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An inventory model for deteriorating items with price dependent demand and time-varying holding cost

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Abstract

In this paper a deterministic inventory model is developed when the deterioration rate is time proportional. Demand rate is a function of selling price and holding cost is time dependent. The model is first solved allowing shortage in inventory. The case of no-shortage is discussed next. The results are illustrated with the help of numerical example. The sensitivity of the solution with the changes of the values of the parameters associated with the model is discussed.

Keywords: EOQ model; deteriorating items; shortage; price dependent demand; time varying holding cost.

1. Introduction

Maximum physical goods undergo decay or deterioration over time. Fruits, vegetables and food items suffer from depletion by direct spoilage while stored. Highly volatile liquids such as gasoline, alcohol and turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain, etc. deteriorate through a gradual loss of potential or utility with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature and inventory researchers felt the necessity to use this factor into consideration.

[Whitin, 1957] considered the deterioration of the fashion goods at the end of a prescribed shortage period. [Ghare and Schrader, 1963] developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was presented by [Shah and Jaiswal, 1977], [Aggarwal, 1978], [Dave and Patel, 1981]. Inventory models with a time dependent rate of deterioration were considered by [Covert and Philip, 1973], [Philip, 1974], [Mishra, 1975] and [Deb and Chaudhuri, 1986]. Some of the recent work in this field has been done by [Chung and Ting, 1993], [Fujiwara, 1993], [Hariga, 1996], [Hariga and Benkherouf, 1994], [Wee, 1995], [Jalan, et al. 1996], [Su, et al. 1996], [Chakraborty and chaudhuri, 1997], [Giri and Chaudhuri, 1997], [Chakraborty, et al. 1998] and [Jalan and Chaudhuri, 1999].

In classical inventory models the demand rate is assumed to be a constant. In reality demand for physical goods may be time dependent, stock dependent and price dependent. Selling price plays an important role in inventory system. [Burwell, 1997] developed economic lot size model for price-dependent demand under quantity and freight discounts. An inventory system of ameliorating items for price dependent demand rate was considered by [Mondal, et. al 2003]. [You, 2005] developed an inventory model with price and time dependent demand.

In most models, holding cost is known and constant. But holding cost may not always be constant. In generalization of EOQ models, various functions describing holding cost were considered by several researchers like [Naddor, 1966], [Van der Veen, 1967], [Muhlemann and Valtis Spanopoulos, 1980], [Weiss, 1982], and [Goh, 1994].

In this present paper, I have developed a generalized EOQ model for deteriorating items where deterioration rate and holding cost are expressed as linearly increasing functions of time and demand rate is a function of selling price. Shortages are allowed here and are completely backlogged.

2. Assumptions and Notations

The fundamental assumptions of the model are as follows:

(i) The demand tare is a function of selling price.

(ii) Shortages are allowed and are fully backlogged.

(iii) The deterioration rate is time proportional.

(iv) Holding cost h(t) per item per time-unit is time dependent and is assumed as

$h(t) = h + \alpha t$ where $\alpha > 0$, $h > 0$.

(v) Replenishment is instantaneous and lead time is zero.

(vi) T is the length of the cycle.

(vii) The order quantity in one cycle is q.

(viii) A is the cost of placing an order.

(ix) The selling price per unit item is p.

(x) C is the unit cost of an item.

(xi) The inventory holding cost per unit per unit time is h(t).

(xii) C_1 is the shortage cost per unit per unit time.

(xiii) $\theta(t) = \theta t$ is the rate of deterioration, $0 < \theta << 1$.

(xiv) During time t_1, inventory is depleted due to deterioration and demand of the item. At time t_1 the inventory becomes zero and shortages start occurring.

(xv) Selling price p follows an increasing trend, and demand rate possess the negative derivative through out its domain where demand rate is $f(p)=(a-p)>0$

3. Mathematical formulation and solution

Let Q(t) be the inventory level at time t ($0 \le t \le T$). The differential equations for the instantaneous state over (0, T) are given by

$$\frac{dQ(t)}{dt} + \theta t Q(t) = -(a-p), \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dQ(t)}{dt} = -(a-p), \qquad t_1 \le t \le T \tag{2}$$

with condition Q(t)=0 at $t=t_1$

Solving (1) and (2) and neglecting higher powers of θ

$$Q(t) = (a-p)[(t_1-t)+\theta\left(\frac{t_1^3}{6} - \frac{t^3}{3} - \frac{t_1 t^2}{2}\right) + \theta^2\left(\frac{t_1^5}{40} - \frac{t^5}{15} - \frac{t^2 t_1^3}{12} + \frac{t_1 t^4}{8}\right)] \quad 0 \le t \le t_1$$

$$= -(a-p)(t-t_1), \qquad t_1 \le t \le T$$

Now stock loss due to deterioration

$$D = (a-p)\int_0^{t_1} \exp\left(\frac{\theta t^2}{2}\right)dt - (a-p)\int_0^{t_1} dt = (a-p)\left[\frac{\theta t_1^3}{6} + \frac{\theta^2 t_1^5}{40}\right]$$

$$q = D + \int_0^T (a-p)dt$$

$$= (a-p)\left[\frac{\theta t_1^3}{6} + \frac{\theta^2 t_1^5}{40}\right] + (a-p)T \tag{3}$$

Holding cost is

$$H = \int_0^{t_1} (h+\alpha t)\exp\left(-\frac{\theta t^2}{2}\right)\left\{\int_t^{t_1} (a-p)\exp\left(\frac{\theta u^2}{2}\right)du\right\}dt$$

$$= (a-p)\int_0^{t_1} (h+\alpha t)\left(1 - \frac{\theta t^2}{2} + \frac{\theta^2 t^4}{8}\right)\left\{\int_t^{t_1}\left(1 + \frac{\theta u^2}{2} + \frac{\theta^2 u^4}{8}\right)du\right\}dt$$

$$= (a-p)h\left[\frac{t_1^2}{2} - \frac{\theta t_1^4}{12} + \frac{\theta^2 t_1^6}{90}\right] + \alpha(a-p)\left[\frac{t_1^3}{6} - \frac{13\theta t_1^5}{120} - \frac{9\theta^2 t_1^7}{560}\right] \tag{4}$$

Now shortage during the cycle, let S

$$=-\int_{t_1}^{T}[-(a-p)(t-t_1)]dt$$

$$=\frac{1}{2}(a-p)(T-t_1)^2 \qquad (5)$$

Total profit per unit time is given by

$$P(T, t_1, p)=p(a-p)-\frac{1}{T}(A+Cq+H+C_1S) \qquad [\text{from } (3), (4), (5)] \ (6)$$

$$=p(a-p)-\frac{1}{T}[A+C(a-p)\{T+\frac{\theta t_1^3}{6}+\frac{\theta^2 t_1^5}{40}\}+h(a-p)[\frac{t_1^2}{2}-\frac{\theta t_1^4}{12}+\frac{\theta^2 t_1^6}{90}]$$

$$+\alpha(a-p)(\frac{t_1^3}{6}-\frac{13\theta t_1^5}{120}-\frac{9\theta^2 t_1^7}{560})+\frac{C_1}{2}(a-p)(T-t_1)^2]$$

Let $t_1=\beta T$, $0<\beta<1$

Hence I have the profit function

$$P(T, p)=p(a-p)-\frac{1}{T}[A+C(a-p)\{T+\frac{\theta\beta^3T^3}{6}+\frac{\theta^2\beta^5T^5}{40}\}+\frac{C_1}{2}(a-p)T^2(1-\beta)^2$$

$$+h(a-p)(\frac{\beta^2T^2}{2}-\frac{\theta\beta^4T^4}{12}+\frac{\theta^2\beta^6T^6}{90})+\alpha(a-p)(\frac{\beta^3T^3}{6}-\frac{13\theta\beta^5T^5}{120}-\frac{9\theta^2\beta^7T^7}{560})] \qquad (7)$$

Our objective is to maximize the profit function P(T, p).The necessary conditions for maximizing the profit are

$\frac{\partial P(T,p)}{\partial T}=0$ and $\frac{\partial P(T,p)}{\partial p}=0.$

which implies

$$A-h(a-p)\frac{\beta^2T^2}{2}-\frac{1}{3}\alpha(a-p)\beta^3T^3-\frac{1}{2}C_1T^2(a-p)(1-\beta)^2$$

$$+\theta[-\frac{1}{3}\theta C(a-p)\beta^3T^3+\frac{1}{4}\theta h(a-p)\beta^4T^4+\frac{13}{30}\theta\alpha(a-p)\beta^5T^5]$$

$$+\theta^2[-\frac{1}{10}C\beta^5T^4(a-p)-\frac{1}{18}h\theta^2\beta^6T^6h(a-p)+\frac{27}{280}\alpha\theta^2\beta^7T^7(a-p)]=0 \qquad (8)$$

and

$$a-2p+C+\frac{1}{2}hT\beta^2+\frac{1}{6}\alpha\beta^3T^2+\frac{1}{2}C_1T(1-\beta)^2$$

$$+\theta[\frac{1}{6}C\beta^3T^2-\frac{1}{12}\beta^4T^3h-\frac{13}{120}\alpha\beta^5T^4]+\theta^2[\frac{1}{40}\beta^5T^4C+\frac{1}{90}h\beta^6T^5-\frac{9}{560}\beta^7T^6\alpha]=0$$

(9)

The solutions of (8) and (9) will give T^* and p^*. The values of T^* and p^*, so obtained, the optimal value $P^*(T, p)$ of the average net profit is determined by (7) provided they satisfy the sufficient conditions for maximizing P(T, p) are

$$\frac{\partial^2 P(T,p)}{\partial T^2}<0,\ \frac{\partial^2 P(T,p)}{\partial p^2}<0 \tag{10}$$

and

$$\frac{\partial^2 P(T,p)}{\partial T^2}\frac{\partial^2 P(T,p)}{\partial p^2}-(\frac{\partial^2 P(T,p)}{\partial T\partial p})^2>0 \text{ at } p=p^* \text{ and } T=T^*. \tag{11}$$

If the solutions obtained from equations (8) and (9) do not satisfy the sufficient conditions (10) and (11) I may conclude that no feasible solution will be optimal for the set of parameter values taken to solve equations (8) and (9). Such a situation will imply that the parameter values are inconsistent and there is some error in their estimation.

4. Numerical example

Example 1. A = 200, a = 100, C = 20, h(t) = 0.4, $C_1=1.2, \beta=0.95, \alpha=0.1, \theta=0.01$ in appropriate units. Based on these input data, the computer outputs are as follows:

Profit = 1497.5922, T^* = 3.3501 and p^* = 60.5321.

Example2.A = 200, a = 100, C = 20, h(t) = 0.4, $\beta=1, \alpha=0.1, \theta=0.01$ in appropriate units. Based on these input data, the computer outputs are as follows:

Profit = 1419.1151, T^* = 1.1862 and p^* = 60.1530.

5. Sensitivity Analysis

To study the effects of changes of the parameters on the optimal profit derived by proposed method, a sensitivity analysis is performed considering the numerical example given above. Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 20% and 50% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are shown in table1and table 2 for with shortage case and without shortage case respectively.

A careful study of table1 reveals the following

(i) $P^*(T, p)$ is slightly sensitive to changes in the values of parameters θ, C_1, h, A, β, α and it is moderately sensitive to changes in C and highly sensitive to changes in a.

(ii) p is slightly sensitive to changes in the values of parameters θ, C_1, h, A, β, α and it is moderately sensitive to changes in C and highly sensitive to changes in a.

(iii) From table1, it is clearly seen that q is slightly sensitive to changes in the values of parameters C_1, α and it is moderately sensitive to changes in θ, C, h, A and highly sensitive to changes in a, β.

(vi) T and t_1 are insensitive to changes in the values of the parameter C_1 and slightly sensitive to changes in the values of parameters α and it is moderately sensitive to changes in θ, h, A, C and highly sensitive to changes in a, β.

Table1

Parameter	**%change**	**%change in profit**	**%change in p**	**%change in q**	**%change in T**	**%change in t_1**
θ	-50	0.4388	-0.0337	8.5958	9.2537	9.2537
	-20	0.1674	-0.0139	3.0585	3.2835	3.2835
	20	-0.1584	0.0108	-2.7975	-2.9850	-2.9850
	50	-0.3816	0.0273	-6.4479	-6.8656	-6.8656
C_1	-50	0.0066	0.0026	-0.0032	0	0
	-20	0.0026	0.0008	-0.0012	0	0
	20	-0.0026	-0.0008	0.0012	0	0
	50	-0.0066	-0.0020	0.0032	0	0

C	-50	28.5153	-8.3416	19.0680	5.3731	5.3731
	-20	10.9888	-3.3410	6.7450	1.4925	1.4925
	20	-10.4351	3.3413	-6.2942	-1.1940	-1.1940
	50	-25.0524	8.3584	-14.9668	-2.3880	-2.3880
h	-50	0.8127	-0.1651	8.3181	7.7611	7.7611
	-20	0.3176	-0.0635	3.1871	2.9850	2.9850
	20	-0.3084	0.0584	-3.1647	-2.9850	-2.9850
	50	-0.7551	0.1453	-7.2731	-6.8656	-6.8656
A	-50	2.2616	-0.2807	-24.3756	-24.1791	-24.1791
	-20	0.8334	-0.0526	-8.7550	-8.6567	-8.6567
	20	-0.7695	0.0948	7.5770	7.4626	7.4626
	50	-1.8344	0.2258	17.6262	17.3134	17.3134
a	-50	-89.5072	-40.6025	-45.6898	49.5522	49.5522
	-20	-46.0998	-16.3625	-16.1165	12.2388	12.2388
	20	59.4249	16.4148	14.3266	-8.6567	-8.6567
	50	173.7306	41.0882	33.6455	-17.9104	-7.9104
β	-50	-4.5184	-1.4892	332.2884	314.3283	107.1641
	-20	1.0118	-0.2916	30.0770	29.8507	2.5137
	20	-1.4337	0.1576	-16.3741	-16.4477	0.2626
	50	-4.0231	0.2510	-32.8485	-33.1343	0.2985
α	-50	0.2035	-0.0174	4.0422	3.8805	3.8805
	-20	0.0797	-0.0067	1.5526	1.4925	1.4925
	20	-0.0776	0.0070	-1.4274	-1.3731	-1.3731
	50	-0.1904	0.0135	-3.7123	-3.5820	-3.5820

A careful study of table2 reveals the following

(i) $P^*(T, p)$ is slightly sensitive to changes in the values of parameters θ, h, α and it is moderately sensitive to changes in C and A and highly sensitive to changes in a.

(ii) p is slightly sensitive to changes in the values of parameters θ, h, A, α and it is moderately sensitive to changes in C and highly sensitive to changes in a.

(iii) From table2, it is clearly seen that q is slightly sensitive to changes in the values of parameters θ, h, α and it is moderately sensitive to changes in C and A and highly sensitive to changes in a.

(iv) T is slightly sensitive to changes in the values of the parameters θ, h, α , and it is moderately sensitive to changes in A, C and a.

Table2

Parameter	%change	%change in profit	%change in p	%change in q	%change in T
θ	-50	0.0834	-0.0187	0.0798	0.1686
	-20	0.0352	-0.0074	0.0488	0.0843
	20	-0.0351	0.0074	-0.0490	-0.0843
	50	-0.0831	0.0186	-0.0806	-0.1686
C	-50	30.7038	-8.3097	21.5116	7.9258
	-20	11.7727	-3.3261	7.3347	2.1922
	20	-11.1512	3.3274	-6.5517	-1.6020
	50	-26.7210	8.3208	-15.2717	-3.0860
h	-50	0.3886	-0.0973	0.6559	0.5059
	-20	0.1513	-0.0388	0.2282	0.1686
	20	-0.1599	0.0384	-0.3120	-0.2529
	50	-0.3853	0.0963	-0.6528	-0.5059
A	-50	5.5226	-0.0238	-7.7562	-7.7571
	-20	2.1185	-0.0091	-2.9509	-2.9512
	20	-2.0137	0.0088	2.8331	2.8330
	50	-4.8872	0.0213	6.7459	6.7453
a	-50	-94.4108	-41.4939	-55.1673	20.5228
	-20	-48.6205	-16.6095	-21.6282	4.6374
	20	62.8809	16.6149	21.4014	-2.9510
	50	183.7986	41.5424	53.2575	-5.8178

α	-50	0.0417	-0.0094	0.0989	0.0843
	-20	0.0130	-0.0038	0.0058	0
	20	-0.0221	0.0035	-0.0901	-0.0843
	50	-0.0417	0.0094	-0.0988	-0.0843

6. Conclusion

In the present paper a deterministic inventory model is developed for deteriorating items. The principle features of the model are as follows:

The deterministic demand rate is assumed to be a function of selling price. Selling price is the main criterion of the consumer when he/she goes to the market to buy a particular item.

Shortages are allowed and are completely backlogged in the present model. In many practical situations, stock out is unavoidable due to various uncertainties. There are many situations in which the profit of the stored item is high than its backorder cost. Consideration of shortages is economically desirable in these cases.

The deterioration factor has been taken into consideration in the present model as almost all items undergo either direct spoilage (like fruits, vegetables etc.) or physical decay (in case of radioactive substances, volatile liquids etc.) in the course of time, deterioration is a natural feature in inventory system. There are many items like perfumes, photographic films etc. which incur a gradual loss of potential or quality over time.

The traditional parameters of holding cost is assumed here to be time varying. As the changes in the time value of money and in the price, index, holding cost cannot remain constant over time. It is assumed that the holding cost is linearly increasing functions of time.

I can make a comparative study between the results of the with-shortage case and without-shortage case. In the numerical examples, it is found that the optimum average profit in with-shortage case is 5.53% more than that of the without-shortage case. Hence the model with-shortage is considered to be better economically.

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