Comparison between LBFGS-B and SPG with quadratic interpolation in line search

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In this work I present the numerical comparisons of LBFGS-B versus SPG (quadratic interpolation) on a collection of 730 simple bounded optimization test problems.

Spectral Projected Gradient is implemented by Birgin, Martinez and Raydan in MSPG.FOR package. Limited memory BFGS with simple bounds is implemented by Ciyou Zhu in collaboration with Byrd, Lu and Nocedal in MLBFGSB.FOR package

The comparison is considered in the following format. Let f_i^{ALG1} and f_i^{ALG2} be the optimal value found by ALG1 and ALG2, for problem $i=1,\ldots,730$, respectively. We say that, in the particular problem i, the performance of ALG1 was better than the performance of ALG2 if:

$$|f_i^{ALG1} - f_i^{ALG2}| < 10^{-3}$$

and the number of iterations, or the number of function-gradient evaluations, or the CPU time of ALG1 was less than the number of iterations, or the number of function-gradient evaluations, or the CPU time corresponding to ALG2, respectively. Out of 730 test simple bounded optimization problems only 546 problems satisfies the above comparison criteria.

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Performance Profile: August 31, 2010
  Results lbfgsb versus mspg1 , valeps= 0.1000000000000E-02
                nexp= 546
 nexptot= 730
 Total Number of iterations for lbfgsb =
 Total Number of iterations for mspg1 =
 Total Number of function evaluations for lbfgsb =
 Total Number of function evaluations for mspg1 =
 Total Time (centeseconds) for lbfgsb = Total Time (centeseconds) for mspg1 =
                                                     459 problems
54 problems
            lbfgsb achieved minimum # of iter in
            mspgl achieved minimum # of iter in 54 problems

lbfgsb and mspgl achieved the same # of iter in 33 problems
  Iterations Performance Profile for lbfgsb
Iterations Performance Profile for mspq1
=[0.16, 0.54, 0.73, 0.83, 0.86, 0.88, 0.89, 0.90, 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.97];
                   achieved minimum # of fg in
                                                     410 problems
            lbfqsb
            mspgl achieved minimum # of fg in 85 problems
lbfgsb and mspgl achieved the same # of fg in 51 problems
  Function Evaluations Performance Profile for lbfgsb
```

```
lbfgsb
= [\,0.84\,,0.93\,,0.95\,,0.99\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,1.00\,,
                             Function Evaluations Performance Profile for mspg1
                            mspq1
=[0.25, 0.54, 0.64, 0.76, 0.82, 0.84, 0.87, 0.89, 0.89, 0.89, 0.89, 0.90, 0.90, 0.91, 0.91, 0.92];
                                                                                                                   lbfgsb
                                                                                                                                                                                                         achieved minimum time in
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              problems
                                                                                                                                                                                                         achieved minimum time in
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       310
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             problems
                                                                                                                   mspg1
                                                                                                                   lbfgsb
                                                                                                                                                                                                         and mspg1
                                                                                                                                                                                                                                                                                                                                    achieved the same time in
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         151
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     problems
                             Time Performance Profile for lbfgsb
                             lbfasb
 = [\, 0\,.\,43\,,\,0\,.\,68\,,\,0\,.\,82\,,\,0\,.\,89\,,\,0\,.\,94\,,\,0\,.\,96\,,\,0\,.\,99\,,\,0\,.\,99\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,00\,,\,1\,.\,0
                             Time Performance Profile for mspg1
 = [0.8\overline{4}, 0.94, 0.95, 0.96, 0.96, 0.97, 0.97, 0.97, 0.97, 0.97, 0.97, 0.97, 0.97, 0.97, 0.98, 0.98, 0.99];
```

The performance profile is illustrated in Figure 1.

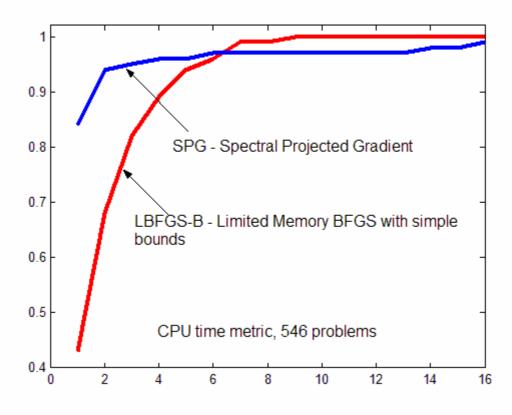


Fig.1. SPG (quadratic) versus LBFGS-B

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