# Fusion modeling to analyze the asymmetry as continuous feature

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**Abstract**

Recognition of symmetrical features, and their degrees into the shapes, is a priority aspect towards obtaining the essential structure of real world problems. Through minimizing redundancies, it is possible to reduce the computational complexity of a task. So, the Computational Symmetry is an emerging and more and more increasing in the futur new area of research, advancing in different directions, as the reasoning, detection and representation about symmetries on computers. It must be considered that until now, relatively few computational tools exist to solve real world situations. A computational model for symmetry will be very useful in fields as Machine Intelligence, Robotics or Computer Vision, for instance. Being, therefore, interesting not only theoretical, from the mathematical viewpoint (as group theory), but also when we apply A. I.

Here, we will attack the Asymmetry as a continuous Feature, and its more essential computational aspects: the possibility of obtaining a geometrical construct which give us an efficient measure of the Level Asymmetry of shapes and in general, for any fuzzy set.

So, it will be possible to attempt the solution of some interesting open problems. As for instance, the discontinuity introduced by Temporal Asymmetry in Causality Theory.

**Keywords:** Fuzzy set theory, Fuzzy real analysis, Fuzzy Measures, Probabilistic Methods in Group Theory, Reasoning under uncertainty, Graph Theory, Bayesian Nets, Probabilistic Graphical Models, Computer Vision, Pattern Recognition.

**MSC Classification:** 03E72, 26E50, 28E10, 68T37, 20P05, 68R10.

## 1. Introduction to Causality

Let us consider the typical question of Virtual or Counterfactual History: "What would have happened had Hitler died in the July 1944 assassination attempt?" Logically, Marshal Rommel and other leaders involved in the plot would have survived to the subsequent revenge, and surely, it would have produced an armistice, ending promptly with World War II. Therefore, these are questions like: "what if...?"

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Another famous example is this: "What would have happened had Napoleon win in the battle of Waterloo?" Then, we would be possibly speaking and writing in French, instead of English.

Although some examples of counterfactual history can be found in the Victorian period, it was in 1931 at London, when J. C. Squire edited a collection of essays of this kind: *If It Had Happened Otherwise* [20], counting with contributing authors as Sir Winston Churchill: "If Lee had not Won the Battle of Gettysburg" Gilbert K. Chesterton: "If Don John of Austria had Married Mary Queen of Scots"Ȯr the famous historian George M. Trevelyan, with the aforementioned topic, on Napoleon as winner in Waterloo. And some really curious for Spanish people, as: "If the Moors in Spain had Won "

This book was modified and published the same year in America, under the name: *If: or, History Rewritten.*

The Counterfactual Theory starts with the work of the Scottish philosopher *David Hume (1711-1776).* Such initial theory is taken up again by *John Stuart Mill*, in 1843. Later, *David Kellogg Lewis (1941-2001)* developed successive, improved versions of their *Counterfactual Theory.* In 1999 were exposed the last of such versions: it was into the *Whitehead Lectures,* at Harvard University.

The supposition on Lewis [12], according which: an asymmetry of causal dependence characterizes our world is basic into the Lewisian framework. But criticism appeared against the explanation given by Lewis, in some authors, as: Horwich [9], Price [16] and [17] and Hausman [8].

One of the main arguments of the critics is based on supposing that this explanation of Lewis suffers from a certain psychological implausibility. This can be found in Horwich [9]. Lewis admits that this asymmetry is possibly a contingent characteristic of the actual world, not present in other worlds.

So, in a world populated by only one atom such asymmetry on the overdetermination does not hold. For this reason, there exists a possible discontinuity problem in the boundary. Because if we consider a contractive sequence of subworlds, each of them asymmetric, converging to the monoatomic world, denoted W, where asymmetry does not hold, we would have a weakness in the theory.

**2. Symmetrical Features**

In our world, there are many temporal asymmetries.

The supposition on Lewis, according which an asymmetry of causal dependence characterizes our world, is basic in the framework of the Lewisian Theory.

Usually, the Symmetry, and in parallel, the Asymmetry, can be considered as the two sides of the same coin: an object will be totally symmetric, or totally asymmetric, relative to a pattern object, without intermediate situations, of partial symmetry or partial asymmetry. But this dychotomical classification, because their simplicity, suffer a lack of necessary and realistic grades. For this reason, it is convenient the introduction of "shade regions", modulating the degrees (a fuzzy concept).

So, defining the symmetry as a continuous feature, many more complex than in the discrete definition, but more convenient solving many problems [14]: a computational model for symmetry will be very useful in fields as Machine Intelligence, Robotics or Computer Vision, for instance. Being, therefore, interesting not only theoretical, from the mathematical viewpoint (as group theory), but also when we apply A. I.

Because the profound understanding of Symmetrical features is very fundamental in basic sciences. It is very usual which it gives support to scientific discoveries. So, for instance:

- the double helix, in the human DNA structure,

- the symmetry of time and space, in Relativity,

- the apparition of the quasicrystals field of study, and their mathematical translation, known as Penrose Tile. It is a nonperiodic tiling generated by an aperiodic set of prototiles. Because their nonperiodic character, it lacks any translational symmetry. Therefore, a shifted copy will never match the original exactly.

Because recognition of symmetrical features, and their degrees into the shapes, is a prioritary aspect towards obtaining the essential structure of real world problems. Through minimizing redundancies, it is possible to reduce the computational complexity of a task. So, the Computational Symmetry is an emerging and more and more increasing in the future new area of research, advancing in different directions, as the reasoning, detection and representation about symmetries on computers. We must consider that until now, relatively few computational tools exist for to solve real world situations.

*Remember* [21]*:*

Let O be a general object (image, signal...). For instance, in dimension one, two or three (1D-, 2D-, 3D-).

The *Symmetry Group of O,* denoted as $G(O)$, is composed of *all the isometries under which is preserved their invariance,* considering the composition as group operation.

Therefore, *the Symmetry Group will be a subset of the Isometry Group:*

$$G(O) \subset Iso(O)$$

The *mathematical study of the Symmetry, into the flat* (therefore, for 2D-shapes), make partial or total use (for monochrome patterns) of:

- *seven frieze groups* along one dimension (so called *strip patterns*), where 2-dim patterns are repeated along one dimension. They are the 1-dim crystallographic groups.

- *seventeen distinct crystallographic planar groups: wallpaper patterns,* i.e., 17 wallpaper groups, describing patterns generated by two linearly independent translations. They are the 2-dim crystallographic groups.

In abrigde analysis, we have the subsequent classification:

*First Case:* There exist Rotations:

If the minimal rotational angle is: $\alpha = 60^o$ :

$p6$ : No Reflections.

$p6m$ : Reflections are present.

The number 6 indicates which acting six times, consecutively, rotations of angle $\pi/3$ *radians*, we can reach the identity transformation.

If $\alpha = 90^o$, then:

$p4$ : No Reflexions.

$p4g$ : There are Reflections, but its edges no necessarily passing through the rotation center of $90^o$.

$p4m$ : There are Reflections, with reflection axis passing through rotation centers of $90^o$. For each of such rotation centers must to pass at least a reflection axis.

(4 indicate that applying four times a rotation of $\pi/2$ *radians*, we reach to the identity transformation).

If $\alpha = 120^o$, then:

$p3$ : No Reflections.

$p31m$ : Reflections, but there exist rotation centers of $\alpha = 120^o$ through does not pass anything reflection axis.

$p3m1$ : There are Reflections and for each rotation center of $\alpha = 120^o$ crosses some reflection axis.

(3 because applying three times a rotation of $2\pi/3$ *radians*, it reaches the identity).

If $\alpha = 180^o$, then:

$p2$ : No Reflection. Neither Glide-Reflection (that is composition of translation and reflection).

$cmm$ : There are Reflections. Its axis passing through the rotation centers. But there are also rotation center through does not pass reflection axis.

$pmm$ : There are also Reflections. For each rotation center of $180^o$ ever passing reflection axis. They intersect orthogonally, in the rotation centers. Such aforementioned reflections are of parallel axis. Glide- Reflections can also exist. Its edges are orthogonal to the precedent ones.

$pgg$ : No Reflections, but Glide-Reflections: their respective edges are orthogonal among them.

(2 indicate that applying two times a rotation of angle $\pi$, we will reach the identity).

In the particular cases of $cmm$ and $pmm$, Glide-Reflections are also allowed. But then, their axis coincides with the reflection axis.

*Second Case:* No Rotations. Therefore, it is not necessary to distinguish a minimal rotation angle.

According to reflections and glide-reflections which shown, it should be:

$cm$ : Reflections and Glide-Reflections.

$pm$ : Reflections, but no Glide-Reflections.

$pg$ : No Reflections, but Glide-Reflections.

$p1$ : Only translations appears among the symmetries.

So, for both precedent questions, this will be the answer: No Reflections neither Glide-Reflections

As you can see, we are described the seventeen crystallographic planar groups different that there exists.

In the *spatial case* (for 3D-shapes), we need some of the *230 spatial groups, generated by three linearly independent translations.* So called *regular crystal patterns.*

Or any other general symmetry group. Observe that for every n-dimensional euclidean space, in spite of the existence of infinite possible periodic patterns, the cardinal number of the set of symmetry groups for a symmetrical pattern is always finite.

The symmetry group of a repeated pattern is a good descriptor, in Artificial Vision research.

Given a n-dimensional object, O, some essential tools will be:

− *mirror-symmetry,* or invariance under a reflection about an hyperplane (n-1)-dimensional, passing through their center of mass,

− *rotational-symmetry of order n,* showing invariance under rotation of angle $2\pi/n$ radians, about their center of mass (2-dim case), or a line (the *rotational symmetry axis*) passing through the center of mass of the object (3-dim case). It will be denoted $C_n$*-symmetry.* Observe that the $C_\infty$ would be the *circular symmetry,*

− *radial symmetry* :the symmetry of a 2-dimensional object where are combined both types of such precedent symmetries: *mirror-symmetries* and $C_n$-*symmetries.* The *radial-symmetry of order n* is denoted $D_n$*-symmetry.*

So, it is possible to construct new plausible computational tools which permets the progressive translation from theoretical concepts on:

$$Symmetry/Asymmetry$$

to interesting applications in the real world.

And with this, the apparition of a new collection of nearest shapes. Because given an object O, we will define SD, the Symmetry Distance of the shape to their reference pattern.

In this way, quantifying the amount of distance departure from Symmetry in shape, as continuous feature, instead of discrete feature: not only the total coincidence neither the absolute difference, but gradual, with their Symmetrical shape.

This distance from Symmetry in shape will be defined as the minimum mean squared distance required to move points from the original shape, in order to obtain a symmetrical shape.

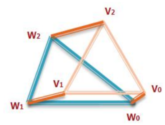
So, SD is the minimum effort required to turn a given shape into a symmetric shape.

Every pair of such shapes ($V$ and $W$, for instance) will be represented by its respective sequence of points.

So, for instance, when $n = 3$ :

$$\{V_j\}_{j=0}^{j=n-1} \ and \ \{W_j\}_{j=0}^{j=n-1}$$

See for this the Fig. 1, going from a high level of asymmetry to another lowest, or vice versa.




Then, the aforementioned metric, $m$, will be defined as:

$$m : \Psi \times \Psi \rightarrow R_+ \cup \{0\}$$
$$m\left(V,\ W\right) = m\left(\{V_j\}_{j=0}^{j=n-1},\ \{W_j\}_{j=0}^{j=n-1}\right) = \frac{\sum \|V_j - W_j\|^2}{n}$$

Also we will define the *Symmetric Transform of* $V$, denoted $ST(V)$, as the closest symmetric shape to V, relative to such metric.

In the particular aforementioned case, corresponding precisely to Fig 1, we have:

$$m\left(V,\ W\right) = m\left(\{V_j\}_{j=0}^{j=2},\ \{W_j\}_{j=0}^{j=2}\right) = \frac{\|V_0 - W_0\|^2 + \|V_1 - W_1\|^2 + \|V_2 - W_2\|^2}{3}$$

By this tool, it is possible to introduce the *SD of a shape,* $V$*,* as the distance measured between such V and their Symmetry Transform, $ST(V)$.

We will shown the *Algorithm* necessary to evaluate such *Symmetry Transform (ST):*

We depart of $n$ original points: $\{V_j\}_{j=0}^{n-1}$, which conforms the shape of $O_i$.

*First step:* Fold $\{V_j\}_{j=0}^{n-1}$ into $\left\{V_j^*\right\}_{j=0}^{n-1}$. For instance, in the $C_n$ case, rotating each point counterclockwise about the centroid by $2\pi\frac{j}{n}$ radians.

*Second step:* Average this new set of points: $V_0^{\diamondsuit} = \frac{1}{n}\sum_{j=0}^{n-1} V_j^*$

*Third step:* Unfold such average point, so obtaining: $\left\{V_j^{\diamondsuit}\right\}_{j=0}^{n-1}$. In the aforementioned example, of $C_n - symmetry$, it consists in maintain $V_0^{\diamondsuit}$, and then we rotate the points $2\pi\frac{j}{n}$ radians.

In this way, we can reach:

$$ST\left(\{V_j\}_{j=0}^{n-1}\right) = \left\{V_j^{\diamondsuit}\right\}_{j=0}^{n-1}$$

Corresponding one-to-one with the points of the precedent shape, but in "more symmetrical" position now.

Therefore, the *SD of a shape* $V$ will be evaluated passing firstly through their Symmetry Transform, and then, computing their respective distance:

$$SD\left(V\right) = m\left(V,\ ST\left(V\right)\right)$$

This measure is invariant under translation and rotation [23].

If the shape V is totally symmetric, then coincides with their symmetric transform, and so, SD is null.

The symmetry has been defined on a sequence of points. Therefore, a subjacent problem of election of a subset of points.

Given a general shape, $O$, it is necessary the transformation which departing from their boundary, $\partial O$, go until a finite sequence of points. This permets to apply the precedent algorithm:

$$Folding \rightarrow Averaging \rightarrow Unfolding$$

Such selection can proceeds in different ways:

We can obtain a polyhedral (ever improved) approximation to $O$. Suppose that the $\partial O$ is a closed planar curve of length $L$.

Then, to introduce (for instance) five points: $\{V_i\}_{i=0}^{4}$, it will be sufficient with to fix an initial point, say $V_0$, and from here, applying a distance equal to $L/5$ over the curve, $V_1$, and so on, until $V_4$.

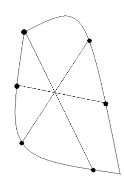
From then, turning out $V_0$ $(see : Fig.\ 2)$ :

$$V_0\,(+L/5) \rightarrow V_1\,(+L/5) \rightarrow V_2\,(+L/5) \rightarrow V_3\,(+L/5) \rightarrow V_4\,(+L/5) \rightarrow V_0$$

The difficulty can appear when the shapes are partially occluded, or perhaps noisy data set.

In such case, it requires a previous process of smoothing.

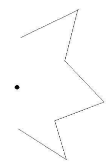
For example, by the equiangular selection (see Fig. 3).



In a very common situation, into the real-world: when the shape is partially occluded, we need to recompose the missing region by supporting in symmetrical features. It is possible to determine a centroid, which by successive approximations, can give us their center of symmetry.

See Fig.4, with a partially occluded shape.

It is definable the symmetry center as the point which minimizes the total of symmetry distances:



$$min \sum SD$$

Their location is possible applying iteratively a procedure of hill-climbing: the gradient descent method. For this, we depart from the centroid of the shape. The position of each new points would be modeled by a Gaussian distribution, which by standardization can be considered a $N(0, 1)$.

There exists also a valuable method evaluating such probable positions, given a set of measurements. Their theoretical basis will be the *Maximum Likelihood Criterion.* So,we can depart of n ordered points: $\{W_i\}_{i=0}^{n-1}$ , each one of them with locations described by a Gaussian:

$$W_i \sim N(V_i,\ \Lambda_i),\ \forall i = 0, 1, 2, ..., n-1$$

being $V_i$ their expected position and $\Lambda_i$ the covariance matrix.

Finally, the probability distributions of SD values corresponds to a *chi-square* with $(n-1)$ freedom degrees: $\chi^2_{n-1}$. But, as known, this would be approximated by a Gaussian distribution.

**3. Our geometrical model**

For each vertex or node, representing into the graph a random variable, we dispose of the probability distribution value associated with their position. So, each possible situation of the node, into the corresponding slice, must possess a numerical image of the random variable, that jointly with the symmetry distance value until the pattern object, $O$, provides of a pair, describing probabilistically their position and how far is of symmetrical final place. Because we don´t know previously the exact position of each node, into each slide, advancing onto the development structure, but only known the probability distribution of such position: with what non-deterministic value such node goes to fill a place.

It is possible to define a Markovian Decision Process, from this model, as a sequential chain of steps, to carry through such randomized Markov process: where each node only depends of the corresponding vertex, that belongs to shapes into the same or the precedent slice (markovian property).

Such shapes can be supposed:

- polyhedral of $n$ vertices in the first step, $\{V_j\}_{j=0}^{n-1}$
- *n-1* vertices in the second shape, $\{V_j\}_{j=0}^{n-2}$
- and so on, until to reach the triangular shape: $\{V_j\}_{j=0}^{2}$
- the line, $\{V_j\}_{j=0}^{1}$

And finally, the monoatomic world: a point, $W = V_0$.

Every one of such shapes would be included into their corresponding slice.

Furthermore, it is possible to suppose associate with them an asymmetry level decreasing, by applying in each step on its points the algorithm to obtain the Symmetry Transform, before acting to delete the corresponding point:

$$\{V_j\}_{j=0}^{n-1} \to ST\left(\{V_j\}_{j=0}^{n-1}\right) \to \{V_j\}_{j=0}^{n-2} \to ST\left(\{V_j\}_{j=0}^{n-2}\right) \to ...$$

$$... \to ST\left(\{V_j\}_{j=0}^{2}\right) \to ST\left(\{V_j\}_{j=0}^{1}\right) \to ST\left(W = V_0\right) = V_0$$

The elimination order will be given by the natural decreasing order of the indices, according the prefixed order of vertices in the original shape.

We can to take as *Total Expectancy Reward (TER),* for their minimization (instead of maximization) process the previous defined Symmetry Distance (SD) between the succesive shapes.

Also it is possible to introduce a *new Reward function* as inversely proportional to such SD translated in 1:

$$TER = \frac{1}{1 + SD\ (O_i,\ O)}$$

In such case, will be logical to apply the procedure of maximization, without the final problem of discontinuity.

According the observability of system states, we construct a FOMDP (Fully Observable Markovian Decision Process), being described without hidden variables.

Associated with each step of this process, we have the "transition probabilities": in the instant temporal $t$, the system is in the state $S_i$, after to take the action, or decision, $a_i$ :

$$do(X = x_i)$$

When it was in the state $S_{i-1}$.

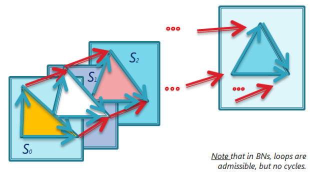
Such probability of transition will be expressed as:

$$P_t\ \left(S_i\ /\ S_{i-1},\ a_i\right)$$

But omitting the typical restriction of Markov Process, we arrive to Bayesian Nets (BNs). These will be expanded to Dynamic Bayesian Nets (DBNs), modelyzing explicitly the time. So, it generalizes many other models, as the HMMs (Hidden Markovian Models).

The subjacent (and basical) idea is the replication of the shapes (therefore, the set of its nodes-vertices, representing random variables), on a sequence of temporal points. Because in our case, the random variables can be the succesive shapes, in the evolution, or its nodes.

Then, we can reach a *Foliation of Bayesian Nets, F* , where each BN belongs to a temporal slice, and so, the total construct will be a Dynamic Bayesian Net:




$$Foliation\ of\ BNs = S\left(T\right) = \cup_{t \in T}\ S\left(t\right)$$

It contains its corresponding slices. So, we can consider each shape immersed in their parallel plate (when we consider the particular case of dimension two), into the global Foliation defined on BNs.

So, will be a *Dynamic Model,* and concretely, a $DBN$, composed by succesive temporal BNs, where the vertices are the nodes in each shape and plate. See for this Fig. 5, which shows their temporal evolution.

But allowing the possibility of existence of arcs among the nodes of different slices: *temporal edges.* Such slices are not necessarily each one connected with the nearest slice, except in the Markovian particular case.

Jointly with another type: the classical *synchronal arcs,* connecting nodes of BNs that belongs to the same slice.

Also we need to comment that such directed edges never will be pointing to the past, because their dynamical character.

We will depart of a model, that is, an idealized representation of reality that highlights some aspects, but ignoring others.

Geometrically, the situation (relative to such symmetric character) should be: a contractive set, or decreasing collection, of subworlds, each one inserted in the precedent, where each one, but the last, shows asymmetries, whereas in the limit, finally, the symmetry appears.

To solve this problem, either we can admit the symmetry as discontinuous function, and so we see without problems that:

$$ASYM \rightarrow ASYM \rightarrow ASYM \rightarrow ... \rightarrow ASYM \rightarrow SYMMETRY$$

Or we may assign a certain value, as a level of symmetry or of asymmetry (complementarity), with a definition suggested by the belonging degree of elements to fuzzy sets; or equivalently, as a level of satisfaction of some condition or property, defined so in the limit it is possible to obtain the state of complete symmetry.

$$A_1 \supseteq A_2 \supseteq A_3 \supseteq\ ...\ \supseteq A_n \supseteq\ ...\ \supseteq A = \{a\}$$

So, for instance: with the contractivity condition taken from the concept of cardinality:

$$c\left(A_1\right) \geq c\left(A_2\right) \geq c\left(A_3\right) \geq\ ...\ \geq c\left(A_n\right) \geq\ ...\ \geq c\left(A\right) = 1$$

Also we can suppose, simplifying, that each world has a cardinal number one less than the precedent world's.

Once classified in decreasing order, reaching some degree of homogeneity among its elements, it is possible to introduce the function "symmetry level" (or asymmetry level, by complementarity). Respectively, denoted $L_s$ and $L_a$.

It is possible to consider each subworld immersed in a different slice. So, we will advance through a progressive and contractive sequence of subworlds. Then, it can appears as a Foliation of BNs as described, so generating a Dynamic Model.

With an increasing sequence of values in a succession, depending on the cardinality of the selected world at each step, until converging to one from the left (as symmetry value, corresponding with the totally symmetrical scenario), in the limit, when we "arrive" to the monoatomic and totally symmetrical world:

$$\{A_n\}_{n \in N} \to A$$

**4. Conclusion**

From this construction it is posssible to introduce a new Normal Fuzzy Measure, named *Asymmetry Level Measure,* denoted $L_a$, or equivalently, the *Symmetry Level Measure,* denoted $L_s$, as we described in detail by our recent paper [5] . Such framework permets the ascension until the only perceived summit: the possible solution of temporal asymmetry problem, in Causality Theory.

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