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Algorithms for Short-Term Production-Planning of Cogeneration Plants

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Abstract

A cogeneration plant, feeding its output water into a district-heating network, may include several types of energy producing units. The most important being the Cogeneration unit, which produces both heat and electricity. Most plants also have a Heat water storage. Finding the optimal production of both heat and electricity and the optimal use of the storage is a challenging mixed integer nonlinear optimization problem. The calculations may be divided into two sub-problems. The unit commitment problem is the problem to determine when a unit should be producing (on) or not (off). To solve the economic dispatch problem is to find the optimal production plan given which units are producing in each time interval. Together the solution of these two problems form the solution of the short-term production-planning problem.

In this thesis a general approach for the mathematical modeling of a cogeneration plant is presented. The model objective function is nonlinear, with nonlinear constraints. Internal plant temperatures, mass flows, storage losses, minimal up and down times and time dependent start-up costs are considered. The demand for heat, the forward temperature from the plant, the return temperature to the plant and the price of electricity are assumed to be known quantities. The net electric power produced is sold for the estimated current market price of electricity.

The unit commitment problem is solved with an algorithm based on Lagrangian relaxation. The dual search direction is computed by the subgradient method and the step length by the Polyak rule II. A heuristic method for the generation of primal feasible solutions is developed.

The economic dispatch problem is solved using a combination of dynamic programming and general-purpose solvers. Algorithms to find starting values for the problem are developed.

The model and the algorithms are implemented in MATLAB, illustrated with numerical examples and analyzed with numerical tests.

Contents

Chapter 1

Introduction

This thesis treats the problem of how to run a cogeneration plant as efficiently as possible. Questions that have to be answered are for instance; if we want to minimize the production cost and the district-heating demand is fulfilled, when shall we turn on and turn off our production units? How much shall a unit produce when it is running? When shall we charge and discharge the Heat water storage? All these questions get their answers when the short-term production-planning problem is solved.

The development of a fast and robust solution algorithm for the short-term production-planning problem is a difficult and time consuming project, of which this thesis can be considered as a first step. The aim of the thesis is to develop a general mathematical model of a cogeneration plant and to develop and implement a working solution algorithm. The idea is that the material presented should form a platform for further development of a fast, robust and practical algorithm for daily use in the control room of a cogeneration plant.

The thesis is outlined as follows. Section 2 presents the most important mathematical theories used and Section 3 gives the technical background of cogeneration for district-heating. In Section 4 the short-term production-planning problem is formulated. A literature review is given in Section 5 and the economic dispatch problem, which appears as a sub-problem, is treated in Section 6. The solution algorithm is developed in Section 7.

In Section 8 we analyze the problem from four different viewpoints. The analysis shows the relevance of using mathematical programming methods for this type of problem. Furthermore, it shows the need for the advanced type of modeling presented in this thesis. Finally, Section 8 gives some concluding remarks and a discussion of further work.

Chapter 2

Mathematical Programming

Very generally, a *mathematical program* is an optimization problem subject to constraints of the form

$$\begin{array}{lll} \min_x f(x) & & \\ \quad s.t. & g_i(x) \leq 0 & i = 1, ..., m \\ & h_i(x) = 0 & i = 1, ..., m' \\ & x \in R^n \times Z^{n'}. & \end{array} \tag{2.1}$$

The vector $x \in R^n \times Z^{n'}$ has continuous components $x_1, ..., x_n$ and integer components $x_{n+1}, ..., x_{n+n'}$ which are the *decision variables* of the problem. The function $f \in R$ is called the *objective function* and the set of conditions $g_i(x) \leq 0$, $i = 1, ..., m$, and $h_i(x) = 0$, $i = 1, ..., m'$, the set of *constraints.* Which method that is best suitable to solve the problem is dependent on its nature. In this section we present the most important methods used in the algorithms developed in this thesis. This will include a discussion of dynamic programming in Section 2.1 and a brief presentation, in Section 2.2, of the Lagrangian relaxation technique. Section 2.3 describes a method for non-smooth optimization.

2.1 Dynamic Programming

Dynamic programming, see for example [Min86] or [Win94], can be applied to many constrained optimization problems with a certain property of being decomposable. The term 'dynamic programming' is derived from the fact that the method was first applied to the optimization of dynamic systems, that is systems which evolve in the course of time and whose evolution can be controlled by decision variables.

The method is to break up a large problem into a series of smaller ones. The characteristics of problems that might be solved are that they can be divided into *stages* with a decision required at each stage. In a dynamic system these stages normally are discrete time intervals of the time horizon considered. Associated with each stage is a number of *states*. The decision chosen at any stage describes how the state at the current stage is transformed into the state at the next stage.

Recursively define the *value function*

$$f_i(v) = \min_u \{g_{uv} + f_{i-1}(u)\}, \tag{2.2}$$

where i is the stage, u is a state in stage $i-1$ and v is a state in stage i. g_{uv} is the cost associated with the decision of moving from u in stage $i-1$ to v in stage i. The technique in dynamic programming is to, stage by stage, compute $f_i(v)$ for every state v using (2.2) recursively. At the first stage begin at a known single state with a zero cost, i.e. $f_0(u) = 0$. After reaching the desired state in the last stage it is easy to back trace the path and hence find the optimal state at every stage.

2.2 Lagrangian Relaxation

Lagrangian relaxation is a decomposition technique used to solve large-scale optimization problems, see for example [Min86] or the classical paper by Marshall Fisher [Fis85]. The method applies preferably to mathematical programs where the objective function and at least some constraints are additive separable, i.e. when (2.1) is on the form

$$\begin{array}{lll} \min\limits_x \sum\limits_{i=1}^{n+n'} f_i(x_i) & & \\ \quad s.t. \quad \sum\limits_{i=1}^{n+n'} g_{i,j}(x_i) \le 0 & j = 1, ..., m & \\ \quad\quad\quad \sum\limits_{i=1}^{n+n'} h_{i,j}(x_i) = 0 & j = 1, ..., m' & \\ \quad\quad\quad x_i \in X_i \subset R^n \times Z^{n'} & i = 1, ..., n+n'. & \end{array} \tag{2.3}$$

The set X_i is given by the constraints only considers the unknown variable x_i. The optimal value of (2.3) will be denoted by f^* and the optimal solution by x^*.

Associate with a constraint $\sum g_{i,j}(x_i) \leq 0$ the real number $\lambda_j \geq 0$, and with a constraint $\sum h_{i,j}(x_i) = 0$ the real number μ_j. The parameters λ_j and μ_j are the so-called *Lagrange multipliers*. Now define the *Lagrange function* associated with problem (2.3) as

$$L(x, \lambda, \mu) = \sum_{i=1}^{n+n'} f_i(x_i) + \sum_{j=1}^{m} \lambda_j \sum_{i=1}^{n+n'} g_{i,j}(x_i) + \sum_{j=1}^{m'} \mu_j \sum_{i=1}^{n+n'} h_{i,j}(x_i). \tag{2.4}$$

Using the Lagrange function, the Lagrangian relaxation of problem (2.3) becomes

$$\begin{aligned} \min_x \; & L(x, \lambda, \mu) = \sum_{i=1}^{n+n'} f_i(x_i) + \sum_{i=1}^{n+n'} \sum_{j=1}^{m} \lambda_j g_{i,j}(x_i) + \sum_{i=1}^{n+n'} \sum_{j=1}^{m'} \mu_j h_{i,j}(x_i) \\ s.t. \; & x_i \in X_i \subset R^n \times Z^{n'} \qquad i = 1, ..., n+n', \end{aligned} \tag{2.5}$$

and since all variable-linking constraints are relaxed, it is possible to separate (2.5) into $n + n'$ independent sub-problems,

$$\begin{aligned} \min_{x_i} \; & f_i(x_i) + \sum_{j=1}^{m} \lambda_j g_{i,j}(x_i) + \sum_{j=1}^{m'} \mu_j h_{i,j}(x_i) \\ s.t. \; & x_i \in X_i \subset R^n \times Z^{n'}. \end{aligned} \tag{2.6}$$

Denote the global optimal value of (2.5) by $\Phi(\lambda, \mu)$. Because the Lagrange multipliers λ_j are non-negative and $\sum g_{i,j}(x_i)$ non-positive, $\Phi(\lambda, \mu)$ will underestimate the optimal value of (2.3) for any $\lambda \geq 0$ and μ, i.e. $f^* \geq \Phi(\lambda, \mu)$. It is therefore natural to define the problem

$$\begin{aligned} \max_{\lambda,\mu} \; & \Phi(\lambda, \mu) \\ s.t. \; & \lambda \geq 0, \end{aligned} \tag{2.7}$$

with the optimal value denoted by Φ^*. In mathematical programming contexts, problem (2.3) is defined as the *primal problem* and problem (2.7) as the *dual problem*. A feasible solution of the primal problem is called *primal feasible solution* and a feasible solution of the dual problem is called *dual feasible solution*.

An important property relating the primal and the dual problem is

$$f(x) \geq f^* \geq \Phi^* \geq \Phi(\lambda, \mu). \tag{2.8}$$

The difference $f^* - \Phi^*$ is called the *duality gap*. For a generally defined function f, a zero duality gap is not guaranteed. In this case the duality gap is indeed important as a measure of how good an obtained feasible solution is.

algorithm primal-dual
Let $\overline{f} = \infty$ and $\overline{\Phi} = -\infty$.
Initiate multipliers λ and μ.
while not convergence
  Solve the relaxed problem.
  Generate a primal feasible solution.
  Update $\overline{f}$ and $\overline{\Phi}$.
  if $\overline{f} - \overline{\Phi} \leq \varepsilon$
    convergence
  endif
  Update multipliers λ and μ.
endwhile

Table 2.1: An algorithm based on Lagrangian relaxation.

The concept of duality makes it possible to derive algorithms based on Lagrangian relaxation. One example of such an algorithm is given in Table 2.1. The parameters $\overline{f}$ and $\overline{\Phi}$ are defined as the best primal feasible solution and the best dual feasible solution found so far. The first step in the algorithm is to set $\overline{f}$ to infinity and $\overline{\Phi}$ to minus infinity. The Lagrange multipliers λ and μ are given initial values, often equal to zero. The solution of the relaxed problem (2.5) is computed solving the $n+n'$ independent sub-problems (2.6). Most often the optimal solution found is not a feasible solution of the primal problem (2.3), and must be adjusted to become feasible. If the computed primal feasible solution is the best so far, $\overline{f}$ is updated. The same applies for $\overline{\Phi}$. The termination criterion $\overline{f} - \overline{\Phi} \leq \varepsilon$ is a direct consequence of relation (2.8), implying that the solution found is not guaranteed to be optimal. The difference between $\overline{f}$ and the optimal value f^*, on the other hand, is guaranteed to be less than ε. If the parameter ε is chosen small, say less than one percent of $\overline{f}$, the solution $\overline{f}$ is in most cases a sufficiently good approximation of f^*. If the termination criterion is not fulfilled, the Lagrange multipliers λ and μ are updated, which is a step in the solution procedure for the dual problem (2.7).

If the algorithm in Table 2.1 shall be of any practical use, the sub-problems (2.6) and the dual problem (2.7) must not be too difficult to solve. The dual problem do has the benign properties:

- The objective function Φ is concave.

- The number of variables in the dual problem, $m+m'$, is often much smaller than the number of variables in the primal problem, $n + n'$.
- Since λ is only restricted to be non-negative and μ is unrestricted, the feasible region in the dual problem is quite simple.

A complicating property is that Φ might be non-smooth. This is the case when some or all variables in the primal problem (2.3) are restricted to be integers. The dual problem is then solved using a method for non-smooth optimization.

2.3 Non-Smooth Optimization

If the objective function f in (2.1) is not continuously differentiable, the problem is called *non-smooth*. When $f(x)$, $x \in R^n$, is non-smooth and convex, the subgradient concept generalizes that of the gradient.

Define a *subgradient* of f at the point $\widetilde{x}$ as a vector $\gamma \in R^n$ that satisfies

$$f(x) \geq f(\widetilde{x}) + \gamma^T(x - \widetilde{x}). \tag{2.9}$$

The set of all subgradients at $\widetilde{x}$ is called the *subdifferential* of f at $\widetilde{x}$ and is denoted by $\partial f(\widetilde{x})$. If f is differentiable at $\widetilde{x}$, the subdifferential at $\widetilde{x}$ reduces to a single element, the gradient of f, i.e. $\partial f(\widetilde{x}) = \{\nabla f(\widetilde{x})\}$.

For the solution of a non-smooth optimization problem, a number of methods are available. Some of these are direct generalizations of methods for smooth optimization. For a survey of non-smooth optimization, see [Min86]. An algorithm frequently used for the special case when f is convex (or concave if maximized), is the subgradient method, which is a direct generalization of the steepest descent algorithm for unconstrained smooth optimization. The algorithm is easily extended to handle constrained problems like (2.7), see Table 2.2. In the algorithm, the unknown is $y^k = (\lambda^k, \mu^k)$, ρ_k the step length and Y the feasible region defined by $\lambda \geq 0$. The first step in the algorithm is to find a starting point y^0. The subgradient $\gamma^k \in \partial\Phi(\lambda^k, \mu^k)$ in the context of Lagrangian relaxation is given as the value of the relaxed constraints at the solution x in (2.5). The expression $\widehat{y} = Proj_Y\,[\widetilde{y}]$ denotes the projection of $\widetilde{y}$ onto the set Y, which in this case is

$$\begin{cases} (\widehat{\lambda}, \widehat{\mu}) = (\widetilde{\lambda}, \widetilde{\mu}) & \text{if } \widetilde{\lambda} \geq 0 \\ (\widehat{\lambda}, \widehat{\mu}) = (0, \widetilde{\mu}) & \text{if } \widetilde{\lambda} < 0. \end{cases} \tag{2.10}$$

algorithm subgradient
Find a starting point y^k, $k = 0$.
while not convergence
  Find a subgradient γ^k.
  $y^{k+1} = Proj_Y\left[y^k + \rho_k \frac{\gamma^k}{\|\gamma^k\|}\right]$.
  if termination criterion satisfied
    convergence
  endif
endwhile

Table 2.2: A subgradient algorithm for a constrained maximization problem.

If the problem considered is a maximization problem, the search step in every iteration is taken in the direction of the subgradient. If the problem is a minimization problem, $y^k + \rho_k \gamma^k / \|\gamma^k\|$ should be replaced by $y^k - \rho_k \gamma^k / \|\gamma^k\|$.

The rate of convergence is highly dependent on the step length ρ_k. A strategy known as the *Polyak rule II* has been proven efficient in practice [Fis85]. It is defined as

$$\rho_k = \xi_k \frac{\Phi^* - \Phi(\lambda^k, \mu^k)}{\|\gamma^k\|}, \qquad 0 < \xi_k \leq 2. \tag{2.11}$$

Since the optimal value Φ^* is unknown, we instead use $\overline{f}$, the best primal feasible solution obtained so far. A common choice is to start the sequence ξ_k with $\xi_k = 2$ and reduce ξ_k with a factor of two whenever $\Phi(\lambda^k, \mu^k)$ has failed to increase in a specified number of iterations.

Chapter 3

Technical Background

In this section the main concepts of district-heating techniques are discussed. The purpose is to give a technical background of the problem stated and solved in this thesis. Most information given is taken from the textbook by Fredriksen and Werner [FW93], which is highly recommended for a more technical and detailed description.

3.1 District-Heating

District-heating is a technique used to supply cities or part of cities with heat from a common heating system, which often has water as distribution medium and heat carrier. These heating systems are supplied with heat from one or more heat production plants. Heat from district-heating systems is normally used for house warming and is therefore occurring frequently in countries with cold winters. The district-heating technique is widely spread in the Nordic countries, Eastern Europe and Russia, but it is also used in North America, Western Europe, China, Korea and Japan. Steam is normally used instead of water as distribution medium in Eastern Europe and Russia.

A district-heating system is usually described using a partition into three subsystems:

- Production plants
- Distribution system
- Substations

In the production plants the supplied raw energy is transformed into heat, i.e. hot water or steam with high pressure and high temperature. The work presented in this thesis considers only this part of the district-heating system. The production plant is described in detail in Section 3.2.

The heat is transferred from the production plants to the distribution system by heating the not so hot water $(40 - 65°C)$ in the return pipe of the system to hot water $(70 - 120°C)$, which is transferred to the forward pipe of the system. The exact forward temperature chosen is mainly dependent on the outdoor temperature. Electrical pumps make the hot water continuously circulate in the distribution system.

A substation is a technical system with the purpose to transfer heat from the distribution system to the consumers. Using heat exchangers the hot water in the distribution system is cooled while simultaneously warming the distribution water in the consumers local system. The heat demand, i.e. the total heat load that always must be supplied by the production plants, is the sum of the consumers different heat demands, e.g. heat for house warming or some industrial process, and the losses in the distribution network. The heat demand is further described in Section 3.3.

The district-heating technique was first introduced in USA in the $1870's$ when systems for selling of steam were built in many cities. In Europe people had to wait until the $1920's$ to see bigger district-heating systems, which were built in Germany. After the Second World War, when Europe was rebuilt, many cities were directly adopted for the district-heating technique used for house warming.

The direct and indirect motives for using a district-heating system are:

- Combined electricity and heat production in cogeneration plants gives a good degree of efficiency.
- Lower relative costs for heat production in bigger plants.
- Cheaper and low quality fuels can be used.
- Possibilities of using local surplus heat from industrial processes.
- Better local environment when using better combustion with high chimneys.
- Better global environment when it is easier and cheaper to implement environment-protecting programs in bigger, central production plants.

Unit	Name
HWC	Heat water central
ELH	Electric heater
HEP	Heat exchange pump
CHP	Cogeneration unit
HWS	Heat water storage

Table 3.1: Production units.

- More flexible energy supplies in bigger plants where the production can be changed, e.g. converted to another type of fuel, in a few years.
- Less space for heat producing units in buildings.

Common arguments against the usage of the district-heating technique are:

- More severe consequences of unscheduled production stops in a larger plant.
- People in common loose their knowledge of heating methods when only a few persons are responsible and supporting the heating system.
- People have less influence on their own supply situation.

3.2 The Production Plant

A production plant may include several production units of different types, see Table 3.1. Together these units shall produce the required heat demand. The combustion unit is the totally dominating unit. These units can either be *Heat water central units*, which are units for heat production only, or *Cogeneration units*, which produces both heat and electric power. The Heat water central is described in detail in Section 3.2.1 and the Cogeneration unit in Section 3.2.2. Two other common types of units are *Heat exchange pumps* and *Electric heaters*, which are presented in Section 3.2.3 and Section 3.2.4, respectively. Normally production plants also include a *Heat water storage,* in which it is possible to store energy, i.e. hot water or steam, for later use. The Heat water storage is described in detail in Section 3.2.5. Other types of production units are *nuclear units*, *geothermal units* and *solar energy units*, but since these are unusual in

Sweden, they are not included in the research presented in this thesis. The presentation of the production units in the next subsections is given as a survey. For a more technical description, see [FW93].

3.2.1 Heat water central

In the Heat water central oil, coal or biofuel is burned to heat the district-heating water. Units fired with coal or biofuel are used as *base load units* and units fired with oil as *peak units*. However, when a plant includes the much more cheap-to-run Cogeneration unit, the Heat water central is only used as a peak unit.

In the context of combustion one differentiates between *hot water boilers* and *steam boilers* dependent on the technology used. The *steam boiler* is more expensive, but has the advantage that it can be complemented with a turbine for electricity production, i.e. converted to a Cogeneration unit. As the electricity price is increasing this becomes more and more interesting since a possibility to make profits at the electricity market arises.

3.2.2 Cogeneration unit

The Cogeneration unit produces both heat and electric power by combustion of oil, coal or biofuel. Comparing with a unit for electricity production only, the Cogeneration unit has a much better degree of efficiency. For a Cogeneration unit the degree of efficiency is about $70 - 95$ percent while it is in the range $25 - 45$ percent for an *Electric power unit*.

The Cogeneration unit is a type of Heat water central complemented with a turbine for electricity production. Heating the internal process water in a boiler, steam is produced. The steam is led via the turbine for electricity production to a heat exchanger. In the heat exchanger the district-heating water is heated and the steam is condensed to water, which is led back to the boiler. The mode of operation is often described using the *alpha value*, defined as the ratio of produced electric power to produced heat power.

Running the Cogeneration unit as described above, it is working as a *back-pressure unit*. Leading part of the steam to a cool condenser instead of the heat exchanger, the unit is working as an *extraction unit*. In this case the unit produces more electric power but at a lower degree of efficiency. If all steam is led to the cool condenser, the Cogeneration unit works as an Electric power unit. Leading all steam past the turbine to the heat exchanger, the Cogeneration unit is in the direct heat mode, i.e. it works as a Heat water central.

3.2.3 Heat exchange pump

With a small supply of electric power the Heat exchange pump transfers energy from a heat source with low temperature to the district-heating water. The heat source is normally sea, drain or industrial waste water. To motivate investments in Heat exchange pumps access to cheap electricity is necessary. This was the situation in Sweden in a period in the $1980's$.

Usually the energy production of the Heat exchange pump is described using the *heat factor*, defined as the ratio of produced heat power to supplied electric power. Normally the heat factor is in the order of three. Since the use of a Heat exchange pump usually is the cheapest way to heat the district-heating water, it is almost always used as a base load. However, one disadvantage is the relatively low output temperature of about $80 - 85°C$.

3.2.4 Electric heater

The Electric heater is using electric power to heat the district-heating water. Producing heat with electricity for distribution in a district-heating system instead of directly warming houses with electric power today seems very non-economical, especially since it is much more expensive to distribute hot water in a district-heating network than electricity in an electric power grid. In the Swedish plants most Electric heaters were incorporated in the period of the $1980's$ when the electricity price was low. The Electric heater is still used as base load in periods with a low electricity price, e.g. at nights, but the main utilization is as a *reserve unit* or as a *peak unit* since it has a short start-up time and is extremely easy to run.

3.2.5 Heat water storage

The Heat water storage is a big steel tank in which it is possible to store hot water or steam for later use. In Sweden it is most common to store hot water. The Heat water storage functions as a reservoir up to a few days. Over a longer time horizon the heat loss becomes too large.

The Heat water storage is used for many purposes. The most important being:

- To increase the part of the heat produced by *base load units*. The storage is discharged during short peaks in the heat demand instead of starting a *peak unit*.

- To smooth the production for the other units, i.e. charge or discharge the storage to avoid fast production changes for the production units.
- To maintain a reserve of district-heating water to be used at unscheduled stops of production units.
- To increase the amount of electric power produced at the best possible degree of efficiency. The Cogeneration unit has its best degree of efficiency when the produced electricity and heat are proportional. If the aim is to produce as much electric power as possible at this degree of efficiency it also implies a high production of heat power. When the heat production exceeds the heat demand the solution is to charge the storage.
- As an expander to keep the pressure at the correct level in the district-heating network.

3.3 The Heat Demand

In every moment the production plants must produce at least the energy needed to meet the total heat demand in the district-heating system. The total heat demand is the sum of all consumers heat loads and the losses in the distribution network. The losses in a common Swedish district-heating system are about $10-30$ percent, dependent on the outdoor temperature and the construction of the specific system.

The heat demand is different for different types of consumers. It is important to know which type is the most dominating, since this category will have the greatest influence of the total demand. In Table 3.2 the distribution over different categories for one year is shown. The most dominating category is the apartment blocks. This category uses the consumed heat mostly for house warming and heating of household water. In 1988, 66 percent of all apartment blocks in Sweden were warmed using district-heating techniques. Ground heating is heat used for melting snow and ice. Deliveries of heat to industries and public buildings are mostly used for warming buildings and offices, but also for industrial processes.

Every category of consumers has its own yearly, weekly and daily consumption pattern. The yearly pattern is for all categories depending on the outdoor temperature; i.e. the heat demand is higher in winter than in summer. The weekly and daily patterns differ between the categories. Offices, shops and industries have a lower heat demand at weekends when the demand at apartment

Category	Delivered heat ($TWh/year$)	Percental distribution (%)
Apartment blocks	19.4	54
Individual houses	2.2	6
Industries	4.0	11
Ground heating	0.1	0.3
Public buildings	6.3	17
Other (offices, shops, etc.)	4.2	12
Total	36.1	100

Table 3.2: Heat deliveries from all Swedish district-heating systems to different categories of consumers in 1988.

blocks and individual houses are higher. That is because people spend more time at home when they are not working. The corresponding is also obvious in the daily pattern. At apartment blocks and individual houses the consumption increases when people are coming home from work about four o'clock in the afternoon. At night, when people are sleeping, the heat demand is low.

To operate a district-heating system in an optimal way it is necessary to have a good prediction of the future heat demand. Given that we know the future weather, i.e. outdoor temperature, wind speed and hours with sunshine, it is possible to compute a very good prediction using the consumers demand curves, i.e. the yearly, weekly and daily patterns. As a consequence, it is the accuracy of the weather forecast which limits the quality of the optimized production schedule.

Chapter 4

Problem Formulation

Production-planning of a district-heating system faces decisions with different time horizons. The long-term production-planning problem considers questions like new investments in production capacity or connections to new areas for the next $5-25$ years.

In the medium-term production-planning problem decisions of fuel contracts and firm electricity contracts over the time horizon $1-5$ years are considered.

The problem of finding the optimal use of a district-heating system over a time period of up to one week is known as the short-term production-planning problem. Often it is partitioned into one problem for every single production plant. Normally the short-term production-planning problem is solved for the next twenty-four hours. That is because of the need of a good prediction of the heat demand, see Section 3.3, which is highly dependent on a good prediction of the future weather, and predictions of the weather for more than the next twenty-four hours are very uncertain. To operate a production plant in an optimal way, i.e. to produce heat satisfying the required heat demand so that the total production cost is minimized, is always important. It is specially important for plants with a high production cost, since lowering the production cost with only a few percent here means that a lot of money is saved, even on daily basis. There are not only economical benefits. An optimal use of fuels also means less pollution and thereby a better environment. In this thesis only the short-term production-planning problem of a single production plant is considered.

For a production plant with only a few production units, and without Heat water storage, it is possible to solve the short-term production-planning problem without any special tool. When the number of production units increases or when the plant includes a Heat water storage the need for a computer program becomes obvious.

The most important factors that complicates the solution procedure are:

- When the number of production units increases it is difficult to decide which units should be on, i.e. produce heat, at every moment. Limiting is the units minimal up and down times. Minimal up time is the time a unit must be on when it has been started and minimal down time is the time a unit must be off when it has been turned off. Associated with a start-up is also a cost that has to be considered. The problem of deciding when the units should be on or off is known as *the unit commitment problem*.

- When the plant includes a Cogeneration unit the problem of deciding the production of electrical power arises. Since produced electric power is normally sold at the electricity market, the aim is to produce as much electric power as possible when the electricity price is high. The Cogeneration unit has its best degree of efficiency when the produced electricity and heat are proportional, i.e. producing more electric power at the best possible degree of efficiency necessarily means a higher production of heat power. This is a problem when the heat production exceeds the heat demand.

- When the plant includes a Heat water storage the possibility to produce heat for later use arises. This gives the problem a strong connection in time and it becomes more difficult to solve, especially when time dependent losses must be considered.

- How the production units are connected. To decide the optimal output temperature for an individual unit is hard, especially when it is connected in series to other units. For most units a high input temperature leads to a lower degree of efficiency. This complication arises e.g. when a Heat exchange pump feeding its output water with relatively high temperature into a Cogeneration unit.

To solve the short-term production-planning problem and master these difficulties, it is necessary to use a combination of mathematical models and computer algorithms. In Section 4.1 the equations describing the different production units are given. The equations developed are based on the mathematical model used in a previous project at Värtan, see [GGB85], [GGB86] and [BGG86]. This mostly linear model is modified to become nonlinear (quadratic) in order to give a better description of the process. In Section 4.2 the equations describing the whole plant are given. The short-term production-planning problem is formulated in Section 4.3 and the special case with fixed temperatures is discussed in Section 4.4.

4.1 Modeling the Production Units

When creating the mathematical model it is natural to partition the time horizon over which the problem is to be solved into a finite number of time intervals. Therefore, define I time intervals in the range $[0, T]$ defined by $I+1$ time points $\tau_0, \tau_1, ..., \tau_I$, where $\tau_0 = 0$ and $\tau_I = T$. The length of the ith time interval is $\Delta\tau_i = \tau_i - \tau_{i-1}$.

For production unit k in time interval i define the heat power production $q_{i,k}$ (in MW), the electric power production $p_{i,k}$ (in MW), the output temperature $t^f_{i,k}$ (in $°C$), the input temperature $t^r_{i,k}$ (in $°C$) and the mass flow rate $m_{i,k}$ (in Mg/s). Let $u_{i,k}$ be a binary variable indicating if unit k in time interval i is on or off, i.e. producing or not producing heat and electric power. If the unit is producing (on) then $u_{i,k}$ is equal to one. If the unit is not producing (off) then $u_{i,k}$ is zero. In every time interval the relation

$$q_{i,k} = c_p m_{i,k}(t^f_{i,k} - t^r_{i,k}) \tag{4.1}$$

holds, together with the upper and lower bounds

$$\underline{t}^f_{i,k} u_{i,k} \leq t^f_{i,k} \leq \overline{t}^f_{i,k} u_{i,k} \tag{4.2}$$

and

$$\underline{m}_{i,k} u_{i,k} \leq m_{i,k} \leq \overline{m}_{i,k} u_{i,k}, \tag{4.3}$$

where c_p is the specific heat capacity of water. The production of electricity, $p_{i,k}$, is always zero for a heat only unit.

Define $T_{i,k}$ to be the running time for unit k at the end of time interval i, i.e. the time since the unit was switched on. $T_{i,k}$ is negative if the unit has been off, otherwise positive. The recursive update of $T_{i,k}$ is defined by

$$T_{i,k} = \begin{cases} T_{i-1,k} + \Delta\tau_i & \text{if } T_{i-1,k} > 0 \text{ and } u_{i,k} = 1 \\ \Delta\tau_i & \text{if } T_{i-1,k} < 0 \text{ and } u_{i,k} = 1 \\ -\Delta\tau_i & \text{if } T_{i-1,k} > 0 \text{ and } u_{i,k} = 0 \\ T_{i-1,k} - \Delta\tau_i & \text{if } T_{i-1,k} < 0 \text{ and } u_{i,k} = 0. \end{cases} \tag{4.4}$$

When unit k in time interval i is switched on, i.e. $T_{i-1,k} < 0$ and $u_{i,k} = 1$, a start-up cost $c_{i,k}^{start}$ must be defined. This cost is dependent on the running time $T_{i-1,k}$ as

$$c_{i,k}^{start} = \gamma_k^1 + \gamma_k^2(1 - e^{\gamma_k^3 T_{i-1,k}}). \tag{4.5}$$

The unit specific parameters γ_k^j are non-negative, i.e. $\gamma_k^j \geq 0, \quad j = 1, 2, 3$, and estimated separately for each unit. Since $T_{i-1,k}$ is by definition negative in the case of start-up, the term $(1 - e^{\gamma_k^3 T_{i-1,k}})$ lies between one and zero. If $|T_{i-1,k}| \gg 0$ the term $(1 - e^{\gamma_k^3 T_{i-1,k}})$ is close to one and the resulting start-up cost is almost $\gamma_k^1 + \gamma_k^2$. This situation is called *cold start*. For a *warm start*, when the term $(1 - e^{\gamma_k^3 T_{i-1,k}})$ is close to zero, the cost is approximately γ_k^1. The start-up cost equation (4.5) is widely used, for example by Magnusson [Mag91].

4.1.1 Heat water central

Define $c_{i,e}$ as the current market price of electricity, c_{fuel} as the fuel cost and c_{fuel}^{tax} as the corresponding fuel tax. Express the production cost $c_{i,k}$ as

$$c_{i,k} = (\frac{q_{i,k}}{\eta_{i,k}}(c_{fuel} + c_{fuel}^{tax}) + p_{i,k}^{help}(c_{i,e} + c_{help}^{tax}))\Delta\tau_i u_{i,k}, \tag{4.6}$$

where $\eta_{i,k}$ is the degree of efficiency and $p_{i,k}^{help}$ is the help power. Help power is electric power used for pumps etc. Normally the tax for used help power c_{help}^{tax} is the electricity tax. Both $\eta_{i,k}$ and $p_{i,k}^{help}$ are approximated using a second-degree polynomial, i.e.

$$\eta_{i,k} = \alpha_k^2(q_{i,k})^2 + \alpha_k^1 q_{i,k} + \alpha_k^0 \tag{4.7}$$

and

$$p_{i,k}^{help} = \beta_k^2(q_{i,k})^2 + \beta_k^1 q_{i,k} + \beta_k^0. \tag{4.8}$$

The six polynomial coefficients α_k^j and β_k^j are estimated at the plant for each individual unit. There are restrictions on the production level for the Heat water central described by the inequality bounds

$$\underline{q}_{i,k} u_{i,k} \leq q_{i,k} \leq \overline{q}_{i,k} u_{i,k}. \tag{4.9}$$

4.1.2 Cogeneration unit

All equations modeling the Cogeneration unit are derived from graphs or equations modeling the Cogeneration unit at Värtan [GGB85]. At this moment,

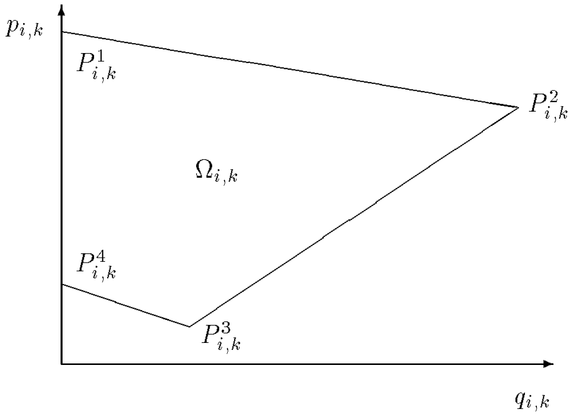

Figure 4.1: The Cogeneration unit heat-electricity constraints.

we can not say if it is necessary to use such an intricate model. We are using it because we believe that a more sophisticated model will give better results. Another motive is that we want to show, from the point of computation time, that it is possible to use an accurate model.

In every time interval the working point $(q_{i,k}, p_{i,k})$ must satisfy

$$(q_{i,k}, p_{i,k}) \in \begin{cases} \Omega_{i,k} & \text{if } u_{i,k} = 1 \\ \{(0,0)\} & \text{if } u_{i,k} = 0. \end{cases} \tag{4.10}$$

The region $\Omega_{i,k}$ is temperature dependent. For a given output temperature $t^f_{i,k}$ and input temperature $t^r_{i,k}$ the region is approximated by the convex hull of four points $P^1_{i,k}$, $P^2_{i,k}$, $P^3_{i,k}$ and $P^4_{i,k}$ in a heat-electricity diagram, see Figure 4.1. The whole region is feasible when the Cogeneration unit is working as an *extraction unit*, see Section 3.2.2. Working as a *backpressure unit* the curve between $P^2_{i,k}$ and $P^3_{i,k}$ acts as an equality constraint. When $t^f_{i,k}$ and $t^r_{i,k}$ are increasing, the Cogeneration unit produces less electric power, i.e. the points $P^2_{i,k}$ and $P^3_{i,k}$ move towards the heat axis (horizontal axis in Figure 4.1). The position of the points $P^1_{i,k}$ and $P^4_{i,k}$ is not dependent on the temperature. Define the weighted temperature average

$$t^m_{i,k} = \frac{v^1_k t^f_{i,k} + v^2_k t^r_{i,k}}{v^1_k + v^2_k}, \tag{4.11}$$

where the weights v_k^1 and v_k^2 are estimated for each individual unit. The two points $P_{i,k}^2$ and $P_{i,k}^3$ are determined using direct measurements using two values for $t_{i,k}^m$, i.e. two values for both $t_{i,k}^f$ and $t_{i,k}^r$. Linear interpolation between the points associated with the two values of $t_{i,k}^m$ gives the current feasible region. In the cool condense mode the working point is positioned at the electricity axis (vertical axis in Figure 4.1) and in the direct heat mode at the heat axis.

The production cost $c_{i,k}$ is the sum of the heat power production cost and the electric power production cost. A complicating issue is to include the taxes. The production cost $c_{i,k}$ is of course zero when the Cogeneration unit is off, i.e. $u_{i,k} = 0$. The procedure to compute $c_{i,k}$ when $u_{i,k} = 1$ is performed as follows:

1. Compute the total fuel cost $c'_{i,k}$ (excluding taxes).
2. Compute the fuel cost for heat power production $c_{i,k}^{heat'}$ (excluding taxes).
3. Use $c'_{i,k}$ and $c_{i,k}^{heat'}$ to compute the electric power production cost $c_{i,k}^{el}$.
4. Compute the income from electricity sold $i_{i,k}^{el}$.
5. Compute the total heat power production cost $c_{i,k}^{heat}$ (including taxes).
6. Finally, compute the total production cost as $c_{i,k} = c_{i,k}^{heat} + c_{i,k}^{el} - i_{i,k}^{el}$.

To compute $c'_{i,k}$ in item one, first determine the total boiler load $L_{i,k}$. The total boiler load is 100 percent at a working point on the line between $P_{i,k}^1$ and $P_{i,k}^2$, i.e. $L_{i,k} = 1.00$. We call this line *the 100 percent load line*. For the working point at the origin the load is zero. To compute the load $L_{i,k}$ for other working points in $\Omega_{i,k}$, use linear interpolation between the origin and the 100 percent load line or its extrapolated part. Approximate the total boiler power $p_{i,k}^{boil}$ using a second order polynomial of the total boiler load as

$$p_{i,k}^{boil} = \sigma_k^2 (L_{i,k})^2 + \sigma_k^1 L_{i,k} + \sigma_k^0. \tag{4.12}$$

The total fuel cost is defined as

$$c'_{i,k} = \frac{p_{i,k}^{boil}}{\eta_{i,k}} c_{fuel} \Delta\tau_i, \tag{4.13}$$

where the degree of efficiency $\eta_{i,k}$ is approximated by a second order polynomial,

$$\eta_{i,k} = \alpha_k^2 (L_{i,k})^2 + \alpha_k^1 L_{i,k} + \alpha_k^0. \tag{4.14}$$

In item two, the fuel cost for heat power production $c_{i,k}^{heat'}$ is computed. First define the heat only load as

$$L_{i,k}^{heat} = \frac{q_{i,k}}{\sigma_k^2 + \sigma_k^1 + \sigma_k^0}. \tag{4.15}$$

Here we use the same parameters σ_k^j, $j = 0, 1, 2$, as in (4.12). The fuel cost for heat power production is then determined by the equation

$$c_{i,k}^{heat'} = (\frac{p_{i,k}^{boil,heat}}{\eta_{i,k}^{heat}} c_{fuel} + p_{i,k}^{help,heat} c_{i,e}) \Delta\tau_i, \tag{4.16}$$

where the boiler power is approximated with

$$p_{i,k}^{boil,heat} = \sigma_k^2 (L_{i,k}^{heat})^2 + \sigma_k^1 L_{i,k}^{heat} + \sigma_k^0. \tag{4.17}$$

As before, second order approximations are used for the degree of efficiency,

$$\eta_{i,k}^{heat} = \alpha_k^2 (L_{i,k}^{heat})^2 + \alpha_k^1 L_{i,k}^{heat} + \alpha_k^0, \tag{4.18}$$

and the help power,

$$p_{i,k}^{help,heat} = \delta_k^2 (L_{i,k}^{heat})^2 + \delta_k^1 L_{i,k}^{heat} + \delta_k^0. \tag{4.19}$$

To compute the electric power production cost $c_{i,k}^{el}$ in item three, we first must compute the help power needed for electricity production $p_{i,k}^{help,el}$. The help power for electricity production depends on the position of the working point in the feasible region $\Omega_{i,k}$. Let $\Psi_{i,k}^{bp}$ be the *backpressure coefficient*. For a point on the line between $P_{i,k}^2$ and $P_{i,k}^3$ (the backpressure line), $\Psi_{i,k}^{bp} = 1$, and on the electric power axis (cool condense), $\Psi_{i,k}^{bp} = 0$. For any combination of heat and electric power, $\Psi_{i,k}^{bp}$ is computed using linear interpolation along a line parallel to the 100 percent load line from the point on the electricity axis to the point on the backpressure line. The expression describing the help power for electricity production is given as

$$p_{i,k}^{help,el} = \beta_k^5 (L_{i,k})^2 + \beta_k^4 L_{i,k} + \beta_k^3 - \Psi_{i,k}^{bp} (\beta_k^2 (L_{i,k})^2 + \beta_k^1 L_{i,k} + \beta_k^0). \tag{4.20}$$

The parameters β_k^j, $j = 0, ..., 5$, are directly determined with linear interpolation using two second-degree polynomials approximating the help power needed at two different values of $\Psi_{i,k}^{bp}$. Normally $\Psi_{i,k}^{bp} = 1$ and $\Psi_{i,k}^{bp} = 0$ are used.

Define the produced net electric power $p_{i,k}^{net}$ as

$$p_{i,k}^{net} = p_{i,k} - p_{i,k}^{help,heat} - p_{i,k}^{help,el}. \tag{4.21}$$

The electric power production cost is then

$$c_{i,k}^{el} = c'_{i,k} - c_{i,k}^{heat'} + p_{i,k}^{help,el} c_{help}^{tax} \Delta\tau_i \tag{4.22}$$

and the corresponding income from selling of electricity, which is computed in item four, is

$$i_{i,k}^{el} = p_{i,k}^{net} c_{i,e} \Delta\tau_i. \tag{4.23}$$

To compute the total heat power production cost $c_{i,k}^{heat}$ in item five, let the specific electricity production cost be

$$\tilde{c}_{i,k,e} = \frac{c_{i,k}^{el}}{p_{i,k}^{net} \Delta\tau_i}. \tag{4.24}$$

The total heat power production cost then becomes

$$c_{i,k}^{heat} = (\frac{p_{i,k}^{boil,heat}}{\eta_{i,k}^{heat}}(c_{fuel} + c_{fuel}^{tax}) + p_{i,k}^{help,heat}(\tilde{c}_{i,k,e} + c_{help}^{tax}))\Delta\tau_i. \tag{4.25}$$

Finally, in item six, the total production cost is computed as the sum of (4.22), (4.23) and (4.25) giving

$$c_{i,k} = c_{i,k}^{heat} + c_{i,k}^{el} - i_{i,k}^{el}. \tag{4.26}$$

Note that in Sweden there is no tax for produced electric power. It is the consumers who pay taxes for used electricity.

4.1.3 Heat exchange pump

Let $t_{i,lake}$ be the heat source temperature, see Section 3.2.3. Define the temperature ratio $\varphi_{i,k}$ as

$$\varphi_{i,k} = \frac{t_{i,k}^f + 273.15}{t_{i,k}^f - t_{i,lake}}. \tag{4.27}$$

The constant term 273.15 is included because normally a Heat exchange pump is described using $t_{i,k}^f$ and $t_{i,lake}$ given in Kelvin. The dependence on temperature for the maximum heat power production is approximated with a linear function of the temperature ratio,

$$\overline{q}_{i,k} = \alpha_k^0 + \alpha_k^1 \varphi_{i,k}. \tag{4.28}$$

As in [GGB86] the *heat factor* $\Phi_{i,k}$ is approximated by

$$\Phi_{i,k} = (\alpha_k^4 + \alpha_k^5 \frac{q_{i,k}}{\overline{q}_{i,k}})\overline{\Phi}_{i,k}, \tag{4.29}$$

where $\overline{\Phi}_{i,k}$ corresponds to an upper bound on the heat factor. This upper bound, like the maximum heat power production, is approximated with a linear function of the temperature ratio,

$$\overline{\Phi}_{i,k} = \alpha_k^2 + \alpha_k^3 \varphi_{i,k}. \tag{4.30}$$

The six parameters α_k^j are estimated at the plant for each individual unit. Using the heat factor, instead of the degree of efficiency, the production cost for the Heat exchange pump is defined as

$$c_{i,k} = \frac{q_{i,k}}{\Phi_{i,k}}(c_{i,e} + c_{hep}^{tax})\Delta\tau_i u_{i,k}. \tag{4.31}$$

There are restrictions on the production level for the Heat exchange pump described as inequality bounds

$$\underline{q}_{i,k} u_{i,k} \leq q_{i,k} \leq \overline{q}_{i,k} u_{i,k}. \tag{4.32}$$

4.1.4 Electric heater

Define the production cost for the Electric heater as

$$c_{i,k} = \frac{q_{i,k}}{\eta_{i,k}}(c_{i,e} + c_{elh}^{tax})\Delta\tau_i u_{i,k}, \tag{4.33}$$

where the degree of efficiency $\eta_{i,k}$ is approximated by a second-degree polynomial

$$\eta_{i,k} = \alpha_k^2 (q_{i,k})^2 + \alpha_k^1 q_{i,k} + \alpha_k^0. \tag{4.34}$$

The polynomial coefficients α_k^j are estimated at the plant for each individual unit. Using electricity to heat, the fuel cost is hence $c_{i,e}$, i.e. the current market price of electricity. The corresponding fuel tax is c_{elh}^{tax}. There are restrictions on the production level for the Electric heater described as inequality bounds

$$\underline{q}_{i,k} u_{i,k} \leq q_{i,k} \leq \overline{q}_{i,k} u_{i,k}. \tag{4.35}$$

4.1.5 Heat water storage

The heat power $q_{i,S}$ from the Heat water storage is by definition positive when discharging. When charging it is negative. No start-up cost is associated with the storage, i.e. $c_{i,S}^{start} = 0$. The heat power contributes to the plant total production cost with a cost $c_{i,S}$. As in [GGB85] costs are computed only when discharging. Let $c_{i,e}$ be the current market price of electricity. Then the cost is defined as

$$c_{i,S} = \begin{cases} 0 & \text{if } q_{i,S} \leq 0 \\ p_{i,S}^{help}(c_{i,e} + c_{help}^{tax})\Delta\tau_i & \text{if } q_{i,S} > 0, \end{cases} \tag{4.36}$$

where the help power $p_{i,S}^{help}$ is modeled using a second order polynomial approximation,

$$p_{i,S}^{help} = \beta_S^2(q_{i,S})^2 + \beta_S^1 q_{i,S} + \beta_S^0. \tag{4.37}$$

The tax for help power, c_{help}^{tax}, is normally the electricity tax.

Define $e_{i,S}$ as the energy content of the Heat water storage at the end of time interval i. Given the storage loss parameters $l_{i,S}$ and $l'_{i,S}$, the energy content $e_{i,S}$ is computed using the recursive equation

$$e_{i,S} = (1 - l_{i,S}\Delta\tau_i)e_{i-1,S} - (q_{i,S} + l'_{i,S})\Delta\tau_i. \tag{4.38}$$

The parameters $l_{i,S}$ and $l'_{i,S}$ may vary over the time horizon. The major reason being that the temperature level in the storage tank may vary over time. Equation (4.38) is a generalization of the storage equation (5.4) used by Ravn and Rygaard [RR94]. There are restrictions of $e_{i,S}$ and $q_{i,S}$ described as the inequality bounds

$$\underline{e}_{i,S} \leq e_{i,S} \leq \overline{e}_{i,S} \tag{4.39}$$

and

$$\underline{q}_{i,S} \leq q_{i,S} \leq \overline{q}_{i,S}. \tag{4.40}$$

4.2 Modeling the Production Plant

To make the model as general as possible, introduce a new type of production unit; the *Dummy unit* (with the abbreviation DUM). The Dummy unit is not producing any heat or electric power. Defining the constraint

$$t_{i,k}^f = t_{i,k}^r \tag{4.41}$$

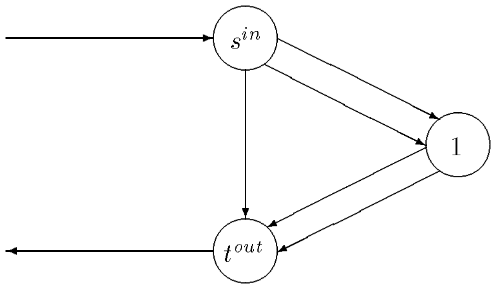

Figure 4.2: A network describing a plant with five units (arcs) and three nodes.

will make the unit fulfill the restriction in (4.1). The production cost for the Dummy unit is equal to zero, i.e.

$$c_{i,k} \equiv 0. \tag{4.42}$$

When modeling a production plant with an arbitrary number of production units in an arbitrary configuration, it is natural to introduce some sort of network description. Define $G = (N, A)$ as the network which describes the unit configuration. The *directed graph* G, see [AMO93], consists of a set N of nodes and a set A of arcs. Every single arc $(s, t) \in A$ from node s to node t corresponds to a production unit. Notice that more than one arc can be connected from node s to node t. The number of nodes depends on the plant specific unit configuration, but there are never less then two, since there always exist one input-node s^{in} and one output-node t^{out}. Figure 4.2 shows a graph describing a plant with five units (arcs) and three nodes.

For every node $n \in N \setminus \{s^{in}, t^{out}\}$ in time interval i define the mass flow equality constraint

$$\sum_{k \in \widetilde{A}(n)} m_{i,k} = \sum_{k \in A(n)} m_{i,k}, \tag{4.43}$$

where $\widetilde{A}(n) = \{k : k \text{ corresponds to } (s, n) \in A \text{ and } s \in N\}$, i.e. k corresponds to an incoming arc of n, and $A(n) = \{k : k \text{ corresponds to } (n, t) \in A \text{ and } t \in N\}$, i.e. k corresponds to an outgoing arc of n. Or equivalently, the mass flow rate into node n equals the mass flow rate out from node n.

Constraints relating the input and output temperatures of a unit are defined in every single node. A unit directly connected to the return pipe of the plant, i.e. a unit corresponding to an outgoing arc of s^{in}, always has an input temperature $t^r_{i,k}$ equal to the plant return temperature $t^r_{i,res}$. The input temperature for a unit not directly connected to the return pipe of the plant is computed as a mixture of output temperatures from units connected into it. This gives

$$\sum_{k \in \tilde{A}(n)} t^f_{i,k} m_{i,k} = \sum_{k \in A(n)} t^r_{i,k} m_{i,k} \tag{4.44}$$

and

$$\begin{cases} t^r_{i,k} = t^r_{i,res} & \text{if } k \in A(s^{in}) \\ t^r_{i,k} = t^r_{i,k'} & \text{if } k \in A(n) \text{ and } k' \in A(n), \end{cases} \tag{4.45}$$

where $n \neq s^{in}$ and $n \neq t^{out}$.

4.3 The Short-Term Production-Planning Problem

We solve the short-term production-planning problem over the time horizon $[0, T]$ partitioned into I time intervals. For every time interval i the following assumptions are made:

- The heat demand $q_{i,D}$ is known.
- The forward temperature demand $t^f_{i,D}$ is known.
- The plant return temperature $t^r_{i,res}$ is known.
- The produced net electric power p^{net}_i is sold at the market electricity price $c_{i,e}$.

How to compute the heat demand $q_{i,D}$ is discussed in Section 3.3. In a similar way, the forward temperature demand $t^f_{i,D}$ and the resulting plant return temperature $t^r_{i,res}$ are computed from a weather forecast. The computation of $q_{i,D}$, $t^f_{i,D}$ and $t^r_{i,res}$ is not considered in the approach presented in this thesis.

The flow of the Heat water storage always comes directly from the return pipe of the plant and goes to the forward pipe of the plant. Using the network description, the storage corresponds to an arc from node s^{in} to node t^{out}. Unlike the other units, it is not possible to control the forward temperature $t^f_{i,S}$ for the Heat water storage. As described in Section 3.2.5, the storage is just a big

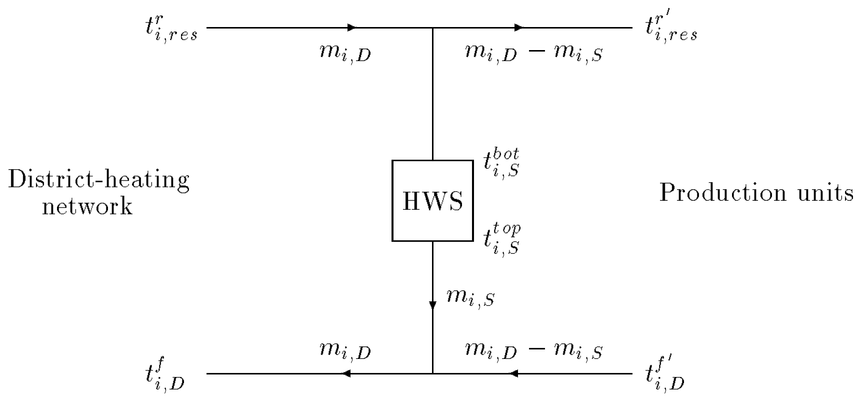

Figure 4.3: The Heat water storage influence on $t^f_{i,D}$ and $t^r_{i,res}$.

tank containing hot water with $t^f_{i,S}$ given as the storage top temperature $t^{top}_{i,S}$. Defining $t^{bot}_{i,S}$ as the storage bottom temperature, the contribution from the Heat water storage may be directly inserted. This gives the corrected temperature demand as

$$t^{f'}_{i,D} = \begin{cases} \frac{t^f_{i,D} m_{i,D} - t^{top}_{i,S} m_{i,S}}{m_{i,D} - m_{i,S}} & \text{if } m_{i,S} \geq 0 \\ t^f_{i,D} & \text{if } m_{i,S} < 0 \end{cases} \tag{4.46}$$

and the corrected plant return temperature as

$$t^{r'}_{i,res} = \begin{cases} t^r_{i,res} & \text{if } m_{i,S} \geq 0 \\ \frac{t^r_{i,res} m_{i,D} - t^{bot}_{i,S} m_{i,S}}{m_{i,D} - m_{i,S}} & \text{if } m_{i,S} < 0. \end{cases} \tag{4.47}$$

Here the mass flow demand $m_{i,D}$ is given by (4.1), i.e. as $m_{i,D} = q_{i,D}/c_p(t^f_{i,D} - t^r_{i,res})$. The constraints (4.46) and (4.47) are illustrated in Figure 4.3. Computing the influence on $t^{top}_{i,S}$ and $t^{bot}_{i,S}$ when charging and discharging the storage, i.e. when $m_{i,S} < 0$ and $m_{i,S} > 0$, is a difficult problem not considered in the research presented in this thesis. In this approach it is assumed that the storage temperature has no influence on the rest of the system, i.e. $t^{f'}_{i,D} = t^f_{i,D}$ and $t^{r'}_{i,res} = t^r_{i,res}$ irrespective of the storage mass flow rate $m_{i,S}$ in (4.46) and (4.47).

The demand constraints that must be fulfilled in time interval i are

$$\sum_{k=1}^{K} q_{i,k} + q_{i,S} \geq q_{i,D} \tag{4.48}$$

and

$$\frac{\sum_{k\in M} m_{i,k} t^f_{i,k}}{\sum_{k\in M} m_{i,k}} \geq t^f_{i,D}, \tag{4.49}$$

where K is the number of production units. The set M is defined as all units directly connected to the forward pipe of the plant, i.e. $M = \widetilde{A}(t^{out})$. The Heat water storage is not included in either the set of K units or the set M.

The decision variables for the production units are the binary variable $u_{i,k}$, the output temperature, the input temperature, the mass flow rate and the heat and electric power production. For the Heat water storage, the heat power production and the energy content are decision variables. Define the decision variable matrix in time interval i for every unit $k = 1, ..., K$ as

$$x_{i,k} = \begin{pmatrix} t^f_{i,k} & t^r_{i,k} & q_{i,k} & p_{i,k} & m_{i,k} \end{pmatrix}^T \tag{4.50}$$

and for the Heat water storage as

$$x_{i,S} = \begin{pmatrix} q_{i,S} & e_{i,S} \end{pmatrix}^T. \tag{4.51}$$

To summarize, define the short-term production-planning problem as the following mathematical program,

$$\begin{array}{ll} \min\limits_{x,u} & \sum\limits_{i=1}^{I}\sum\limits_{k=1}^{K} (c_{i,k}(x_{i,k}, u_{i,k}) + c^{start}_{i,k}(u_k)) + \sum\limits_{i=1}^{I} c_{i,S}(x_{i,S}) \\ s.t. & x_{i,k}, x_{i,S} \in \Omega \\ & u_{i,k} \in \{0,1\} \\ & \text{minimal up and down times.} \end{array} \tag{4.52}$$

The feasible region Ω is defined by the specific unit constraints (4.1), (4.2), (4.3), (4.4), (4.9), (4.10), (4.32), (4.35), (4.38), (4.39), (4.40), (4.41), the configuration constraints (4.43), (4.44), (4.45) and the demand constraints (4.48) and (4.49). The objective function is the total production cost, i.e. the sum of all units start-up costs (4.5) and production costs, defined by either (4.6), (4.26), (4.31), (4.33), (4.36) or (4.42).

4.4 The Special Case with Fixed Temperatures

In the short-term production-planning problem (4.52) the output temperatures from units in parallel are mixed giving a resulting forward temperature. This is formulated in equation (4.44), (4.45) and (4.49). In some production plants there are no possibilities to mix these temperatures. The consequence is that units in parallel must have the same output temperature. If these temperatures are fixed, i.e. given in advance, the problem becomes less complicated.

When the units input temperature $t^r_{i,k}$ and output temperature $t^f_{i,k}$ are given, the input temperature constraints (4.45) are trivially fulfilled. Since all temperatures associated with a single node are equal, these temperatures are eliminated from equation (4.44), implying that constraints (4.43) and (4.44) are equal. Substituting the mass flow rate $m_{i,k}$ from equation (4.1) into constraint (4.43) it becomes

$$\sum_{k\in\tilde{A}(n)} \frac{q_{i,k}}{c_p(t^f_{i,k} - t^r_{i,k})} = \sum_{k\in A(n)} \frac{q_{i,k}}{c_p(t^f_{i,k} - t^r_{i,k})} \tag{4.53}$$

and substituting $m_{i,k}$ into the temperature demand constraint (4.49) it becomes

$$\sum_{k\in M} \frac{q_{i,k}t^f_{i,k}}{c_p(t^f_{i,k} - t^r_{i,k})} \geq \sum_{k\in M} \frac{q_{i,k}t^f_{i,D}}{c_p(t^f_{i,k} - t^r_{i,k})}. \tag{4.54}$$

Notice that both (4.53) and (4.54) are linear in $q_{i,k}$. When the output temperature from units connected directly to the forward pipe of the plant are equal to the forward temperature demand, i.e. when $t^f_{i,k} = t^f_{i,D}$, $k \in M$, (4.54) is trivially fulfilled.

With fixed temperatures the heat power upper bound constraint for the Heat exchange pump (4.28) turns into a simple bound constraint. Also the Cogeneration unit feasible region, see Figure 4.1, gets a nice feature. When the temperatures are given, the region is fixed, i.e. the constraints coupling the heat production $q_{i,k}$ and the electricity production $p_{i,k}$ are all linear. Since there are no additional constraints on $p_{i,k}$, it is possible to eliminate it from the problem. For any feasible heat power production $q_{i,k}$ there is an optimal electricity production $p_{i,k}$, i.e. $p_{i,k}$ could be described as a function of $q_{i,k}$. This implies that the feasible region reduces to simple bounds on $q_{i,k}$. The elimination of $p_{i,k}$, first done by Ravn and Rygaard in [RR94], is not implemented in the approach presented in this thesis.

With these simplifications a production unit is described using only $u_{i,k}$, $q_{i,k}$ and $p_{i,k}$. The decision variables for the Heat water storage are still $q_{i,S}$

and $e_{i,S}$. This implies that the short-term production-planning problem (4.52) for the case when all temperatures are fixed is a mathematical program with nonlinear objective and linear constraints.

Chapter 5

Literature Review

The short-term production-planning problem of a cogeneration plant is a well-known, but difficult mathematical programming problem. In spite of several attempts, no good practical solution has been found. Which solution method is used depends on the mathematical formulation, i.e. if the model used is:

- Deterministic or stochastic
- Continuous or discrete
- Linear or nonlinear
- Convex or non-convex

A linear problem containing both continuous as well as integer variables is called a mixed integer program (MIP). If the problem is nonlinear in both the integer and continuous variables the problem is a mixed integer nonlinear program (MINLP). The short-term production-planning problem (4.52) is a MINLP. Methods that might be applied to the problem formulated as a MIP or MINLP are:

- Heuristic methods
- Dynamic programming
- Branch and bound methods
- Lagrangian relaxation

A closely related problem is the short-term power production-planning problem, which finds the optimal production schedule for the production units in an electric power grid. Normally storing energy is not possible in this case. Since storing energy implies a strong connection in time, this problem is smaller in the sense of time-coupling constraints. However, the common structure gives opportunities of using the same solution strategy.

During the last twenty years, several attempts of finding a practical solution method for the short-term production-planning problem of a cogeneration plant have been made. In the rest of this section, a survey of the most important is presented, grouped according to their basic underlying methodology.

5.1 Dynamic Programming

Hakon Mosbech applied dynamic programming, described in Section 2.1, to the short-term production-planning problem [Mos82]. The plant models studied includes electric power units, heat power units, Cogeneration units and a Heat water storage. The total production of these units shall cover both an electric power and a heat power demand. Each unit is described using a nonlinear cost function and linear constraints. No costs are associated with the storage, and its feasible working points are given by

$$e_{i,S} = e_{i-1,S} - q_{i,S}\Delta\tau_i, \tag{5.1}$$

and the upper and lower bounds of the energy content $e_{i,S}$,

$$\underline{e}_{i,S} \leq e_{i,S} \leq \overline{e}_{i,S}, \tag{5.2}$$

and the heat power $q_{i,S}$,

$$\underline{q}_{i,S} \leq q_{i,S} \leq \overline{q}_{i,S}. \tag{5.3}$$

Time dependent start-up costs are considered but the influences of internal temperatures are neglected.

Solving the problem over a specified time period, the best unit commitment is found by a trial and error method. Given a unit commitment, dynamic programming is used to optimize the economic dispatch. Dynamic programming solves the storage problem in a straightforward way. In every time interval i the energy content $e_{i,S}$ is discretisized, defining the state space. The stages are the time intervals. Using a recursive function corresponding to (2.2), the move from a state (energy level) in stage (time interval) $i-1$ to a state in stage

i is restricted by (5.1), (5.2) and (5.3). Moving from $e_{i-1,S}$ to $e_{i,S}$, the heat power from the storage $q_{i,S}$ is given by (5.1). Then, when the operation of $q_{i,S}$ is known, no time-coupling constraints remain and the economic dispatch for time interval i is separately optimized. The optimal solution to this smaller problem corresponds to g_{uv} in equation (2.2). Using the mathematical model and solution method by Hakon Mosbech, the economic dispatch is optimized in a very natural way, while no satisfactory method for the unit commitment problem is developed.

Hans Ravn and Jens Rygaard goes one step further and augments the state space [RR94]. The mathematical model is similar to Mosbech's, but without any demand of electricity. Produced electric power is instead assumed sold to a predefined electricity price. Moreover, storage losses are considered by replacing (5.1) with

$$e_{i,S} = (1 - l_{i,S})e_{i-1,S} - q_{i,S} - l'_{i,S}, \tag{5.4}$$

where $l_{i,S}$ and $l'_{i,S}$ are storage loss parameters. The length of each time interval is always equal to one, i.e. $\Delta\tau_i = 1$. Defining the number of successive time intervals a unit has been on or off as

$$T_{i,k} = \begin{cases} \min\{\overline{T}_k, T_{i-1,k} + 1\} & \text{if } T_{i-1,k} > 0 \text{ and } u_{i,k} = 1 \\ 1 & \text{if } T_{i-1,k} < 0 \text{ and } u_{i,k} = 1 \\ -1 & \text{if } T_{i-1,k} > 0 \text{ and } u_{i,k} = 0 \\ \max\{\underline{T}_k, T_{i-1,k} - 1\} & \text{if } T_{i-1,k} < 0 \text{ and } u_{i,k} = 0, \end{cases} \tag{5.5}$$

these (discrete) variables, together with the discretisized energy content $e_{i,S}$, are taken as states. Here $u_{i,k}$ is a binary variable indicating if unit k is on, $u_{i,k} = 1$, or off, $u_{i,k} = 0$. $\overline{T}_k$ and $\underline{T}_k$ are the upper and lower limits of $T_{i,k}$, respectively. Using this strategy the best unit commitment also is found in the dynamic programming procedure. One problem is that the state space may be quite large, and therefore the solution procedure will take long time.

Ito et. al., are also using dynamic programming to optimize a model that has a demand of space cooling [IYS91] [IYS92]. The model used is linear and the solution technique is similar to these describe above. A survey of the dynamic programming technique applied to the unit commitment problem without storage possibilities are given in [SPR87] and [HHWS88].

5.2 Branch and Bound

In 1994 Hans Eriksson presented an approach [Eri94] modeling the short-term production-planning problem as a mixed integer linear program, optimized with a branch and bound method using LP-relaxation as a lower bound, see [Win94]. The model covers unit commitment and operation of heat storage devices over a time period of one to several days, accounting for demand of both electricity and heat, ramp-rate constraints, start-up costs and other technical constraints, continuous and logical. Also the intricate structure of the Swedish system for taxes and emission fees is explicitly included. Except for the plant internal temperatures, the model considers most types of costs and limitations needed to describe a production plant. The disadvantage is that the number of discrete variables grows, implying that the branch and bound procedure takes long time. More than 350 discrete and 800 continuous variables are needed when optimizing a normal sized Swedish plant over a time period divided into twenty-four time intervals.

The approach by Seeger and Verstege [SV91] has much in common with the approach by Eriksson. A branch and bound method is also used by von Spakovsky et. al. [vSCB95].

5.3 Lagrangian Relaxation

There has been few attempts to solve the short-term production-planning problem of a cogeneration plant using Lagrangian relaxation, see Section 2.2. In 1987 Mogens Pedersen and Hans Ravn made one [PR87]. They considered a model without storage and with the internal temperatures and flows neglected. The cost model used for each unit was a piecewise continuous function of the heat produced, due to discontinuous changes in the mixture of coal and oil. Also time dependent start-up costs, corresponding to equation (4.5), were considered. Optimizing a system with up to five units, the conclusion was that the Lagrangian relaxation technique is not suitable as a solution method, at least not for the model used.

As a solution method for the closely related problem of an electricity grid, Lagrangian relaxation has been used during the last twenty years. One of the more recent is an approach developed by Torbjörn Magnusson [Mag91], influenced by previous work of Muckstadt and Koenig [MK77], and of Merlin and

Sandrin [MS83]. The problem solved is defined as

$$\begin{aligned}
\min_{p,u} & \sum_{i=1}^{I}\sum_{k=1}^{K} \left(u_{i,k} c_{i,k}(p_{i,k}) + c_{i,k}^{start}(u_k)\right) \\
s.t. \quad & \sum_{k=1}^{K} p_{i,k} = p_{i,D} \\
& \sum_{k=1}^{K} u_{i,k} \min(\overline{p}_k^{inc}, \overline{p}_k - p_{i,k}) \geq p_{i,R} \\
& \underline{p}_k u_{i,k} \leq p_{i,k} \leq \overline{p}_k u_{i,k} \\
& u_{i,k} \in \{0,1\} \\
& \text{minimal up and down times,}
\end{aligned} \tag{5.6}$$

where the production cost $c_{i,k}(p_{i,k})$ is defined as a second order polynomial and the time dependent start-up cost $c_{i,k}^{start}(u_k)$ is given by (4.5). The constraints are the electric power demand constraints $\sum p_{i,k} = p_{i,D}$, the spinning reserve constraints $\sum u_{i,k} \min(\overline{p}_k^{inc}, \overline{p}_k - p_{i,k}) \geq p_{i,R}$, the unit specific constraints $\underline{p}_k u_{i,k} \leq p_{i,k} \leq \overline{p}_k u_{i,k}$ and the minimal up and down times. The binary variable $u_{i,k}$ indicates if the unit is producing (on) or not (off). Since normally no possibility of storing energy is given in an electric power grid, a reserve has to be available to cover unscheduled stops in the production units. This reserve must be available within a very short time, say ten minutes, and therefore only units on-line, whose turbines are already spinning, can contribute. Thereby the name spinning reserve constraint. The parameter $p_{i,R}$ is the required power reserve quantity and $\overline{p}_k^{inc}$ is the amount of power unit k can increase its generation in the specified time. The minimum and maximum electricity production for unit k are $\underline{p}_k$ and $\overline{p}_k$, respectively. The spinning reserve function $\min(\overline{p}_k^{inc}, \overline{p}_k - p_{i,k})$ will henceforth be denoted $r_k(p_{i,k})$.

Problem (5.6) is solved using the procedure based on Lagrangian relaxation described in Section 2.2. The demand constraints are relaxed using the multipliers λ_i, $i = 1, ..., I$, and the spinning reserve constraints using the non-negative multipliers μ_i, $i = 1, ..., I$, giving

$$\begin{aligned}
\min_{p,u} & \sum_{i=1}^{I}\sum_{k=1}^{K} \left(u_{i,k} c_{i,k}(p_{i,k}) + c_{i,k}^{start}(u_k) - \lambda_i p_{i,k} - \mu_i u_{i,k} r_k(p_{i,k})\right) \\
& + \sum_{i=1}^{I} (\lambda_i p_{i,D} + \mu_i p_{i,R}) \\
s.t. \quad & \underline{p}_k u_{i,k} \leq p_{i,k} \leq \overline{p}_k u_{i,k} \\
& u_{i,k} \in \{0,1\} \\
& \text{minimal up and down times.}
\end{aligned} \tag{5.7}$$

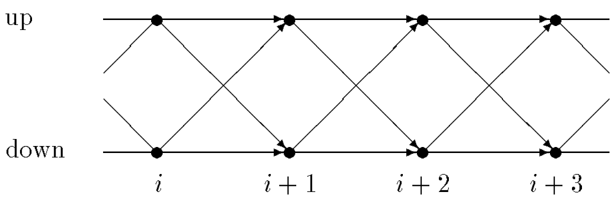

Figure 5.1: Single unit dynamic programming with two states.

It is possible to separate problem (5.7) into one problem for each single unit k in the same way as described in (2.3) - (2.6). This single unit problem becomes

$$\begin{array}{ll} \min\limits_{p_k, u_k} \sum\limits_{i=1}^{I} & (u_{i,k} c_{i,k}(p_{i,k}) + c_{i,k}^{start}(u_k) - \lambda_i p_{i,k} - \mu_i u_{i,k} r_k(p_{i,k})) \\ s.t. & \underline{p}_k u_{i,k} \leq p_{i,k} \leq \overline{p}_k u_{i,k} \\ & u_{i,k} \in \{0, 1\} \\ & \text{minimal up and down times.} \end{array} \tag{5.8}$$

For a successful and rapid solution of problem (5.6), the need for an efficient solver for the unit specific problems (5.8) is necessary. Using the technique first introduced by Muckstadt and Koenig [MK77], Magnusson can manage this difficulty. A natural application of the dynamic programming algorithm for the problem yields a state diagram as in Figure 5.1. The up state corresponds to on-line, i.e. the current unit is producing electric power, and the down state corresponds to off-line, i.e. the current unit is not producing electric power. By introducing additional states corresponding to the up or down time, the minimal up and down time constraints may be incorporated. A state variable $w_{i,k}$ is given the positive value of the up time or the negative value of the down time, whichever applicable. The state space, when the minimal up time is equal to two and the minimal down time is equal to three, is illustrated in Figure 5.2. The recursive function, corresponding to equation (2.2), for the single unit problem reads

$$\begin{aligned} f_{i,k}(w_{i,k}, \lambda, \mu) \quad = \quad & \min_{(u_{i,k}, p_{i,k}) \in Y_k} \{c_{i,k}(p_{i,k}) + c_{i,k}^{start}(u_{i,k}, w_{i,k}) \\ & -\lambda_i p_{i,k} - \mu_i r_k(u_{i,k}) + f_{i+1,k}(u_{i,k}, \lambda, \mu)\}, \end{aligned} \tag{5.9}$$

where $Y_k = (0,0) \cup \left\{(1, p_{i,k}) : \underline{p}_k \leq p_{i,k} \leq \overline{p}_k\right\}$ and the spinning reserve function $r_k(u_{i,k})$ is independent of the generation level. Using this dynamic programming

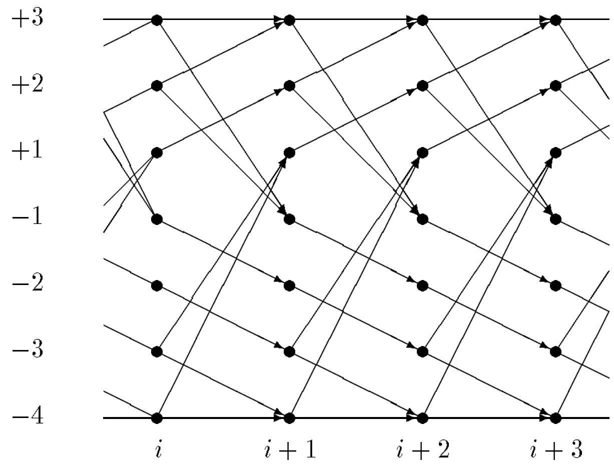

Figure 5.2: Single unit dynamic programming with states corresponding to the up or down time.

procedure, problem (5.8) is solved very efficiently.

The approach by Magnusson is easily augmented to cover a district-heating system by adding a heat demand constraint to (5.6). The objective in (5.6) must be modified to consider units for heat production. Applying Lagrangian relaxation to the problem including a Heat water storage, the problem separates into one problem for each production unit and one problem for the Heat water storage.

An algorithm solving the storage problem is developed by van Amerongen et. al. [BBvA96]. The heat power from the storage $q_{i,S}$ is expressed as the difference between the heat power for discharging $q_{i,S}^{dchr}$ and the heat power for charging $q_{i,S}^{chr}$, i.e. $q_{i,S} = q_{i,S}^{dchr} - q_{i,S}^{chr}$. The upper and lower bounds of $q_{i,S}$ are fulfilled defining the constraints

$$0 \le q_{i,S}^{chr} \le -\underline{q}_{i,S}, \tag{5.10}$$

$$0 \le q_{i,S}^{dchr} \le \overline{q}_{i,S} \tag{5.11}$$

and

$$q_{i,S}^{dchr} \times q_{i,S}^{chr} = 0. \tag{5.12}$$

Equation (5.12) is needed to prevent both $q_{i,S}^{dchr}$ and $q_{i,S}^{chr}$ from being non-zero. The equations describing the energy content $e_{i,S}$ are defined as

$$e_{i,S} = (1 - l_S)e_{i-1,S} - l_S^{dchr} q_{i,S}^{dchr} + l_S^{chr} q_{i,S}^{chr} \tag{5.13}$$

and

$$\underline{e}_{i,S} \leq e_{i,S} \leq \overline{e}_{i,S}, \tag{5.14}$$

where l_S, l_S^{dchr} and l_S^{chr} are parameters modeling storage losses. Using $q_{i,S}^{dchr}$ and $q_{i,S}^{chr}$, the heat demand constraint is expressed as

$$\sum_{k \in K_q \cup K_{qp}} q_{i,k} + q_{i,S}^{dchr} - q_{i,S}^{chr} = q_{i,D}. \tag{5.15}$$

Here K_q is the set of all heat only units and K_{qp} the set of all Cogeneration units in the system. No costs are associated with the storage.

Applying Lagrangian relaxation to problem (5.6) augmented with (5.10) - (5.15), the heat demand constraints (5.15) are relaxed using the multipliers π_i, $i = 1, ..., I$. The problem separates, exactly as in the Magnusson approach, into one problem for each unit. The sub-problem for the Heat water storage becomes

$$\begin{array}{ll} \min\limits_{q_S^{dchr}, q_S^{chr}, e_S} & \sum\limits_{i=1}^{I} \left(\pi_i q_{i,S}^{chr} - \pi_i q_{i,S}^{dchr}\right) \\ s.t. & e_{i,S} = (1 - l_S)e_{i-1,S} - l_S^{dchr} q_{i,S}^{dchr} + l_S^{chr} q_{i,S}^{chr} \\ & 0 \leq q_{i,S}^{chr} \leq -\underline{q}_{i,S} \\ & 0 \leq q_{i,S}^{dchr} \leq \overline{q}_{i,S} \\ & \underline{e}_{i,S} \leq e_{i,S} \leq \overline{e}_{i,S} \\ & q_{i,S}^{dchr} \times q_{i,S}^{chr} = 0. \end{array} \tag{5.16}$$

Without loss of accuracy the nonlinear constraints $q_{i,S}^{dchr} \times q_{i,S}^{chr} = 0$ are dropped. The remaining constraints and objective are linear; i.e. the storage problem (5.16) is a linear program.

The algorithm developed by van Amerongen et. al. solves the linear program (5.16) very efficiently. The solution procedure, based on the Simplex method, is adapted so that the special structure of the problem is exploited. Comparative test results show that a prototype program yields a computational time reduction of about 50 percent compared to the time needed with an advanced general-purpose code.

In [Fel97] Stefan Feltenmark presents an approach solving the short-term power production-planning problem with Lagrangian relaxation. His work is a further development of Magnusson's approach. Feltenmark also solves a cogeneration problem with production of electric power and desalinated water.

5.4 Other Approaches

The equations developed in Section 4 are mainly inspired by the mathematical model used at Värtan, see [GGB85] and [GGB86]. Other approaches have had less influence on the one presented in this thesis. In this section some of these other models and solution methods are shortly commented.

Like Lagrangian relaxation there are other methods separating the problem into a number of smaller sub-problems. In [YI95] an extended Dantzig-Wolfe algorithm is used by Yokoyama and Ito to optimize a mixed integer linear model. The use of Benders decomposition as a solution method for the problem without storage possibilities is presented in [LR]. A decomposition procedure in order to solve the Kuhn-Tucker conditions is given in [RvA94].

The problems discussed so far are all defined as deterministic. In the paper by Palsson and Ravn [PR94], the problem is formulated as a stochastic optimization problem and the solution method used is the Progressive Hedging Algorithm.

Chapter 6

Solving the Economic Dispatch Problem

During the solution procedure of a mixed integer nonlinear program often a series of static problems are solved, i.e. problems where the discrete variables are fixed. In the short-term production-planning problem (4.52) this continuous problem is named *the economic dispatch problem.* Setting the binary variables $u_{i,k}$ to either one or zero, the economic dispatch problem is defined as

$$\begin{aligned} \min_x \ & \sum_{i=1}^{I} \sum_{k=1}^{K} (c_{i,k}(x_{i,k}, u_{i,k}) + c_{i,k}^{start}(u_k)) + \sum_{i=1}^{I} c_{i,S}(x_{i,S}) \\ s.t. \ & x_{i,k}, x_{i,S} \in \Omega, \end{aligned} \tag{6.1}$$

where the feasible region Ω is defined as in (4.52). In this section a numerical solution algorithm for the economic dispatch problem (6.1) is presented. Section 6.1 gives an algorithm based on dynamic programming. The algorithm together with the model developed in Section 4 is implemented in MATLAB. The implementation is described in Section 6.2 and illustrated with a numerical example in Section 6.3. Finally, some computational results are given in Section 6.4.

6.1 A Dynamic Programming Algorithm

The economic dispatch problem (6.1) is solved using an algorithm based on dynamic programming, see Section 2.1. As in the approaches by Mosbech [Mos82] and Ravn et. al. [RR94], the energy content of the Heat water storage is discretisized. During the dynamic programming procedure these discrete levels of the energy content work as states and the time intervals as stages. The move from a state (energy level) in stage (time interval) $i-1$, i.e. from $e_{i-1,S}$, to a state in stage i, i.e. to $e_{i,S}$, is achieved by charging or discharging. When discharging, the heat power from the storage $q_{i,S}$ is positive and when charging $q_{i,S}$ is negative. The exact operation of $q_{i,S}$ is given by (4.38). The possibility to charge and discharge is in every time interval restricted by (4.39) and (4.40).

Given the heat power from the storage, $q_{i,S}$, the economic dispatch problem (6.1) separates into smaller sub-problems which are only dependent on a single time interval. A sub-problem for time interval i is defined as

$$\begin{array}{c} \min\limits_{x_i} \sum\limits_{k=1}^{K} c_{i,k}(x_{i,k}, u_{i,k}) \\ s.t. \quad x_{i,k} \in \Omega_i, \end{array} \tag{6.2}$$

where the feasible region Ω_i is given by the constraints in (6.1) which are relevant in time interval i. Since problem (6.2) is not proved to be convex, there is no guarantee that we will find the global optimum when solving it. In this thesis, a general-purpose solver is used to solve problem (6.2), see Section 6.2.

Solving the economic dispatch problem (6.1), the value function (2.2) is

$$c_i(e_{i,S}) = \min_{e_{i-1,S}} \left\{ \widetilde{c}(e_{i-1,S}, e_{i,S}) + c_{i-1}(e_{i-1,S}) \right\}, \tag{6.3}$$

where i is the stage (time interval) and $e_{i,S}$ is a state (energy level) in stage i. The associated cost $\widetilde{c}(e_{i-1,S}, e_{i,S})$ when moving from $e_{i-1,S}$ to $e_{i,S}$ is defined as the plant operation cost in time interval i. Given $e_{i-1,S}$ and $e_{i,S}$, $\widetilde{c}(e_{i-1,S}, e_{i,S})$ is computed as the charging or discharging cost (4.36) added to the plant production cost corrected for the known storage utilization. The plant production cost corrected for the known storage utilization is the optimal solution of (6.2).

Solving (6.2) for any $q_{i,S}$ satisfying (4.40), the heat demand that has to be met by the production units is in the range $q_{i,D} - \overline{q}_{i,S}$ to $q_{i,D} - \underline{q}_{i,S}$. Moreover, a discretization in $e_{i-1,S}$ and $e_{i,S}$ implies in practice a discretization in $q_{i,S}$, where the discrete levels of $q_{i,S}$ are given by (4.38). When the number of discretisized energy levels $e_{i-1,S}$ and $e_{i,S}$ grows, i.e. when there are many different levels $q_{i,S}$, the number of problems (6.2) to be solved is large. To shorten the total evaluation time, it is in advance possible to restrict the maximum number of problems to be solved. This implies that (6.2) is only solved for a smaller number of levels $q_{i,S}$. To approximate the production cost for the remaining $q_{i,S}$, linear interpolation is used. Finally, when the dynamic programming procedure is completed

algorithm start
Compute the output temperature $t^f_{i,k}$.
Compute the input temperature $t^r_{i,k}$ using $t^f_{i,k}$ and $t^r_{i,res}$.
for $k = 1, ..., K$
  Compute the maximum production cost $\overline{c}_{i,k}$ using $\overline{q}_{i,k}$, $t^f_{i,k}$ and $t^r_{i,k}$.
  Compute the minimum production cost $\underline{c}_{i,k}$ using $\underline{q}_{i,k}$, $t^f_{i,k}$ and $t^r_{i,k}$.
  $q_{i,k} = \underline{q}_{i,k}$
endfor
while $\sum_{k=1}^{K} q_{i,k} < q_{i,D} - q_{i,S}$
  Find unit u so $\frac{\overline{c}_{i,u}-\underline{c}_{i,u}}{\overline{q}_{i,u}-\underline{q}_{i,u}} = \min_{k \in K} \frac{\overline{c}_{i,k}-\underline{c}_{i,k}}{\overline{q}_{i,k}-\underline{q}_{i,k}}$, where $K = \left\{ k : q_{i,k} = \underline{q}_{i,k} \right\}$.
  $q_{i,u} = \overline{q}_{i,u}$
endwhile
Compute the mass flow rate $m_{i,k}$ using $q_{i,k}$, $t^f_{i,k}$ and $t^r_{i,k}$.
for $k = 1, ..., K$
  Find $p_{i,k}$ that minimizes the total production cost using $q_{i,k}$,
    $m_{i,k}$, $t^f_{i,k}$ and $t^r_{i,k}$.
endfor
$x^{start}_{i,k} = \begin{pmatrix} t^f_{i,k} & t^r_{i,k} & q_{i,k} & p_{i,k} & m_{i,k} \end{pmatrix}^T, \quad k = 1, ..., K.$

Table 6.1: Algorithm start.

and the optimal levels $q_{i,S}$ has been determined for every time interval, one problem (6.2) is solved to obtain a solution satisfying all constraints.

Since we have to solve problem (6.2) many times with an iterative algorithm, the need for good starting values is obvious. To ensure convergence in practice and shorten the total evaluation time a starting value algorithm has to be fast and give starting values x^{start}_i fairly close to optimum. We have developed a new algorithm, Algorithm start, which fulfills these demands. A first version was presented in Dotzauer and Holmström [DH97a]. In Table 6.1 Algorithm start is described.

The first step in the algorithm is to approximately compute the output temperature $t^f_{i,k}$ for each unit. The procedure is heuristic and computes the output temperature by linear interpolation in a three-dimensional table. The table contains empirically reasonable $t^f_{i,k}$ for some specified values of the return temperature $t^r_{i,res}$, some specified values of the forward temperature demand $t^f_{i,D}$ and some unit configurations. The output temperature of a Dummy unit is trivially given as the output temperature from units connected into it. The input temperature $t^r_{i,k}$ of each unit is approximately given as the average of the output temperatures from units connected with the current unit. The input temperature for units directly connected to the return pipe of the plant is always $t^r_{i,res}$.

When computing the maximum production cost $\overline{c}_{i,k}$ and the minimum production cost $\underline{c}_{i,k}$ in Algorithm start, evaluate the cost equation (4.6), (4.26), (4.31) or (4.33) for the unit with the heat production set to the maximum heat production $\overline{q}_{i,k}$ and to the minimum heat production $\underline{q}_{i,k}$, respectively. The working point for the Cogeneration unit is assumed to be the point $P^2_{i,k}$, see Figure 4.1, when computing $\overline{c}_{i,k}$. The minimum production cost $\underline{c}_{i,k}$ for the Cogeneration unit is determined by optimizing the production cost $c_{i,k}$ with respect to the electricity production $p_{i,k}$, setting the heat production to the constant value $\underline{q}_{i,k}$.

In the next step, set the heat production $q_{i,k}$ to its maximum value $\overline{q}_{i,k}$ for units with a low relative production cost $(\overline{c}_{i,k} - \underline{c}_{i,k})/(\overline{q}_{i,k} - \underline{q}_{i,k})$ and to $\underline{q}_{i,k}$ for the others. This is done recursively to fulfill the heat demand constraint (4.48). The mass flow rate $m_{i,k}$ for each unit is computed using (4.1). Finally, the electricity production $p_{i,k}$ is set to the value minimizing the total production cost for each unit.

6.2 A MATLAB Implementation

The model for the economic dispatch problem developed in Section 4 and formulated in (6.1) is implemented in MATLAB. Using the dynamic programming technique discussed in Section 6.1 as a solution method, the sub-problem (6.2) is solved several times. The objective in (6.2) is implemented as six MATLAB m-files; one for each type of production unit and one extra routine computing the objective-function. Correspondingly the objective-gradient, the constraint-function and the constraint-gradient are implemented as six m-files each.

The nonlinear programming problem (6.2) is solved using the general-purpose solver NPSOL [GMSW86]. NPSOL is implemented in FORTRAN, and called from MATLAB using a MEX-file interface. The main algorithm is a sequential quadratic programming (SQP) algorithm, see [GMW82]. The MEX-file interface is a part of TOMLAB [Hol97b] and the MATLAB toolbox NLPLIB [Hol97a].

As an alternative to the dynamic programming algorithm developed, the implementation gives opportunities to directly solve the economic dispatch problem (6.1) with NPSOL or the large-scale programming solver MINOS [MS95]. Like NPSOL, MINOS is implemented in FORTRAN, and called from MATLAB using the TOMLAB MEX-file interface. MINOS is using a projected augmented Lagrangian algorithm [GMW82] to solve nonlinear problems.

Algorithm start in Table 6.1 is implemented as two m-files and the dynamic programming algorithm is implemented as six m-files. To summarize, the mathematical model and the solution algorithm for the economic dispatch problem (6.1) are implemented as 32 m-files, in total about 3000 lines of code.

6.3 A Numerical Example

To illustrate the use of the dynamic programming algorithm presented in Section 6.1, we minimize the production cost for a cogeneration plant with five units. This is done for a twenty-four hour period partitioned into twenty-four one-hour time intervals. We define a model with one unit of each type in Table 3.1. The unit parameters are described in Table 6.2. To define the feasible region for the Cogeneration unit shown in Figure 4.1, first define the fixed points $P^1_{i,k} = (0.0, 247.0)$ MW and $P^4_{i,k} = (0.0, 58.6)$ MW. The two temperature dependent points have the values $P^2_{i,k} = (330.0, 216.0)$ MW and $P^3_{i,k} = (95.0, 44.0)$ MW for the temperatures $t^f_{i,k} = 90$ °C and $t^r_{i,k} = 50$ °C and the values $P^2_{i,k} = (324.0, 190.6)$ MW and $P^3_{i,k} = (89.0, 35.4)$ MW for the temperatures $t^f_{i,k} = 120$ °C and $t^r_{i,k} = 80$ °C. Assuming that no start-ups occur during the time horizon, the parameters in equation (4.5) are of no interest.

No cost or loss is associated with the Heat water storage, i.e. $l_{i,S} = 0$ and $l'_{i,S} = 0$ in (4.38) and $\beta^2_S = 0$, $\beta^1_S = 0$ and $\beta^0_S = 0$ in (4.37). At the beginning of the time period the energy content of the storage is $e_{0,S} = 750$ MWh and at the end the same, i.e. $e_{24,S} = 750$ MWh. The storage is discretisized in steps of 10 MWh.

When modeling problem (6.2) one Dummy unit is introduced. The unit configuration is shown in Figure 6.1 and the corresponding directed graph is the one given in Figure 4.2. The sets used in (4.43) and (4.44) are $\widetilde{A}(1) = \{DUM, HEP\}$ and $A(1) = \{CHP, ELH\}$. Constraint (4.43) is in this case $m_{i,dum} + m_{i,hep} = m_{i,chp} + m_{i,elh}$ and constraint (4.44) is $t^f_{i,dum} m_{i,dum} +$

	HWC	ELH	HEP	CHP	HWS		HEP	CHP
α_k^0	0.9451751	0.99	22.8	0.95026	-	α_k^3	0.39	-
α_k^1	$-3.91\cdot10^{-5}$	0	9.57	0.02936	-	α_k^4	0.75	-
α_k^2	$-1.3\cdot10^{-6}$	0	0.94	-0.04	-	α_k^5	0.25	-
β_k^0	0.7810143	-	-	-1.3636	-	β_k^3	-	0.1248
β_k^1	$1.3835\cdot10^{-3}$	-	-	2.68335	-	β_k^4	-	1.19567
β_k^2	$2.25\cdot10^{-5}$	-	-	3.39695	-	β_k^5	-	10.068
$\overline{q}_{i,k}$	115	50	-	-	100	υ_k^1	-	3
$\underline{q}_{i,k}$	40	15	0	-	-100	υ_k^2	-	1
$\overline{t}_{i,k}^f$	120	120	80	120	-	σ_k^0	-	34
$\underline{t}_{i,k}^f$	50	50	50	50	-	σ_k^1	-	518
$\overline{m}_{i,k}$	∞	∞	∞	∞	-	σ_k^2	-	0
$\underline{m}_{i,k}$	0	0	0	0	-	δ_k^0	-	3.35
$\overline{e}_{i,S}$	-	-	-	-	1000	δ_k^1	-	6.25
$\underline{e}_{i,S}$	-	-	-	-	500	δ_k^2	-	0

Table 6.2: Unit parameters ($1 \leq i \leq 24$).

$t_{i,hep}^f m_{i,hep} = t_{i,chp}^r m_{i,chp} + t_{i,elh}^r m_{i,elh}$. Equation (4.45) gives $t_{i,hwc}^r = t_{i,res}^r$, $t_{i,dum}^r = t_{i,res}^r$, $t_{i,hep}^r = t_{i,res}^r$ and $t_{i,chp}^r = t_{i,elh}^r$. The set in (4.49) is in this case $M = \{HWC, CHP, ELH\}$.

The problem is solved using the heat demand, electricity price and temperature values in Table 6.3. The specific heat capacity of water is $c_p = 4.18$ J/(gC°). The specific costs are $c_{fuel} = 90$, $c_{fuel}^{tax} = 70$, $c_{help}^{tax} = 50$, $c_{elh}^{tax} = 60$ and $c_{hep}^{tax} = 50$. All costs are given in SEK/MWh. The optimal use of the Heat water storage is shown in Figure 6.2. The result is not surprising. The solution is to discharge the storage to its lower bound when the electricity price is low and charge it to its upper bound when the price is high. It is easy to see the reason why, looking at the heat and electricity production for the units in Figure 6.3. When the electricity price is high it is optimal to produce as much electric power as possible. Since the Cogeneration unit (CHP) has its best degree of efficiency for a working point at the backpressure line, it has to produce more heat power. When producing more heat than the demand, the solution is to charge the storage. In time intervals 7, 8, 17 and 18, the heat production for the Cogeneration unit is limited by the maximum storage charge capacity $\underline{q}_{i,S}$.

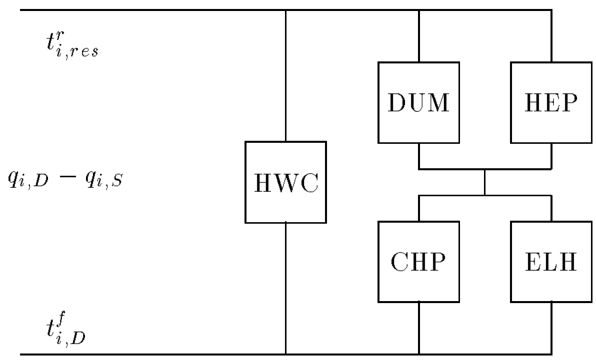

Figure 6.1: The unit configuration.

The Heat exchange pump (HEP) runs at maximum production capacity during the entire time period, see Figure 6.3. The production for the Electric heater (ELH) is at its upper bound when the electricity price is low and at its lower bound when the price is high. It is easy to see that the production for the Heat water central (HWC) should be as low as possible during the whole period. Note the somewhat higher production in time intervals 15 and 16 which is a consequence of the storage discretization.

The output temperatures of interest in our example are those from the Cogeneration unit, the Heat water central and the Electric heater. Mixed together they shall cover the forward temperature demand. The optimal solution has the output temperatures for the Heat water central and the Electric heater at their upper bounds, i.e. $t^f_{i,hwc} = 120$ °C and $t^f_{i,elh} = 120$ °C, $1 \leq i \leq 24$. The output temperature for the Cogeneration unit is varying between 92 °C and 98 °C during the time period. Having this temperature as low as possible implies the possibility to produce more electric power with the same amount of heat running the Cogeneration unit in the backpressure mode.

6.4 Computational Results

In this section we present some results on solving the economic dispatch problem (6.1) using the implementation described in Section 6.2. Four tests are

	unit	$1 \leq i \leq 4$	$5 \leq i \leq 6$	$7 \leq i \leq 8$
$q_{i,D}$	MW	300	350	350
$c_{i,e}$	SEK/MWh	100	100	200
$t^f_{i,D}$	°C	100	100	100
$t^r_{i,res}$	°C	55	55	55
$t_{i,lake}$	°C	7.5	7.5	7.5

	unit	$9 \leq i \leq 16$	$17 \leq i \leq 18$	$19 \leq i \leq 24$
$q_{i,D}$	MW	450	350	350
$c_{i,e}$	SEK/MWh	200	200	100
$t^f_{i,D}$	°C	100	100	100
$t^r_{i,res}$	°C	55	55	55
$t_{i,lake}$	°C	7.5	7.5	7.5

Table 6.3: Heat demand, electricity price and temperature values.

performed:

Test 1. Compares results when using and not using the starting value algorithm in Table 6.1. The test is made on a single time interval, i.e. problem (6.2) is solved.

Test 2. Compares results when and when not optimizing the input and output temperatures for each unit. The test is made on a single time interval, i.e. problem (6.2) is solved.

Test 3. Shows the performance of the dynamic programming part of the algorithm.

Test 4. Shows the behavior of two general-purpose solvers applied to the economic dispatch problem (6.1). The test is made on 12 time intervals.

The tests are made for four different unit configurations called Case A, B, C and D. The parameters for all units are the same for all cases and are the ones used in the numerical example, see Table 6.2. The difference between the test cases is the unit configuration and the number of units assumed to be producing (on). The unit configuration in Case A is identical with the one used in Section 6.3, see Figure 6.1. The total number of units of each type and how these are

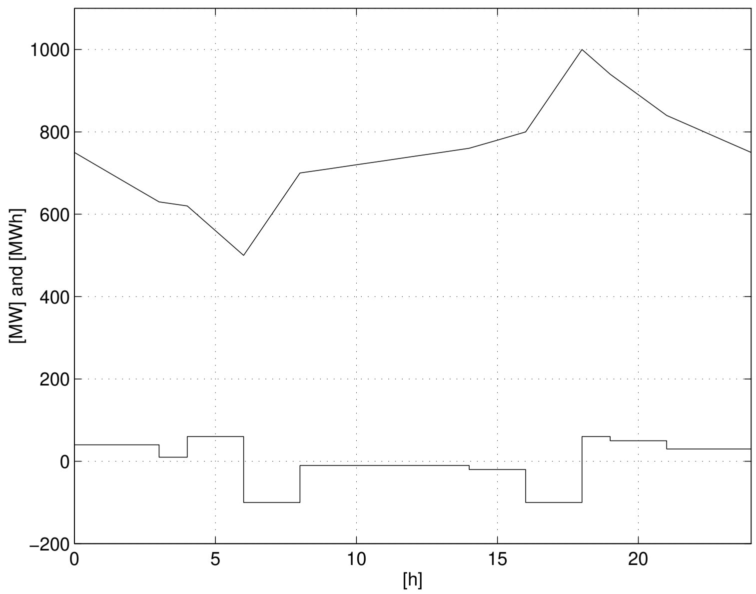

Figure 6.2: Optimal energy content (upper line) and heat power from the storage.

connected in Case B, C and D are given in Table 6.4 - 6.6, respectively. The unit commitment is in all cases given such that all units are producing.

In all test cases the heat demand $q_{i,D}$ is 90 percent of the total maximum heat production. Unless otherwise stated, all other parameters needed are the same as those used in the numerical example. The calculations are made on a PC with Windows 95 and a Pentium 150 MHz processor.

Test 1. When the economic dispatch problem (6.1) is solved using dynamic programming, problem (6.2) appears as a sub-problem. In the first test, problem (6.2) is solved both with and without using our starting value algorithm in Table 6.1. During the test, the electricity price $c_{i,e}$ is 200 SEK/MWh. When problem (6.2) is solved without the starting value algorithm a random and feasible starting point x_{rand} is computed; feasible with respect to the working point interval for each unit. No remarks can be made on the feasibility of the demand constraints (4.48) and (4.49) or the unit configuration constraints (4.43)

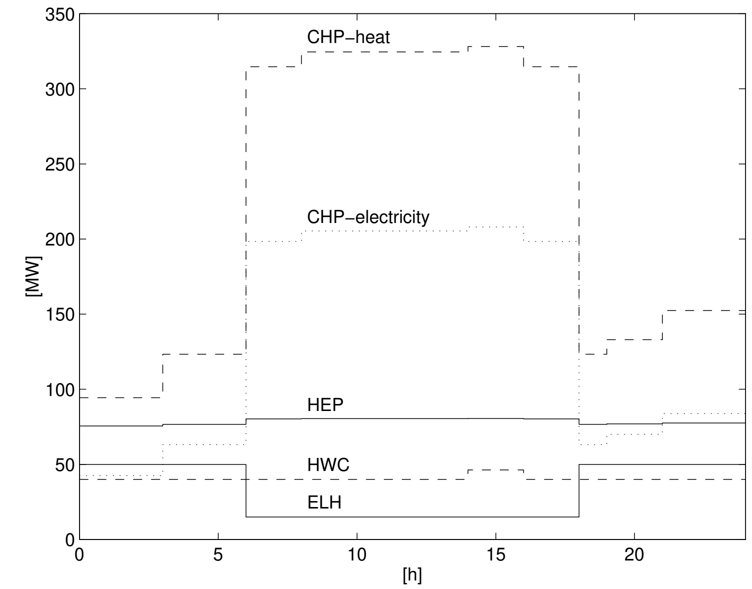

Figure 6.3: The optimal heat and electric power production for all units.

- (4.45). The choice of input and output temperatures at the starting point is given in Table 6.7. The heat production $q_{i,k}$ for a Heat water central and an Electric heater is randomized between $\underline{q}_{i,k}$ and $\overline{q}_{i,k}$. For a Heat exchange pump and a Cogeneration unit it is randomized between $\overline{q}_{i,k}/2$ and $\overline{q}_{i,k}$.

The results when solving with the starting value algorithm are given in Table 6.8 and the results, which are the average of 100 test runs, when solving without the starting algorithm are given in Table 6.9. Here C^* is the optimal production cost in kSEK, $||x^* - x_{start}||$ the distance between the optimal point x^* and the starting point x_{start}, τ_{eval} the computation time in seconds, f_{eval} the number of function evaluations and $kflops$ the number of floating point operations used, displayed in thousands of operations. The number of constraint evaluations is always equal to f_{eval} in our algorithm. In all cases, the 100 test runs using random starting points converge to the same solution as when using the starting value algorithm.

The computation time and the number of function evaluations needed for each random starting point are depicted in Figure 6.4 and Figure 6.5. The

	HWC	ELH	HEP	CHP
From node s^{in} to node t^{out}	2	2	-	1
From node s^{in} to node 1	-	-	2	-
From node 1 to node t^{out}	-	-	-	1

Table 6.4: Unit configuration in Case B.

	HWC	ELH	HEP	CHP
From node s^{in} to node t^{out}	4	4	-	2
From node s^{in} to node 1	-	-	4	-
From node 1 to node t^{out}	-	-	-	2

Table 6.5: Unit configuration in Case C.

solid line shows the result when using the starting value algorithm. In Case A, C and D the starting points given by the starting value algorithm always give a shorter computation time and fewer number of function evaluations. In Case B about 20 percent of the random starting points give better results, but many points give much worse results. From these four cases it is clear that more robust results are obtained using the starting value algorithm. As computation time is a limiting factor, it is necessary to use a starting value algorithm.

Test 2. The purpose of the second test is to examine the behavior of the algorithm when or when not optimizing the input and output temperatures in problem (6.2). Table 6.10 gives the results when optimizing the temperatures. The results are the average of five test runs using the different electricity prices 100, 125, 150, 175 and 200 SEK/MWh. In the next case we fix the input and output temperatures at the optimal temperatures determined in the first case. The results, which are the average of five test runs using the different electricity prices, are given in Table 6.11. The conclusion is that the computation time is about 80 percent shorter when not optimizing the temperatures.

Using dynamic programming to find the optimal use of the Heat water storage implies that problem (6.2) is solved several times in every time interval. If this technique shall be of any practical use, i.e. to be able to solve the economic dispatch problem (6.1) in a reasonable time, the solution procedure must be

	HWC	ELH	HEP	CHP
From node s^{in} to node t^{out}	3	1	-	4

Table 6.6: Unit configuration in Case D.

	Node s^{in}	Node t^{out}	Node 1
$t^f_{i,k}$ and $t^r_{i,k}$	$t^r_{i,res} = 55$	$t^f_{i,D} = 100$	65

Table 6.7: Node temperatures in $°C$.

fast. If we consider a production plant with a large number of units over a large number of time intervals, then it is obvious that the computation time τ_{eval} must be shortened. Possibly a better algorithm to find starting values can solve this problem. Another alternative is to consider optimal solutions from earlier solved problems with similar data. The computation time may also be shortened if the model is implemented in C or FORTRAN. However, in the future faster machines will significantly reduce the computation time. As we can see in Table 6.11, the computation time τ_{eval} is more acceptable when solving problem (6.2) without optimizing the input and output temperatures.

Test 3. The computation time for the dynamic programming part of the algorithm is dependent on the number of stages (time intervals) and on the number of states (discretisized levels of the storage energy content). Here the total number of production units has no influence. The computation times for some different number of stages and states are shown in Figure 6.6. To obtain the solution time for the economic dispatch problem (6.1), one has to add the time needed when solving problem (6.2). In the worst case, problem (6.2) is solved several times at every state in every stage. Since a realistic problem normally has two or more time intervals with similar conditions, the number of times problem (6.2) has to be solved is reduced. For two time intervals with identical conditions the calculation is only done once.

To estimate the worst case solution time for the four test cases, add the time given in Figure 6.6 to the evaluation time τ_{eval} in Table 6.10 multiplied with the number of stages and the square of the number of states. For example, assume that we will solve the economic dispatch problem for Case B expanded with a

	C^*	$\|\|x^* - x_{start}\|\|$	τ_{eval} (sec)	f_{eval}	$kflops$
Case A	66.5605	6.9395	0.66	7	11.1
Case B	134.4284	31.7799	3.46	22	66.0
Case C	268.8568	66.9200	4.78	13	77.6
Case D	196.2431	54.8333	2.09	9	46.7

Table 6.8: Solution using starting values supplied by the starting value algorithm.

	C^*	$\|\|x^* - x_{rand}\|\|$	τ_{eval} (sec)	f_{eval}	$kflops$
Case A	66.5605	130.2445	1.98	22.1	34.9
Case B	134.4284	187.0776	4.55	29.2	87.7
Case C	268.8568	268.3645	13.83	39.2	234.2
Case D	196.2427	262.2338	6.95	30.2	156.7

Table 6.9: Average result using 100 random starting points.

Heat water storage. If we consider 24 time intervals (stages) and we discretisize the storage energy content with 30 levels (states), the estimated worst case solution time is $24 \cdot 30^2 \cdot 3.53 + 38 = 76286$ seconds. Here the value 3.53 is taken from Table 6.10 and the value 38 from Figure 6.6. In this example problem (6.2) is solved $30^2 = 900$ times in each time interval. As discussed in Section 6.1, our approach gives opportunities to restrict in advance the maximum number of problems (6.2) to be solved. Acceptable results are given when the number of problems solved in each time interval is in the range 10 to 20, e.g. if the number of problems to be solved is restricted to 15, the worst case computation time becomes $24 \cdot 15 \cdot 3.53 + 38 = 1308.8$ seconds.

Test 4. The last test considers the direct solution of the economic dispatch problem (6.1) with NPSOL and MINOS. Here the four test cases are expanded to cover 12 one-hour time intervals. The test cases now also includes a Heat water storage, with an end level $e_{12,S}$ chosen to be 750 MWh. In each time interval the electricity price is set to 200 SEK/MWh and the heat demand to 90 percent of the total maximum heat production. All other parameters needed are given in Section 6.3. The initial input and output temperatures are computed

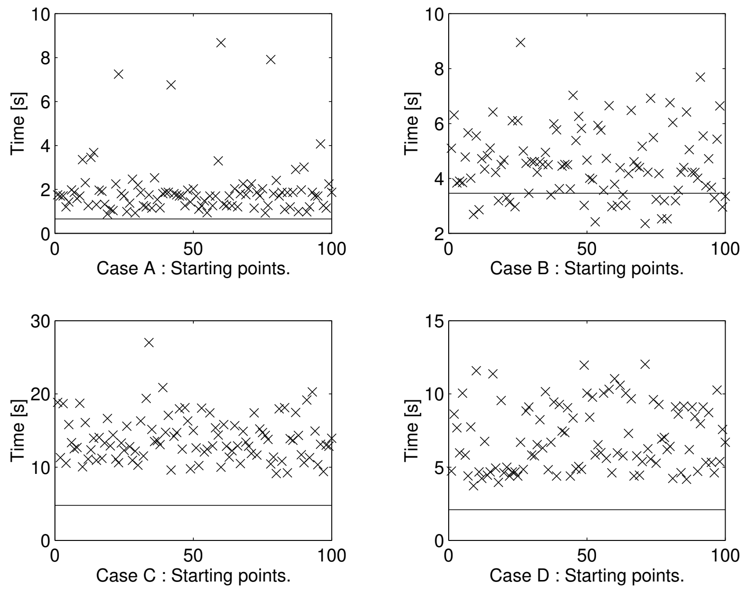

Figure 6.4: Computation time using 100 random starting points. The solid horizontal line shows the time needed using the starting values supplied by the starting value algorithm.

by the starting value algorithm in Table 6.1. As already discussed in Section 4.4, all constraints become linear when the temperatures are given, implying that a phase I LP problem using artificial variables could be solved to generate a feasible starting point [Win94].

Table 6.12 gives the results using NPSOL. Except for the solution time τ_{eval}, which is too long for practical use, NPSOL shows good performance in all cases. When the number of time intervals increases, the solution procedure gets extremely slow. The computation time for our algorithm is less sensitive to the number of time intervals. When we try to solve the economic dispatch problem (6.1) using MINOS, we do not find the optimal solution in any case. It seems that the constraints are too nonlinear in the variables for MINOS to perform well.

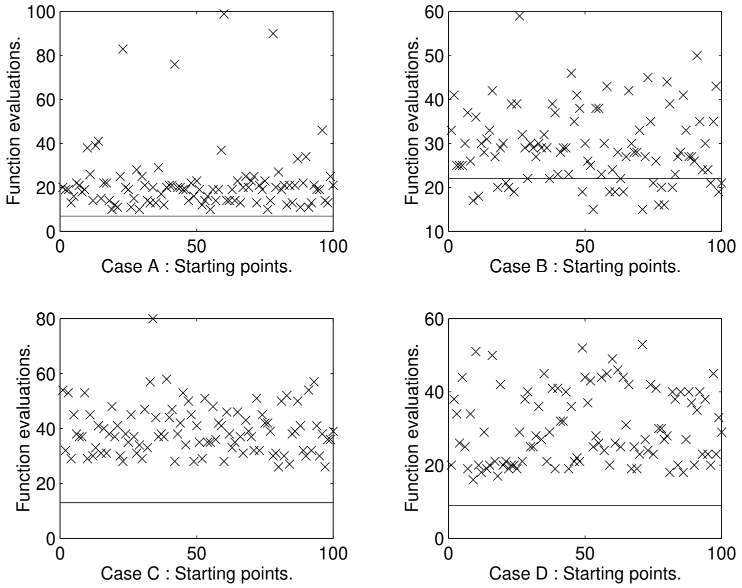

Figure 6.5: Number of function evaluations using 100 random starting points. The solid horizontal line shows the number of function evaluations needed using the starting values supplied by the starting value algorithm.

	C^*	$\|\|x^* - x_{start}\|\|$	τ_{eval} (sec)	f_{eval}	$kflops$
Case A	74.5049	11.3254	0.77	8.0	12.7
Case B	150.3492	52.6056	3.53	21.6	64.8
Case C	300.6984	106.4629	10.74	29.2	174.3
Case D	235.7548	79.5503	3.68	15.4	79.9

Table 6.10: Average result when optimizing the temperatures of each unit.

	C^*	$\|\|x^* - x_{start}\|\|$	τ_{eval} (sec)	f_{eval}	$kflops$
Case A	74.5049	5.7842	0.48	8.6	10.4
Case B	150.3733	11.8530	0.49	5.0	11.7
Case C	300.8024	37.3204	0.93	4.6	21.5
Case D	235.9196	38.8545	0.78	4.6	19.4

Table 6.11: Average result when not optimizing the temperatures of each units.

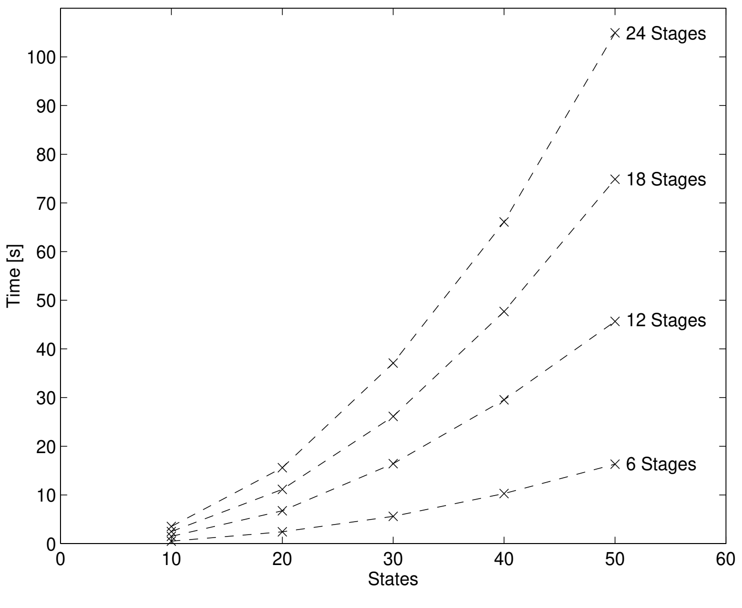

Figure 6.6: The dynamic programming computation time.

	C^*	$\|\|x^* - x_{start}\|\|$	τ_{eval} (min)	f_{eval}	$kflops$
Case A	765.1147	193.7328	2.30	34	669.4
Case B	1545.7792	423.8001	34.13	176	6439.3
Case C	3091.5583	775.5855	310.78	369	26646.9
Case D	2354.9126	813.3175	45.71	176	11071.4

Table 6.12: Solving the economic dispatch problem with 12 time intervals using NPSOL.

Chapter 7

Short-Term Production-Planning

In this section an algorithm solving the short-term production-planning problem (4.52) is developed. A first version was presented in Dotzauer and Holmström [DH97b]. The approach is much influenced by the previous work by Torbjörn Magnusson [Mag91], which solves the closely related problem of an electricity grid. The basic solution strategy is outlined in Section 7.1. In Section 7.2 - 7.6 a detailed description of the different steps in the solution procedure are given. Section 7.7 discusses the implementation. A numerical example is given in Section 7.8 and in Section 7.9 computational results using the algorithm are presented.

7.1 Basic Solution Strategy

The solution procedure is based on Lagrangian relaxation, see Section 2.2. Relaxing all unit-coupling constraints, the short-term production-planning problem decomposes into one separate problem for each single unit. In (4.52) the unit-coupling constraints are the heat demand constraints (4.48), the forward temperature constraints (4.49), the mass flow constraints (4.43) and the temperature constraints (4.44) and (4.45). The exact number of relaxed constraints are dependent on the number of time intervals I, on the number of units K and on the unit configuration, i.e. on the number of nodes $|N|$ in the modeled network, see Section 4.2.

The inequality constraints (4.48) and (4.49) are relaxed using the non-negative multipliers λ_i^1 and λ_i^2, $i = 1, ..., I$. The equality constraints (4.43) and (4.44) are relaxed using the multipliers $\lambda_i^{3,n}$ and $\lambda_i^{4,n}$, $i = 1, ..., I$ and $n = 1, ..., |N| - 2$. They are unrestricted in sign. The reason why $|N|$ is reduced by two in the set describing n is because (4.43) and (4.44) are not defined in the input-node s^{in} or in the output-node t^{out}, see Section 4.2. Finally, the equality constraints (4.45) are relaxed using the unrestricted multipliers $\lambda_i^{5,k}$, $i = 1, ..., I$ and $k = 1, ..., K$.

As an illustrative example, consider a plant giving the directed graph shown in Figure 4.2. The unit configuration is the same as used in the example in Section 6.3, see Figure 6.1. In this case the short-term production-planning problem (4.52) is

$$\begin{array}{rl} \min\limits_{x,u} c(x,u) = & \sum\limits_{i=1}^{I} \sum\limits_{k=1}^{K} (c_{i,k}(x_{i,k}, u_{i,k}) + c_{i,k}^{start}(u_k)) + \sum\limits_{i=1}^{I} c_{i,S}(x_{i,S}) \\ s.t. & q_{i,hwc} + q_{i,dum} + q_{i,hep} + q_{i,chp} + q_{i,elh} + q_{i,S} \geq q_{i,D} \\ & m_{i,hwc} t_{i,hwc}^f + m_{i,chp} t_{i,chp}^f + m_{i,elh} t_{i,elh}^f \\ & \quad \geq m_{i,hwc} t_{i,D}^f + m_{i,chp} t_{i,D}^f + m_{i,elh} t_{i,D}^f \\ & m_{i,dum} + m_{i,hep} = m_{i,chp} + m_{i,elh} \\ & t_{i,dum}^f m_{i,dum} + t_{i,hep}^f m_{i,hep} = t_{i,chp}^r m_{i,chp} + t_{i,elh}^r m_{i,elh} \\ & t_{i,chp}^r = t_{i,elh}^r \\ & x_{i,k}, x_{i,S} \in \Omega' \\ & u_{i,k} \in \{0,1\} \\ & \text{minimal up and down times,} \end{array} \tag{7.1}$$

where all unit-coupling constraints are expressed separately. The feasible region Ω' is defined in the same way as Ω in (4.52), but excluding all unit-coupling

algorithm STPP
Let $\overline{c} = \infty$ and $\overline{\Phi} = -\infty$.
Initiate Lagrange multipliers λ.
while not convergence
  Solve the unit specific problems.
  Generate a primal feasible solution.
  Update $\overline{c}$ and $\overline{\Phi}$.
  if $\overline{c} - \overline{\Phi} \leq \varepsilon$
    convergence
  endif
  Update Lagrange multipliers λ.
endwhile

Table 7.1: The short-term production-planning algorithm STPP.

constraints. The relaxed problem in this example becomes

$$\begin{array}{ll}\min\limits_{x,u} & \sum\limits_{i=1}^{I} \Big(\sum\limits_{k=1}^{K} (c_{i,k}(x_{i,k}, u_{i,k}) + c_{i,k}^{start}(u_k)) + c_{i,S}(x_{i,S}) \\ & +\lambda_i^1 (q_{i,D} - q_{i,hwc} - q_{i,dum} - q_{i,hep} - q_{i,chp} - q_{i,elh} - q_{i,S}) \\ & +\lambda_i^2 (m_{i,hwc} t_{i,D}^f + m_{i,chp} t_{i,D}^f + m_{i,elh} t_{i,D}^f \\ & \quad -m_{i,hwc} t_{i,hwc}^f - m_{i,chp} t_{i,chp}^f - m_{i,elh} t_{i,elh}^f) \\ & +\lambda_i^{3,1} (m_{i,chp} + m_{i,elh} - m_{i,dum} - m_{i,hep}) \\ & +\lambda_i^{4,1} (t_{i,chp}^r m_{i,chp} + t_{i,elh}^r m_{i,elh} - t_{i,dum}^f m_{i,dum} - t_{i,hep}^f m_{i,hep}) \\ & +\lambda_i^{5,chp} (t_{i,chp}^r - t_{i,elh}^r) \Big) \\ s.t. & x_{i,k}, x_{i,S} \in \Omega' \\ & u_{i,k} \in \{0,1\} \\ & \text{minimal up and down times.} \end{array} \tag{7.2}$$

Problem (7.2) decomposes into one problem for each unit.

The complete algorithm for solving the short-term production-planning problem (4.52) is given in Table 7.1. This algorithm, Algorithm STPP, is a direct extension of the primal-dual algorithm in Table 2.1. A condition for the primal-dual algorithm to perform well is that the global optimum for the relaxed problem (2.5) is found in each iteration. Since our relaxed problem (7.2) is not proved to be convex, we can not guarantee that the global optimum will be found. How this affects the algorithm is discussed in Section 7.3.

7.2 Initialization of Lagrange Multipliers

The Lagrange multipliers used to relax the heat demand constraints (4.48), are initially set to the relative production cost when all units are producing at 67 percent of their maximum, i.e.

$$\lambda_i^1 = \frac{\sum\limits_{k=1}^{K} c_{i,k}(\widetilde{q}_{i,k})}{\sum\limits_{k=1}^{K} \widetilde{q}_{i,k}}, \tag{7.3}$$

where $\widetilde{q}_{i,k} = \underline{q}_{i,k} + \frac{2}{3}(\overline{q}_{i,k} - \underline{q}_{i,k})$. The other multipliers, i.e. λ_i^2, $\lambda_i^{3,n}$, $\lambda_i^{4,n}$ and $\lambda_i^{5,k}$, are initially set to zero.

7.3 Solving the Unit Specific Problems

The relaxed problem (7.2) decomposes into one problem for each unit. The unit specific problem for unit k, when k is not the Heat water storage, is

$$\begin{array}{ll} \min\limits_{x_k, u_k} & \sum\limits_{i=1}^{I} \left(c_{i,k}(x_{i,k}, u_{i,k}) + c_{i,k}^{start}(u_k) - \lambda_i^1 q_{i,k} + \lambda_i^2 m_{i,k}(t_{i,D}^f - t_{i,k}^f)\right. \\ & \left. +\lambda_i^{3,n} m_{i,k} - \lambda_i^{3,n'} m_{i,k} + \lambda_i^{4,n} t_{i,k}^r m_{i,k} - \lambda_i^{4,n'} t_{i,k}^f m_{i,k} + \lambda_i^{5,k} t_{i,k}^r\right) \\ s.t. & \text{unit specific constraints} \\ & \text{minimal up and down times.} \end{array} \tag{7.4}$$

Unit k corresponds to an arc connected from node n to node n'. The multiplier λ_i^2 is equal to zero if unit k is not connected directly to the forward pipe of the plant. Given the Lagrange multipliers λ and the binary variable $u_{i,k}$, the optimal production $x_{i,k}$ is computed using a general-purpose solver, see Section 7.7. Moreover, the cost of starting a unit for a given down time, is computed using (4.5). Then the optimal unit commitment, i.e. the optimal values of $u_{i,k}$, can be found using the dynamic programming technique presented in Section 5.3 and illustrated in Figure 5.2.

The unit specific problem for the Heat water storage is

$$
\begin{aligned}
\min_{q_S, e_S} \; & \sum_{i=1}^{I} \left(c_{i,S}(x_{i,S}) - \lambda_i^1 q_{i,S}\right) \\
s.t. \; & e_{i,S} = (1 - l_{i,S}\Delta\tau_i)e_{i-1,S} - (q_{i,S} + l'_{i,S})\Delta\tau_i \\
& \underline{e}_{i,S} \leq e_{i,S} \leq \overline{e}_{i,S} \\
& \underline{q}_{i,S} \leq q_{i,S} \leq \overline{q}_{i,S}.
\end{aligned}
\tag{7.5}
$$

A general-purpose solver is used to solve problem (7.5), see Section 7.7.

Since the relaxed problem (7.2) is not proved to be convex, we can not guarantee that the global optimum will be found when solving it, i.e. when we solve problem (7.4) and problem (7.5). The global optimal value of (7.2) is a lower bound of the optimal value of the short-term production-planning problem (4.52). If the solution obtained when solving (7.2) is not the global optimum, the lower bound of (4.52) might look better than it in fact is. Or equivalently, we might believe that the best primal feasible solution $\overline{c}$ in Algorithm STPP is better than it is. As a consequence, the algorithm might converge to a solution that is not acceptable, but in practice we have not yet found any such case. Another consequence is that the rate of convergence might be affected.

7.4 Generating a Primal Feasible Solution

If the optimal solution of the relaxed problem (7.2) is not primal feasible, a heuristic method is used to create one. The total production cost of the generated primal feasible solution (PFS) will serve as an upper bound of the optimal solution of the primal problem. It is therefore important to generate the PFS as good as possible since this will decrease the number of iterations in the dual algorithm. The method is to first modify the optimal unit commitment of the relaxed problem and make it feasible. In this step the binary variable $u_{i,k}$ in the economic dispatch problem (6.1) is set. Next the production level of each individual unit is computed, i.e. the economic dispatch problem is solved using the algorithm described in Section 6. The first step, which uses a heuristic method, will now be outlined in detail.

When the relaxed problem (7.2) is solved, a trial set of binary variables $u_{i,k}$ is achieved. This unit commitment is primal feasible if the producing units, i.e. units with $u_{i,k} = 1$, together with the Heat water storage are able to satisfy all constraints in the short-term production-planning problem (4.52). If not, additional units are switched on to make the unit commitment primal feasible by the following procedure:

- Additional units are switched on to make
$$\overline{q}_{i,S} + \sum_{k \in \{k : u_{i,k}=1\}} \overline{q}_{i,k} \geq q_{i,D} \tag{7.6}$$
satisfied for every time interval i. Now it is possible to fulfill (4.40) and (4.48) in every time interval.

- Beginning with the first time interval i, additional units are switched on so that the Heat water storage equation (4.38) is fulfilled with
$$q_{i,S} = \max \left\{ \underline{q}_{i,S}, q_{i,D} - \sum_{k \in \{k : u_{i,k}=1\}} \overline{q}_{i,k} \right\}. \tag{7.7}$$
This makes constraint (4.39) satisfied in every time interval.

- Finally, we want to fulfill the configuration constraints (4.43), (4.44), (4.45) and the temperature demand constraint (4.49) in every time interval i. We may have to switch on other units and change the unit commitment to satisfy these constraints.

When the unit commitment in time interval i has to be changed and another unit switched on, which unit should be switched on? Our algorithm makes this choice using a combination of heuristics and priority lists. In the priority list the units are sorted in order of increasing relative production cost. The method is to, as long as possible, avoid new start-ups. To select the unit k to be switched on, we take a unit that fulfills the lowest selection criterion in the following list:

1. Let unit k be a unit which is producing in time interval $i+1$, i.e. $u_{i,k} = 0$ and $u_{i+1,k} = 1$. Unit k is chosen only if its minimal up and down times will be satisfied.

2. Same as item one, but with $i+1$ replaced by $i-1$.

3. Let unit k be a unit which is producing in time interval $i+1$, i.e. $u_{i,k} = 0$ and $u_{i+1,k} = 1$. Unit k must also be switched on or off in some other time intervals to satisfy its minimal up and down times.

4. Same as item three, but with $i+1$ replaced by $i-1$.

5. Let unit k be any unit with $u_{i,k} = 0$. Unit k must also be switched on or off in some other time intervals to satisfy its minimal up and down times.

If there are more than one candidate in the lowest of the steps described above, unit k is chosen using the priority list.

7.5 Termination Criterion

Algorithm STPP terminates when the duality gap is small, say less than one percent of the best primal feasible solution found so far. As discussed in Section 2.2 such small duality gaps can not be guaranteed. The algorithm then terminates after a pre-specified number of iterations. This is normally acceptable since the real problem seldom is to find the true dual optimum, but to find the best possible primal feasible solution within a certain amount of computer time.

7.6 Updating of Lagrange Multipliers

The Lagrange multipliers are updated using the subgradient method presented in Section 2.3. The step length is given by the *Polyak rule II* (2.11).

7.7 A MATLAB Implementation

The short-term production-planning algorithm given in Table 7.1 and described in Section 7.1 - 7.6 is implemented in MATLAB. The solution algorithm for the unit specific problems is partitioned into five m-files. When problem (7.4) is solved using dynamic programming it separates into one sub-problem for each time interval with the binary variable $u_{i,k}$ set to either one or zero. This single unit, single time interval continuous problem is solved using NPSOL [GMSW86]. The unit specific problem for the Heat water storage (7.5) is solved using NPSOL or MINOS [MS95].

The heuristic procedure described in Section 7.4 is implemented as five m-files. Including the implementation of the economic dispatch problem described in Section 6.2, the short-term production-planning algorithm is partitioned into 60 m-files using about 8000 lines of code.

7.8 A Numerical Example

The short-term production-planning algorithm in Table 7.1 is applied to the production plant used in the numerical example in Section 6.3. Parameters describing the start-up cost (4.5) are given in Table 7.2. All units have a minimal up time equal to one hour and a minimal down time equal to two hours. Initially all units are producing with the running time $T_{0,k}$ in (4.4) equal to ten hours. All other parameters needed are given in Section 6.3.

	HWC	ELH	HEP	CHP
γ_k^1	2	2	1	4
γ_k^2	2	2	1	4
γ_k^3	0.4	0.4	0.4	0.4

Table 7.2: Parameters describing the start-up cost.

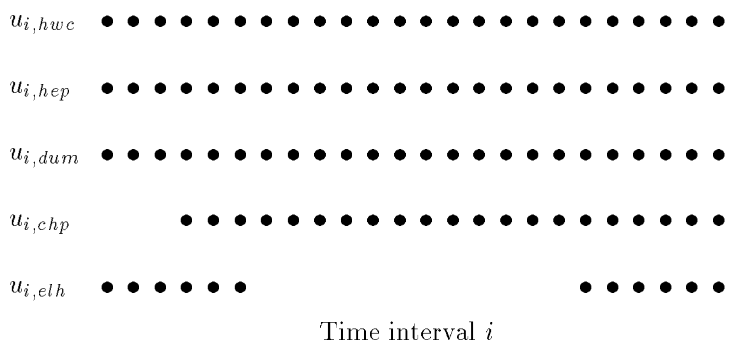

Figure 7.1: The feasible unit commitment found.

Starting with ξ_k set to one in (2.11) and dividing ξ_k by two when the best dual objective has failed to increase in three iterations, the algorithm converges to the unit commitment in Figure 7.1. Here a black dot means that the current unit is running.

Interrupting the calculations after 100 iterations, the duality gap is 3.1 percent of the best primal feasible solution found so far. The primal and dual objective are depicted in Figure 7.2.

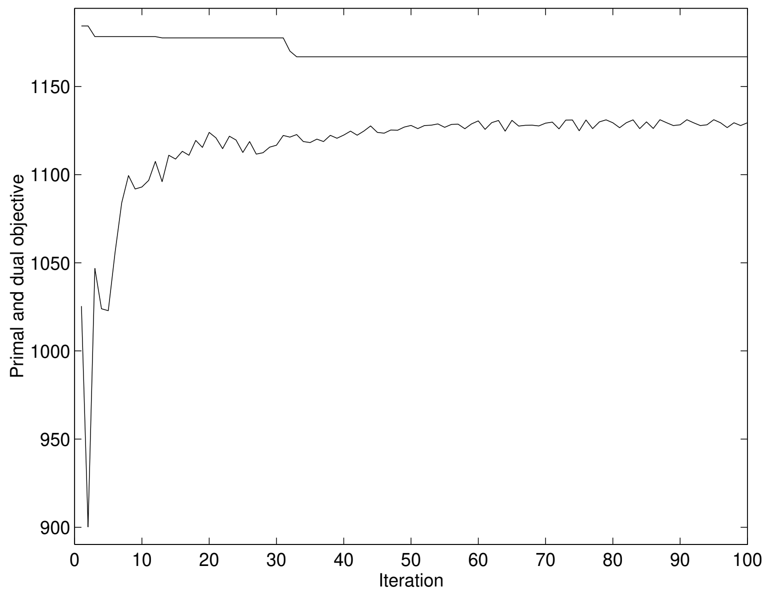

Figure 7.2: Primal and dual objective.

7.9 Computational Results

This section presents some results solving the short-term production-planning problem (4.52) using the implementation described in Section 7.7. All calculations are made on a PC with Windows 95 and a Pentium 150 MHz processor. Tests have been run for the four test cases, Case A - Case D, used in Section 6.4 expanded to consider 12 one-hour time intervals and a Heat water storage. The storage energy content is discretisized with 30 states and the final energy content $e_{12,S}$ is chosen to be 750 MWh. The electricity price $c_{i,e}$ is 200 SEK/MWh and the heat demand $q_{i,D}$ is 70 percent of the total maximum heat production. All other parameters needed are the same as used in the numerical example in Section 7.8.

The parameter ξ_k in (2.11) is initially set to one and divided by two when the best dual objective has failed to increase in three iterations. Interrupting the calculations after 50 iterations gives the results in Table 7.3. Here $\overline{c}$ is the best primal feasible solution found in kSEK, τ_{eval} is the computation time in minutes and $Mflops$ is the number of million floating point operations used.

	$\overline{c}$	Duality gap	τ_{eval} (min)	$Mflops$
Case A	529.0751	0.7052	10.98	11.6610
Case B	1067.1075	0.7250	25.11	28.2586
Case C	2134.2125	1.1062	46.06	52.9381
Case D	1719.1540	0.5803	28.86	38.7263

Table 7.3: Solving the short-term production-planning problem.

The duality gap is given in percent of $\overline{c}$.

The results show that the algorithm is robust. A time consuming part of the algorithm is the solution of the unit specific problems (7.4). When problem (7.4) is solved with the dynamic programming technique, it runs a series of single unit, single time interval problems. In the approach presented in this thesis the general-purpose solver NPSOL is used to solve these problems. In the future more effort has to be made to develop a solution algorithm which speed up these calculations.

Chapter 8

The Need for Production-Planning

This section tries to illustrate the savings that can be made when using the production-planning algorithm developed in this thesis. The intention was to apply the model and algorithm on the cogeneration plant in Västerås, but since it has been difficult to get good data some additional information had to be added. The parameters describing the plant in Västerås are taken from the master thesis by Peter Wiklund and Fredrik Wilhelmsson [WW88]. Additional data needed are taken from the model used at Värtan, see [GGB85] and [GGB86]. The reason why good data are not available from the cogeneration plant in Västerås is because no-one has made any attempts to estimate the parameters and variables needed for an accurate model.

The plant model considered in this section consists of three Cogeneration units (CHP1, CHP2, CHP3), two Electric heaters (ELH1, ELH2), one Heat water central (HWC) and one Heat water storage (HWS). The unit configuration is shown in Figure 8.1 and the parameters describing the different units are given in Table 8.1 and in Table 8.2. Parameters describing the feasible region for the Cogeneration units, see Figure 4.1, are given in Table 8.3. Units CHP1 and CHP3 are using coal as fuel and units CHP2 and HWC are using oil.

To give an idea about the possible savings, assume we could lower the total production cost by one percent. For the plant at Värtan, that has a daily production cost of about 700 kSEK (or about $87'500 in 1997), the savings becomes 7000 SEK ($875) per day. On yearly bases, this implies that the savings

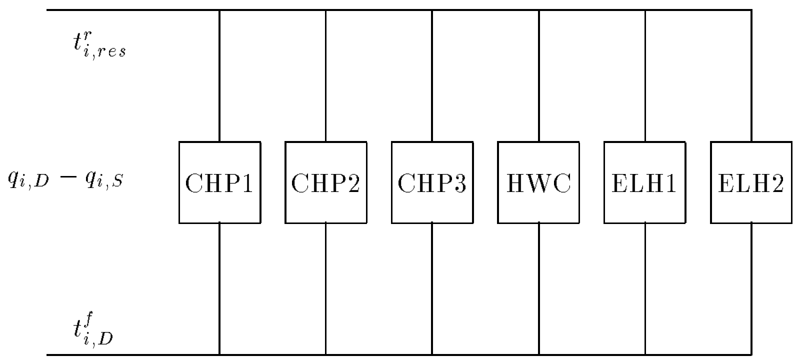

Figure 8.1: The unit configuration.

exceed two million Swedish crowns ($250'000). The following subsection gives an analysis of the algorithm and the mathematical model. In Section 8.2 some conclusions are given and in Section 8.3 possible improvements and further work are discussed.

8.1 Case Studies

The tests performed in this section only consider the economic dispatch problem (6.1). It is assumed that the unit commitment given is such that all units are running. The fuel costs and taxes are $c_{oil} = 90$, $c^{tax}_{oil} = 70$, $c_{coal} = 55$, $c^{tax}_{coal} = 40$, $c^{tax}_{help} = 50$ and $c^{tax}_{elh} = 60$. All costs are given in SEK/MWh. The specific heat capacity of water is $c_p = 4.18$ J/(gC^o).

In the first test we will illustrate the importance of running the Heat water storage efficiently. The time horizon considered is twenty-four hours, partitioned into twenty-four one-hour intervals. The heat demand, temperature demand, plant return temperature and electricity price are given in Table 8.4. At the beginning of the time period the energy content of the storage is $e_{0,S} = 750$ MWh and at the end the same, i.e. $e_{24,S} = 750$ MWh. Discretisizing the storage energy content in steps of 10 MWh, the dynamic programming algorithm described in Section 6.1 gives a total production cost of 1334.2 kSEK. Solving the same problem and not using the storage gives a total production cost of

1347.3 kSEK. To summarize the results:

Cost not using the storage (kSEK)	1347.3
Cost using the storage (kSEK)	1334.2
Savings (kSEK)	13.1
Savings (percent)	1.0,

i.e. the savings are in the order of one percent using the storage.

Looking at Table 8.4 we see that it is possible to fulfill the heat demand during time intervals one to three ($1 \leq i \leq 3$) and during time intervals twenty-two to twenty-four ($22 \leq i \leq 24$) only using the units CHP1, CHP3, ELH2 and HWS. Neglecting the start-up costs, i.e. $\gamma_k^1 = 0$, $\gamma_k^2 = 0$ and $\gamma_k^3 = 0$ in (4.5) for all units, and solving problem (6.1) with CHP2, HWC and ELH1 turned off during time interval one, two, three, twenty-two, twenty-three and twenty-four, the total production cost becomes 1307.5 kSEK. Comparing with the case when all units were running, the cost is 26.7 kSEK lower. Depending on if the units shall produce directly after the last time interval, there might be up to six new start-ups as a consequence of this operation. This means, if the total cost associated with the start-ups is higher than 26.7 kSEK, it is more economical to keep the units running. The start-up cost for a unit is in the range from a few thousand to over hundred-thousand Swedish crowns depending on unit type, type of fuel and the time it has been off. If we assume that a start-up cost for a unit is 5000 SEK, we get

	All units on	Units turned off
Cost ignoring start-up costs (kSEK)	1334.2	1307.5
Possible number of start-ups	0	≤ 6
Cost not ignoring start-up costs (kSEK)	1334.2	$1307.5 - 1337.5$
Savings not ignoring start-up costs (kSEK)	0	$26.7 - (-3.3)$.

Depending on the number of start-ups, there are possible savings, but there are also possible losses. **To get the correct answer you need an accurate mathematical model and a well working solution algorithm**. Notice also that the real problem is even more difficult since the true start-up costs are time dependent.

In the plant model all units are connected directly to the forward pipe of the plant, i.e. all units contribute to the temperature demand (4.49). The question is, how much does the result differ if we do not make use of the opportunity to mix temperatures? Consider the smaller sub-problem (6.2) over a one-hour time interval with the heat demand equal to 80 percent of the maximal production, the temperature demand equal to 100 $°C$ and the plant return temperature equal to 50 $°C$. Solving the problem with the five different electricity prices 125, 150, 175, 200 and 225 SEK/MWh gives an average production cost of 67.28 kSEK when mixing of temperatures is allowed. Now assume that the unit output temperatures are given equal to the temperature demand, i.e. $\bar{t}^f_{i,k} = \underline{t}^f_{i,k} = t^f_{i,D}$. Solving problem (6.2) now gives a higher production cost in all cases. Here the average cost is 68.02 kSEK, i.e. about 1.1 percent higher. The exact results are

						Average
Electricity price (SEK/MWh)	125	150	175	200	225	175
Cost not mixing temp. (kSEK)	84.9	76.4	68.0	59.6	51.2	**68.02**
Cost mixing temp. (kSEK)	84.7	76.0	67.3	58.6	49.9	**67.28**
Savings (SEK)	240	447	723	999	1276	**737**
Savings (percent)	0.28	0.59	1.06	1.68	2.49	**1.08**.

Further savings that can be done depend on how well the mathematical model describes the production plant. With a less accurate model, i.e. a model where all equations relating the variables are linear or a model neglecting the need for help power, storage losses, time depending start-up costs etc., you may not find the best real-life working point, even if you get the optimal solution for the mathematical model. To illustrate the consequence of convergence to an incorrect working point, consider a situation where the heat demand is 80 percent of the maximal production and the electricity price is 100 SEK/MWh. Solving problem (6.2) gives a solution where all Cogeneration units are working at the backpressure line, see Section 4.1.2. Now consider what happens if the need for help power is neglected in the model, i.e. $\beta^j_k = 0$, $j = 0, ..., 5$, and $\delta^j_k = 0$, $j = 0, ..., 2$, for all units. Solving the problem under these conditions, the production for unit CHP2 moves along the backpressure line, see Figure 8.2. In the first case the working point is $(207.4, 121.7)$ MW and in the second case it is $(239.6, 144.2)$ MW. Also the production for the Heat water central is changed,

Electricity price (SEK/MWh)	80	90	100	110	120
Production ignoring help power (MW)	75	**59**	**18**	**15**	15
Production not ignoring help power (MW)	75	**75**	**50**	**27**	15.

As we can see, there are for all electricity prices but the extremes dramatic changes in the optimal working point. In our example the convergence to an incorrect working point increases the total production cost with about 0.1 percent. Together with the contribution from neglecting nonlinearities, storage

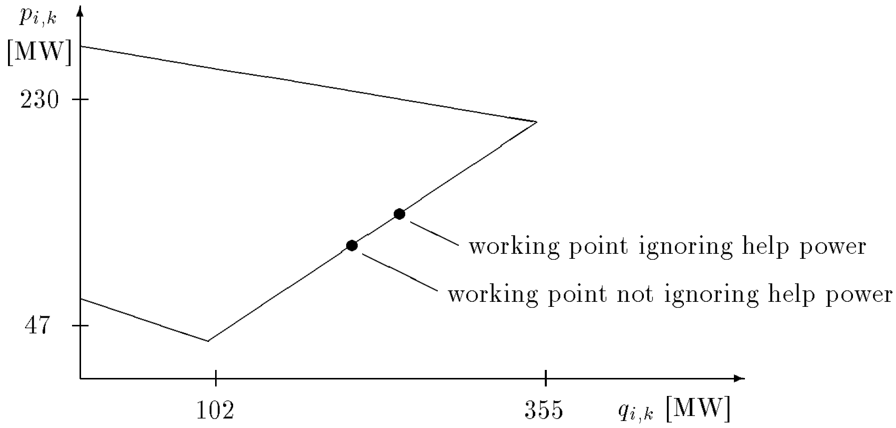

Figure 8.2: The heat-electricity constraints for unit CHP2.

losses etc., it is reasonable to estimate the total savings using our accurate model to more than one percent.

8.2 Conclusions

The numerical examples given in Section 6.3 and Section 7.8, the computational results given in Section 6.4 and Section 7.9 and the analysis performed in Section 8.1 show that a good utilization of the cogeneration plant implies significant savings. For the decision of a production schedule, a well working production-planning computer program is an invaluable tool. The tests show that the need for an accurate mathematical model not only is desirable, it is necessary. In such a model one can not neglect the influence of internal temperatures, the need for help power or start-up costs. Of course an accurate model requires an efficient solution algorithm, but the algorithm developed in this thesis shows that this goal is not unreachable. The algorithm developed should not be seen as the final solution, but as a first step towards an efficient production-planning tool. A discussion of further improvements are given in Section 8.3.

8.3 Further Work

The mathematical model and algorithm presented in this thesis contributes as a first step towards the development of a fast, robust and accurate production-planning tool. In some cases the material presented may be exactly what you need. In other cases a more accurate model and a faster algorithm is desirable. To cover all cases and fulfill all demands some improvements and further developments have to be done.

To make the mathematical model more accurate, the influence of temperatures for the Heat water storage should be incorporated. Also the structure of the Swedish system for taxes and emission fees must be included. So far no good method to estimate the end level of the storage energy content $e_{I,S}$ has been developed. A simple suggestion is just to add the term $-\nu^{e}_{I,S} e_{I,S}$ to the objective in (4.52). Here $\nu^{e}_{I,S}$ is the value of the energy content at the end of the time horizon. With this term in the objective, the end level of the energy content is computed in the solution procedure of the short-term production-planning problem.

In the model, start-up costs are included. There are also a cost associated with a shut-down. This cost is probably dependent on the time the actual unit has been on. If the shut-down cost is not time dependent, it can be included in the start-up cost, since there must be a start-up for every shut-down. If it is time dependent, it must be expressed separately.

Also the possible appearance of common steam headers must be included in the model. They are units sharing the same heat exchanger, turbine and cool condenser, see Section 3.2.2. This could be done by splitting the model describing the Cogeneration unit into four pieces; one modeling the boiler, one modeling the heat exchanger, one modeling the turbine and one modeling the cool condenser.

Using a Cogeneration unit or a Heat water central it is possible to mix different kinds of fuel. In our model the fuel costs and fuel taxes are assumed known, i.e. the mixture of fuel is given in advance. If decision variables describing the mixture are included in the model, it should be possible to decide the optimal mixture during the calculations. This should lower the total production cost even more.

The modifications discussed so far only consider the mathematical model. There are also opportunities to improve the solution algorithm. In the approach presented, general-purpose solvers are used to solve problem (6.2), problem (7.5) and a sub-problem to problem (7.4). Maybe it is possible to develop more efficient solvers that make use of the special structure of these problems. The dual problem is solved using the subgradient method, see Section 7.6. There are other techniques that might perform better. Interesting methods are used by Magnusson [Mag91] and Feltenmark [Fel97]. The further development of the dual solver should also include heuristics to estimate better starting values for the dual parameters λ in Section 7.2.

In the procedure for generating primal feasible solutions, given in Section 7.4, heuristics are used to choose units to be switched on. Here corresponding heuristics choosing units to be switched off should be incorporated.

	CHP1	CHP2	CHP3	HWC	ELH1	ELH2
α_k^0	0.95	0.95	0.95	0.940625	0.99	0.99
α_k^1	0	0	0	0	0	0
α_k^2	-0.02	-0.02	-0.02	$-2.7778 \cdot 10^{-6}$	0	0
β_k^0	0	0	0	0.73125	-	-
β_k^1	0.152198	1.859341	0.68	$2.3334 \cdot 10^{-3}$	-	-
β_k^2	1.197802	3.340659	3.20	$1.6667 \cdot 10^{-5}$	-	-
β_k^3	0.30	1.20	0.90	-	-	-
β_k^4	0.16978	1.989011	0.805495	-	-	-
β_k^5	2.78022	9.010990	7.494505	-	-	-
υ_k^1	3	3	3	-	-	-
υ_k^2	1	1	1	-	-	-
σ_k^0	10	37	28	-	-	-
σ_k^1	130	555	402	-	-	-
σ_k^2	0	0	0	-	-	-
δ_k^0	0.95	3.6	2.75	-	-	-
δ_k^1	1.8	6.7	5.15	-	-	-
δ_k^2	0	0	0	-	-	-
$\overline{q}_{i,k}$	-	-	-	75	40	35
$\underline{q}_{i,k}$	-	-	-	15	12	4
$\overline{t}_{i,k}^f$	120	120	120	120	120	120
$\underline{t}_{i,k}^f$	50	50	50	50	50	50
$\overline{m}_{i,k}$	∞	∞	∞	∞	∞	∞
$\underline{m}_{i,k}$	0	0	0	0	0	0

Table 8.1: Unit parameters.

	β_S^0	β_S^1	β_S^2	$\overline{q}_{i,S}$	$\underline{q}_{i,S}$	$\overline{e}_{i,S}$	$\underline{e}_{i,S}$	$l_{i,S}$	$l'_{i,S}$
HWS	0	0	$6.667 \cdot 10^{-6}$	100	-100	1000	500	0.001	0.01

Table 8.2: Heat water storage parameters.

			$t^f_{i,k}=90,\ t^r_{i,k}=50$		$t^f_{i,k}=120,\ t^r_{i,k}=80$	
	$P^1_{i,k}$	$P^4_{i,k}$	$P^2_{i,k}$	$P^3_{i,k}$	$P^2_{i,k}$	$P^3_{i,k}$
CHP1	$0,50$	$0,10$	$95,40$	$27,8$	$93,35$	$26,6$
CHP2	$0,265$	$0,63$	$355,230$	$102,47$	$348,203$	$96,38$
CHP3	$0,190$	$0,42$	$270,155$	$78,32$	$265,137$	$73,25$

Table 8.3: Cogeneration unit parameters.

	unit	$1 \le i \le 3$	$4 \le i \le 6$	$7 \le i \le 9$	$10 \le i \le 12$
$q_{i,D}$	MW	450	500	550	600
$c_{i,e}$	SEK/MWh	100	150	175	175
$t^f_{i,D}$	°C	100	100	100	100
$t^r_{i,res}$	°C	50	50	50	50

	unit	$13 \le i \le 15$	$16 \le i \le 18$	$19 \le i \le 21$	$22 \le i \le 24$
$q_{i,D}$	MW	650	600	550	450
$c_{i,e}$	SEK/MWh	175	150	125	100
$t^f_{i,D}$	°C	100	100	100	100
$t^r_{i,res}$	°C	50	50	50	50

Table 8.4: Heat demand, electricity price and temperatures.

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