < l \le t_0 + \frac{1}{r}\log_e \frac{S}{R} \\ &= R \quad \text{if} \quad l > t_0 + \frac{1}{r}\log_e \frac{S}{R}. \end{aligned} \tag{2}$$

The excess items are also sold at a salvage value, the functional form of which is as follows:

$$R(l) = R \ -\beta\ (t_0 - l) \ \text{ if } \ 0 \le l \le t_0$$
$$= R \qquad\qquad\qquad \text{ if } \ l > \ t_0. \tag{3}$$

Clearly, $0 < R - \beta t_0 \le R \le S(l) \le S$ and $\int_0^\infty R(l)dG(l) \le R$.

It is justified to assume that the average selling price per unit must exceed the per unit cost price and the average carrying cost together, which should be greater than the salvage value per unit, *i.e.*,

$$R < C + C_1 \int_0^{t_0} (t_0 - l)dG(l) < \int_0^\infty S(l)dG(l).$$

3 Optimal Ordering Policy

Let the order of quantity q be placed t_0-time prior to the point-of-demand, the point of time at which the demand arrives. To handle the uncertainty in demand due to its randomness, we have to make an optimal decision about the order quantity q. Because of the randomness of lead-time l, the supply may arrive before the arrival of demand, or after. In case it arrives before the point-of-demand, i.e., if $l < t_0$, then there will incur a holding cost of $C_1 q(t_0 - l)$ for holding the items up to the time-point of demand, and the items will also start deteriorating until they are sold. On the other hand, if the supply arrives after the arrival of demand, the items have to be sold at some reduced price, as given in (1) or (2) above, that too up to a certain point of time, after which the items possess only the salvage value, R. It is to be noted that the excess items, if any, will also be sold at the salvage value, as given in (3). Let $\psi(q)$ be the expected profit function, maximizing which we can determine the optimal solution. Now we make an attempt to derive the expression for $\psi(q)$, which comes out to be as follows:

$$\begin{aligned}
\psi(q) \quad = \quad & -Cq - C_1 q \int_0^{t_0} (t_0 - l)dG(l) - C_2 \int_q^\infty (x - q)dF(x) \\
& + \int_q^\infty [\int_0^{t_0} S(l)qdG(l) + \int_{t_0}^{t'} S(l)qdG(l) + \int_{t'}^\infty RqdG(l)]dF(x) \\
& + \int_0^q [\int_0^{t_0} S(l)xdG(l) + \int_{t_0}^{t'} S(l)xdG(l) + \int_{t'}^\infty RxdG(l) \\
& \quad + \int_0^{t_0} \{R - \beta(t_0 - l)\}(q - x)dG(l) + \int_{t_0}^{t'} R(q - x)dG(l) \\
& \qquad\qquad\qquad\qquad + \int_{t'}^\infty R(q - x)dG(l)]dF(x),
\end{aligned}$$

where

$$
\begin{aligned}
t' &= t_0 + \frac{S-R}{b} && \text{for linlin } S(l) \\
&= t_0 + \frac{1}{b}\log_e \frac{S}{R} && \text{for linex } S(l).
\end{aligned}
$$

The expression for $\psi(q)$ reduces to

$$
\begin{aligned}
\psi(q) &= -Cq - C_1 q \int_0^{t_0} (t_0 - l) dG(l) - C_2 \int_q^{\infty} (x-q) dF(x) \\
&+ \int_q^{\infty} q\{\int_0^{\infty} S(l) dG(l)\} dF(x) + \int_0^q x\{\int_0^{\infty} S(l) dG(l)\} dF(x) \\
&+ \int_0^q (q-x)\{\int_0^{\infty} R(l) dG(l)\} dF(x).
\end{aligned}
$$

Finally we have

$$
\begin{aligned}
\psi(q) &= -C_2 E(X) + [\{\int_0^q x dF(x)\}\{\int_0^{\infty} S(l) dG(l) + C_2 - \int_0^{\infty} R(l) dG(l)\}] \\
&- [qF(q)\{\int_0^{\infty} S(l) dG(l) + C_2 - \int_0^{\infty} R(l) dG(l)\}] + [q\{\int_0^{\infty} S(l) dG(l) + C_2 - C \\
&- C_1 \int_0^{t_0} (t_0 - l) dG(l)\}]. \qquad (4)
\end{aligned}
$$

Now we prove the following result which helps us determine the optimal order quantity:

Result 3.1. The expected profit function, $\psi(q)$, is a concave function of q.
Proof. Here,

$$
\begin{aligned}
\frac{d\psi(q)}{dq} &= -F(q)\{\int_0^{\infty} S(l) dG(l) + C_2 - \int_0^{\infty} R(l) dG(l)\} + \{\int_0^{\infty} S(l) dG(l) + C_2 - C \\
&\quad -C_1 \int_0^{t_0} (t_0 - l) dG(l)\}.
\end{aligned}
$$

$$
\frac{d^2\psi(q)}{dq^2} = -\{\int_0^{\infty} S(l) dG(l) + C_2 - \int_0^{\infty} R(l) dG(l)\} f(q),
$$

which is negative.
Hence the result [Roberts and Varberg (1973)]. ■

By virtue of the above result we can get the optimal order quantity, q_0, of q by solving $\frac{d\psi(q)}{dq} = 0$, which implies,

$$
F(q) = 1 - \frac{C + C_1 \int_0^{t_0} (t_0 - l) dG(l) - \int_0^{\infty} R(l) dG(l)}{\int_0^{\infty} S(l) dG(l) + C_2 - \int_0^{\infty} R(l) dG(l)}. \qquad (5)
$$

and hence the maximum expected profit, $\psi(q_0)$, is given by

$$Max\ \psi(q) = \psi(q_0) = -C_2E(X)+[\{\int_0^{q_0} xdF(x)\}\{\int_0^{\infty} S(l)dG(l)+C_2-\int_0^{\infty} R(l)dG(l)\}]. \tag{6}$$

Special cases of different demand and lead-time distributions are discussed here.

In particular, for uniform demand and uniform lead time the optimal order quantity will be determined as follows:

If X is uniform between m and $m+D$ and l is uniform between a and $a+T$ $(0 < q < D,\ 0 < t_0 < T)$, with $S(l)$ having linlin form as given in (1), the optimal order quantity q_0 is obtained as follows:

Case 1. If $t_0 + \frac{S-R}{b} < T + a$,

$$q_0 = m + D[1 - \frac{C - R + (C_1+\beta)\{\frac{(t_0-a)^2}{2T}\}}{C_2 + \frac{1}{T}\{(t_0 + \frac{S-R}{b})(S-R) - aS\}}]. \tag{7}$$

and the maximum expected profit is

$$Max\ \psi(q) = \psi(q_0) = -C_2(m + \frac{D}{2}) + \frac{q_0^2 - m^2}{2D}\{C_2 + \frac{S-R}{T}(t_0 + \frac{S-R}{2b}) - as\}. \tag{8}$$

Case 2. If $t_0 + \frac{S-R}{b} > T + a$,

$$q_0 = m + D[1 - \frac{C - R + (C_1+\beta)\{\frac{(t_0-a)^2}{2T}\}}{(S - R + C_2) - \frac{b}{2T}(a + T - t_0)^2}]. \tag{9}$$

and the maximum expected profit is

$$Max\ \psi(q) = \psi(q_0) = -C_2(m+\frac{D}{2})+\frac{q_0^2 - m^2}{2D}\{(S-R+C_2) - \frac{b}{2T}(a+T-t_0)^2\}. \tag{10}$$

The following numerical example is provided to illustrate the result:

Example 1: Suppose that the demand is uniform over (700,1700) and the lead-time is uniform over (2,12). Let the selling price function be linlin with $S = 1000, \beta = 50, b = 200, R = 400$ and the cost parameters be $C_1 = 10, C_2 = 10, C = 500$. Let $t_0 = 6$. The optimal order quantity is $q_0 = 1277.14$. The maximum expected profit is 187691.43 (all values in appropriate units).

A sensitivity analysis is performed in the next section by changing the values of β and b. Table 1 shows how the optimal values of order quantity and the expected profit change with the change in the values of b for some fixed β-values, and Table 2 shows how the optimal values change with the change in β-values for some fixed values of b.

For linex selling price $S(l)$, as given in (2), the optimal order quantity q_0 will be as follows:

Case 1. If $t_0 + \frac{1}{r}\log_e \frac{S}{R} < T + a$,

$$q_0 = m + D[1 - \frac{C - R + (C_1+\beta)\{\frac{(t_0-a)^2}{2T}\}}{C_2 + \frac{S-R}{T}(t_0 - a + \frac{1}{r}) - \frac{R}{rT}\log_e \frac{S}{R}}]. \tag{11}$$

and the maximum expected profit is

$$Max\ \psi(q) = \psi(q_0) = -C_2(m + \frac{D}{2}) + \frac{q_0^2 - m^2}{2D}\{C_2 + \frac{S-R}{T}(t_0 - a + \frac{1}{r}) - \frac{R}{rT}\log_e \frac{S}{R}\}. \tag{12}$$

Case 2. If $t_0 + \frac{1}{r}\log_e \frac{S}{R} > T + a$,

$$q_0 = m + D[1 - \frac{C - R + (C_1+\beta)\{\frac{(t_0-a)^2}{2T}\}}{C_2 - R + \frac{S}{T}\{t_0 - a + \frac{1}{r}(1 - e^{-r(T+a-t_0)})\}}]. \tag{13}$$

and the maximum expected profit is

$$Max\ \psi(q) = \psi(q_0) = -C_2(m + \frac{D}{2}) + \frac{q_0^2 - m^2}{2D}\{C_2 - R + \frac{S}{T}\{t_0 - a + \frac{1}{r}(1 - e^{-r(T+a-t_0)})\}\}. \tag{14}$$

The following numerical example illustrates the results:

Example 2: Suppose that the demand is uniform over (700,1700) and the lead-time is uniform over (2,12). Let the selling price function be linex with $S = 1000, \beta = 50, r = 0.305, R = 400$ and the cost parameters be $C_1 = 10, C_2 = 10, C = 500$. Let $t_0 = 6$. The optimal order quantity is $q_0 = 1246.78$ and the maximum expected profit is 161800.74 (all values in appropriate units).

Tables 3 and 4 show how the optimal order quantity and the expected profit change with the change in the values of β and r.

Now we obtain the optimal solution for uniform demand, exponential lead-time with linlin selling price function.

If $X \sim U(m, m+D)$, $l \sim \exp(\lambda)$, then the optimal order quantity, q_0, is obtained as

$$q_0 = m + D[1 - \frac{C - R + (C_1+\beta)\{t_0 - \frac{1}{\lambda} + \frac{1}{\lambda}e^{-\lambda t_0}\}}{S - R + C_2 - \frac{b}{\lambda}e^{-\lambda t_0}\{1 - e^{-\lambda(\frac{S-R}{b})}\}}]. \tag{15}$$

and the maximum expected profit is

$$Max\ \psi(q) = \psi(q_0) = -C_2(m + \frac{D}{2}) + \frac{q_0^2 - m^2}{2D}\{S - R + C_2 - \frac{b}{\lambda}e^{-\lambda t_0}(1 - e^{-\lambda(\frac{S-R}{b})})\}. \tag{16}$$

A numerical example is provided to illustrate the solution as follows:

Example 3: Suppose that the demand is uniform over (700,1700) and the lead-time is exponential with mean 7. Let the selling price function be linlin with $S = 1000, \beta = 50, b = 200, R = 400$ and the cost parameters be $C_1 = 10$, $C_2 = 10$, $C = 500$. Let $t_0 = 6$. The optimal order quantity is $q_0 = 1158.35$. The maximum expected profit is 159596.22 (all values in appropriate units).

Now we obtain the optimal order quantity if the demand follows a beta distribution. The beta distribution is a more flexible probability distribution compared to the other demand distributions we have already discussed, because it can accommodate different ranges of the variate value and different shapes of the distribution. Its shape depends on the values of its parameters. Thus, in practice, from the knowledge of the shape of actual demand distribution, we can choose the appropriate values of m and n and proceed.

For a beta demand and an exponential lead-time with a linlin selling price function, the optimal order quantity is obtained as follows:

If $X \sim B(m,n)$ with $\alpha < x < \gamma$ (then $\alpha < q < \gamma$), $l \sim \exp(\lambda)$, the optimal order quantity q_0 is given by

$$I_{\frac{q_0-\alpha}{\gamma-\alpha}}(m,n) = 1 - \frac{C - R + (C_1+\beta)\{t_0 - \frac{1}{\lambda} + \frac{1}{\lambda}e^{-\lambda t_0}\}}{S - R + C_2 - \frac{b}{\lambda}e^{-\lambda t_0}\{1 - e^{-\lambda(\frac{S-R}{b})}\}}, \tag{17}$$

where $I_{\frac{q-\alpha}{\beta-\alpha}}(m,n)$ can be obtained from Karl Pearson's 'Tables of the Incomplete Beta Function'(1934).

The maximum expected profit is

$$\begin{aligned} Max\ \psi(q) = \psi(q_0) = -C_2\{\alpha + (\gamma-\alpha)\frac{m}{m+n}\} + \frac{1}{B(m,n)}\{\alpha I_{\frac{q_0-\alpha}{\gamma-\alpha}}(m,n) \\ + (\gamma-\alpha)I_{\frac{q_0-\alpha}{\gamma-\alpha}}(m+1,n)\}\{S - R + C_2 - \frac{b}{\lambda}e^{-\lambda t_0}(1 - e^{-\lambda(\frac{S-R}{b})})\}. \end{aligned} \tag{18}$$

Example 4: Suppose that the demand follows a beta distribution with m =1 and n =2 (positively skewed), with a minimum demand of 700 units and maximum demand of 1700 units, i.e., $\alpha = 700$, $\gamma = 1700$, and the lead-time is exponential with mean 7. Let the selling price function be linlin with $S = 1000, \beta = 50, b = 200, R = 400$ and the cost parameters be $C_1 = 10$, $C_2 = 10$, $C = 500$. Let $t_0 = 6$. Then the optimal order quantity comes out to be $q_0 = 964$. The maximum expected profit is 387017.56 (all values in appropriate units).

4 Sensitivity Analysis

To study the effect of variations in the values of the parameters or the form of selling price function or the lead-time distributions on the optimal order quantity and the expected profit, sensitivity analyses are carried out. Here, in this section, Table 1 and Table 2 show how the optimal values of order quantity and the expected profit change with the change in the values of b and β, when the demand and the lead-time both are considered to be uniform with linlin selling price function. Let $m = 700$, $D = 1000$, $a = 2$, $t_0 = 6$, $T = 10$, $C = 500$, $C_1 = 10$, $C_2 = 10$, $S = 1000, R = 400$. Here the expected profit is found to be more sensitive to the change in b-value compared to the change in β-value. In Table 1 and Table 2 we observe that the increase in b-value results in a significant percentage decrease in expected profit, whereas an increase (even a large increase) in β-value results in a small percentage decrease in the expected profit. The optimal order quantity is also more sensitive to the change in b-value, compared to the change in β-value.

Table 3 and Table 4, show how the optimal values of order quantity and expected profit change with the change in values of β and r, when the demand and lead-time both are assumed to be uniform, with linex selling price. Let $m = 700$, $D = 1000$, $a = 2$, $T = 10$, $t_0 = 6$, $C = 500$, $C_1 = 10$, $C_2 = 10$, $S = 1000$, $R = 400$.

Table 5 shows the effect of the form of the selling price function on the optimal values of the order quantity and expected profit. Here in the course of our study we assume linlin and linex form of the selling price function $S(l)$. Considering uniform demand and uniform lead-time we examine how the optimal order quantity and the expected profit vary for various values of the other parameters, As before, we consider $m = 700$, $D = 1000$, $a = 2$, $T = 10$, $t_0 = 6$, $C = 500$, $C_1 = 10$, $C_2 = 10, S = 1000$, $R = 400$. Note that the time $t^{'}$ is such that after $(t^{'} - t_0)$-time since the arrival of demand, the items possess only salvage value, i.e., the items can be sold at R, the salvage value, if the lead-time exceeds the time $t^{'}$. In Table 5 we observe that the profits are more sensitive than the order quantities to the change in form of the selling price function. Optimal values of the order quantity and the expected profit are higher for linlin selling price function.

Table 6 shows the sensitivity of the optimal order quantity and the expected profit to the change in lead-time distribution with a linlin form of the selling price function $S(l)$. We examine the change for uniform and exponential form of lead-time distribution having mean lead-time 7, considering $m = 700$, $D = 1000$, $a = 2$, $T = 10$, $t_0 = 6$, $C = 500$, $C_1 = 10$, $C_2 = 10$, $S = 1000$, $R = 400$. In Table 6 we see that the profit is more sensitive than order quantity to the change in the distribution of lead-time.

Table 1. Optimal order quantity and maximum expected profit for different values of b for some fixed values of β

β	b	% change in b	q_0	% change in q_0	Max.expected profit	% change in profit
25	120	-	1427.66	-	351829.79	-
	150	25	1387.80	-2.79	282380.49	-19.74
	200	33.33	1334.29	-3.86	213805.71	-24.28
50	120	-	1385.11	-	323702.13	-
	150	25	1339.02	-3.33	255112.20	-21.19
	200	33.33	1277.14	-4.62	187691.43	-26.43
60	120	-	1368.09	-	312689.36	-
	150	25	1319.51	-3.55	244478.05	-21.81
	200	33.33	1254.29	-4.94	177565.71	-27.37
65	120	-	1359.57	-	307234.04	-
	150	25	1309.76	-3.66	239219.51	-22.14
	200	33.33	1242.86	-5.11	172571.43	-27.86

Table 2. Optimal order quantity and maximum expected profit for different values of β for some fixed values of b

b	β	% change in β	q_0	% change in q_0	Max.expected profit	% change in profit
120	25	-	1427.63	-	351829.79	-
	50	100	1385.11	-2.98	323702.13	-7.99
	60	20	1368.09	-1.23	312689.36	-3.40
	65	8.33	1359.57	-0.62	307234.04	-1.74
150	25	-	1387.80	-	282380.49	-
	50	100	1339.02	-3.51	255112.20	-9.66
	60	20	1319.51	-1.46	244478.05	-4.17
	65	833	1309.76	-0.74	239219.51	-2.15
200	25	-	1334.29	-	213805.71	-
	50	100	1277.14	-4.28	187691.43	-12.21
	60	20	1254.29	-1.79	177565.71	-5.39
	65	8.33	1242.86	-0.91	172571.43	-2.81

Table 3. Optimal order quantity and maximum expected profit for different values of r for some fixed values of β

β	r	% change in r	q_0	% change in q_0	$Max.expected\ profit$	% change in $profit$
25	0.183	-	1361.01	-	245199.75	-
	0.229	25.14	1336.32	-1.81	216025.05	-11.90
	0.305	33.19	1308.03	-2.12	187348.80	-13.27
50	0.183	-	1308.04	-	218509.33	-
	0.229	25.14	1279.50	-2.18	189866.89	-13.11
	0.305	33.19	1246.78	-2.56	161800.74	-14.78
60	0.183	-	1286.85	-	208129.78	-
	0.229	25.14	1256.77	-2.34	179721.85	-13.65
	0.305	33.19	1222.28	-2.74	151924.50	-15.47
65	0.183	-	1276.26	-	203003.57	-
	0.229	25.14	1245.40	-2.42	174717.52	-13.93
	0.305	33.19	1210.03	-2.84	147059.87	-15.83

Table 4. Optimal order quantity and maximum expected profit for different values of β for some fixed values of r

r	β	% change in β	q_0	% change in q_0	$Max.expected\ profit$	% change in $profit$
0.183	25	-	1361.01	-	245199.75	-
	50	100	1308.04	-3.89	218509.33	-10.89
	60	20	1286.85	-1.62	208129.78	-4.75
	65	8.33	1276.26	-0.82	203003.57	-2.46
0.229	25	-	1336.32	-	216025.05	-
	50	100	1279.50	-4.25	189866.89	-12.11
	60	20	1256.77	-1.78	179721.85	-5.34
	65	833	1245.40	-0.90	174717.52	-2.78
0.305	25	-	1308.03	-	187348.80	-
	50	100	1246.78	-4.68	161800.74	-13.64
	60	20	1222.28	-1.96	151924.50	-6.10
	65	8.33	1210.03	-1.00	147059.87	-3.20

Table 5. Sensitivity of optimal order quantity and maximum expected profit to the change in the nature of the selling price function

β	t'	Order quantity			Max.expected profit		
		Linlin $S(l)$	Linex $S(l)$		Linlin $S(l)$	Linex $S(l)$	
		q_L	q_E	$\frac{q_E}{q_L}$	p_L	p_E	$\frac{p_E}{p_L}$
25	9	1334.29	1308.03	0.98	213805.71	187348.80	0.88
	10	1387.80	1336.32	0.96	282380.49	216025.05	0.77
	11	1427.66	1361.01	0.95	351829.79	245199.75	0.70
50	9	1277.14	1246.78	0.98	187691.43	161800.74	0.86
	10	1339.02	1279.50	0.96	255112.20	189866.89	0.74
	11	1385.11	1308.04	0.94	323702.13	218509.33	0.68
60	9	1254.29	1222.28	0.97	177565.71	151924.50	0.86
	10	1319.51	1256.77	0.95	244478.05	179721.85	0.74
	11	1368.09	1286.85	0.94	312689.36	208129.78	0.67
65	9	1242.86	1210.03	0.97	172571.43	147059.87	0.85
	10	1309.76	1245.40	0.95	239219.51	174717.52	0.73
	11	1359.57	1276.26	0.94	307234.04	203003.57	0.66

Table 6. Sensitivity of optimal order quantity and maximum expected profit to the change in lead-time distribution

β	b	Order quantity			Max.expected profit		
		exponential lead-time	uniform lead-time		exponential lead-time	uniform lead-time	
		q_{0e}	q_{0u}	$\frac{q_{0e}}{q_{0u}}$	p_{0e}	p_{0u}	$\frac{p_{0e}}{p_{0u}}$
25	200	1280.62	1334.29	0.96	219674.63	213805.71	1.03
	150	1293.86	1387.80	0.93	234312.24	282380.49	0.83
	120	1305.24	1427.66	0.91	247742.65	351829.79	0.70
50	200	1158.35	1277.14	0.91	159596.22	187691.43	0.85
	150	1175.45	1339.02	0.88	173486.76	255112.20	0.68
	120	1190.14	1385.11	0.86	186274.81	323702.13	0.58
60	200	1109.44	1254.29	0.88	137251.53	177565.71	0.77
	150	1128.08	1319.51	0.85	150790.00	244478.05	0.62
	120	1144.10	1368.09	0.84	163275.35	312689.36	0.52
65	200	1084.99	1242.86	0.87	126440.61	172571.43	0.73
	150	1104.40	1309.76	0.84	139791.65	239219.51	0.58
	120	1121.09	1359.57	0.82	152115.83	307234.04	0.50

5 Summary and Conclusion

The model considered in this paper incorporates some realistic features that are likely to be associated with an inventory of any deteriorating material. Deterioration over time is a natural feature, maybe in terms of its quality or maybe in terms of its usability. Naturally the selling price decreases with the deterioration of the items. The problem of finding optimal ordering policies for deteriorating items is addressed in this paper, where selling price decreases with time, as well as with the deterioration of the item. A special nature of selling price function has been considered here, which accommodates deterioration aspect of the items and delay in delivery time in its functional form. A random demand comes for a specific time-point when the items possess the maximum value. They are sold at a reduced price if the supply arrives late. This can happen due to the uncertainty of lead-time. The stochastic variability of demand as well as of lead time can have a great impact on the optimal order quantity. A closed form solution has been obtained here. The sensitivity of the solution to the change in the values of different parameters, and also the change in the form of selling price function or lead-time distribution has been studied. We have seen that the optimal order quantity and the expected profit are more sensitive to the change in b-value compared to the change in β-value, i.e., the rate of decrease in selling price per unit per unit time due to deterioration of the items is less than the rate of its decrease per unit time due to delay in delivery (when lead-time exceeds t_0). We have observed that the profits are more sensitive than the order quantity to the change in the form of the selling price function, also to the change in the distribution of lead-time.

While the focus of this paper has been on the order quantity of deteriorating items, there are other issues that have not been considered in the present paper, for example, it could be interesting to extend the results to the multi-product inventory models, where some of the items are perishable, some are not, or, the rate of deterioration is not same for all items. An extension of this problem to a multi-period model can also be of interest for further investigation. Demand can be considered to be selling price-dependent. This remains a challenging problem for future research.

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