Comment on G. Eichfelder (2010): Multiobjective bilevel optimization

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Abstract

Recently, Prof. Dr. Gabriele Eichfelder in her paper (Multiobjective bilevel optimization, Math., Progra., Ser. A (2010), 123: 419-449, doi: 10.1007/s10107-008-0259-0) discussed nonlinear non convex complex bilevel optimization problems using an optimistic approach and presented well structured algorithm for the solution of the problems. However, In general we have no comment over the content of the paper which has sufficient research on the topic. But we raised a question about use of phrase Multiobjective bi-level optimization as main title instead of the phrase bi-level multiobjective optimization. In this paper, we claim and prove that the current main title of the mentioned paper is not correct to the content of the paper and it must be corrected to bi-level multiobjective optimization. The proposed comment will be useful in order to avoid confusion over the problem for future researchers.

Keywords: Bilevel multiobjective optimization, Multi-level programming problem, Bilevel programming problem, Multiobjective programming problem.

1. Introduction

Multi-level programming problems (MLPPs) are complex hierarchical decentralized programming problems with multiple decision makers (DMs) in multi level or hierarchical organizations, where decisions have interacted with each other. Bi-level programming problem (BLPP) is a special case of MLPPs in which only two levels exists namely leader's level (FL-First level) and follower's level (SL-Second level). The basic concept of the bi-level programming technique is that a first level decision maker (FLDM) sets his goals (objectives) and then asks each subordinate level (second level-SLDM) for their optima which are calculated in their isolation; the second level decision maker (SLDM) decisions are submitted and modified by the FLDM with consideration of the overall benefit for the organization. This process continued until a satisfactory solution is reached. A bibliography of references on bi-level linear and non linear programming which is annually updated can be found in Vincent et al. (1994), Dempe et al. (2013), Bialas et al. (1984) etc.

Bi-level programming problem (BLPP) with multiple objectives at each level frequently encountered in any two level hierarchical organizations. Such types of problems in hierarchical organization with conflicting objectives at each level formulate the bi-level multiobjective programming problems (BL-MOPPs).

In literature, most of the developments on BLPP problems focus on bi-level linear programming problem (Bialas and Karwan, 1984; Ben-Ayed, 1993; Pyng and Lin, 1996; Abo-Sinna, 2001) and many other for bi-level nonlinear programming and bi-level multiobjective programming etc. (Anandilingam, 1983; Shih and Xia, 2001; Abo-Sinna[1], 2001; Abo-Sinna[2], 2001; Abo-Sinna and Baky, 2003; Abo-Sinna and Baky, 2005; Abo-Sinna and Baky, 2007; Abo-Sinna and Baky, 2010).

Mathematically, if we consider bi-level maximization type bi-level multiobjective programming problem with m objectives at each level as introduced in literature (as by Shih and Xia, 2001; Abo-Sinna and Baky, 2010; Abo-Sinna[1], 2001; Abo-Sinna and Baky, 2003; Baky, 2009; Baky and Abo-Sinna, 2013; Baky et al. 2014 and by many researchers) can be defined as:

$$\underset{\overline{X_1}}{Max}\{Z_{11}, Z_{12}, ..., Z_{1m}\} \qquad \text{(Leader's level)}$$

$$\underset{\overline{X_2}}{Max}\{Z_{21}, Z_{22}, ..., Z_{2m}\} \qquad \text{(Follower's level)}$$

Subject to, $\overline{A_{l1}}\ \overline{X_1} + \overline{A_{l2}}\ \overline{X_2} (\leq, =, \geq) b_l \quad \forall l = 1, 2, ..., p$

and $\overline{X_1} \geq 0, \overline{X_2} \geq 0.$ (1)

Where $Z_{tr}(\overline{X}) \quad \forall t = 1, 2, r = 1, 2, ..., m$ is the *tr*-th objective function at *t*-th level DM.

$\overline{X_1} = \left\{X_1^1, X_1^2, ..., X_1^{N_1}\right\}'$ decision variables under the control of first level DM.

$\overline{X_2} = \left\{X_2^1, X_2^2, ..., X_2^{N_2}\right\}'$ decision variable under the control of second level DM.

Where $'$ denotes transposition, $\overline{A_{lt}}\ l = 1, 2, ..., p,\ t = 1, 2$ are m row vectors, each of dimension $(1 \times N_j)$. $\overline{A_{lt}}\ \overline{X_t},\ t = 1, 2$ is a column vector of dimension $(n \times 1)$. We take $\overline{X} = \overline{X_1} \cup \overline{X_2}$ and $N = N_1 + N_2$. Here one DM is located on each level. Decision vector $\overline{X_t}, t = 1, 2$ is control of *t*- th level DM having N_t number of decision variables.

Keeping in view the above literatures on bi-level multiobjective programming problem and general mathematical definition (1) of the problem, these are defined as bi-level multiobjective programming problems.

Recently G. Eichfelder (2010) published paper titled 'multiobjective bilevel optimization' and discussed the quite similar optimization problems which are defined as bi-level multiobjective optimization problems in literature. If we

analysis the current title of the paper of Eichfelder (2010), it seems the researcher have done work mainly on multiobjective optimization problems which are further classified as bilevel structure. But in her paper, G. Eichfelder (2010) discussed the mainly two level (bi-level) programming problems with multiple objectives at each level. Therefore it is incorrect to mention these problems as multiobjective bilevel optimization problems and consequently to mention the main title of the manuscript as 'multiobjective bilevel optimization'. Hence in order to avoid ambiguity in problem statement, the main title of the paper must be corrected accordingly as 'bi-level multiobjective optimization'.

2. Conclusion

We comment on the main title of the previously published article and propose minor rectification in the title of the manuscript because the current title of the manuscript is misleading and creates confusions in the minds of the young researchers about the problem definition and problem formulatiom etc. The proposed comment will be useful for the development of new methodologies and techniques on bi-level multiobjective programming problems in near future.

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