

# Pressure Distribution Problem

## 25 Conjugate Gradient Algorithms Comparison between conjugate gradient algorithms and their accelerated variants

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### 1. Conjugate Gradient Algorithms

```
*-----  
| Fastest Conjugate Gradient Algorithm Project  
| April 1, 2009  
| stoptest = 1  
| maxiter = 10000  
| Conjugate Gradient with Powell restart.  
*-----  
  
1 CG-HS Algorithm. Function:Pressure Distribution Problem  
Hestenes and Stiefel. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 1092 308 1717 624 2177 -.2828927135341E+00 .3094556267870E-04  
-----  
TOTAL 1092 308 1717 624 21.77 (seconds) proc= 28.21%  
  
2 CG-FR Algorithm. Function:Pressure Distribution Problem  
Fletcher and Reeves. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 1137 339 1792 654 2221 -.2828929493795E+00 .2289865251513E-04  
-----  
TOTAL 1137 339 1792 654 22.21 (seconds) proc= 29.82%  
  
3 CG-PRP Algorithm. Function:Pressure Distribution Problem  
Polak-Ribiere and Polyak. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 881 275 1410 528 1738 -.2828927675034E+00 .3353412105454E-04  
-----  
TOTAL 881 275 1410 528 17.38 (seconds) proc= 31.21%  
  
4 CG-PRP+ Algorithm. Function:Pressure Distribution Problem  
Polak-Ribiere and Polyak plus. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 881 275 1410 528 1729 -.2828927675034E+00 .3353412105454E-04  
-----  
TOTAL 881 275 1410 528 17.29 (seconds) proc= 31.21%
```

5 CG-CD Algorithm. Function:Pressure Distribution Problem  
Conjugate Descent - Fletcher. stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	1272	376	2031	758	2485	- .2828926551976E+00	.3122189548122E-04
TOTAL	1272	376	2031	758	24.85 (seconds)	proc=	29.56%

6 CG-LS Algorithm. Function:Pressure Distribution Problem  
Liu and Storey. stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	1022	310	1617	594	1991	- .2828924337707E+00	.4631531989285E-04
TOTAL	1022	310	1617	594	19.91 (seconds)	proc=	30.33%

7 CG-DY Algorithm. Function:Pressure Distribution Problem  
Dai and Yuan. stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	880	232	1383	502	1694	- .2828927621973E+00	.3710896825946E-04
TOTAL	880	232	1383	502	16.94 (seconds)	proc=	26.36%

8 CG-DL Algorithm. Function:Pressure Distribution Problem  
Dai and Liao (t=1). stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	1051	296	1642	590	2027	- .2828925290751E+00	.3677239976280E-04
TOTAL	1051	296	1642	590	20.27 (seconds)	proc=	28.16%

9 CG-DL+ Algorithm. Function:Pressure Distribution Problem  
Dai and Liao plus (t=1). stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	854	236	1346	491	1654	- .2828928619864E+00	.2362264546372E-04
TOTAL	854	236	1346	491	16.54 (seconds)	proc=	27.63%

10 CG-SDC Algorithm. Function:Pressure Distribution Problem  
Andrei - Sufficient Descent Condition. stoptest= 1  
aCGSD: beta = gtg/yts - ytg\*stg/(yts\*yts)

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	10001	9992	10009	8	10792	.1269510947732E+02	.1842346800380E+01
TOTAL	10001	9992	10009	8	107.92 (seconds)	proc=	99.91%

11 CG-hDY Algorithm. Function:Pressure Distribution Problem  
hDY:Hybrid Dai and Yuan. max(c\*Dy,min(HS,DY)). stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	1232	355	1940	707	2409	- .2828928692524E+00	.2865598941648E-04
TOTAL	1232	355	1940	707	24.09 (seconds)	proc=	28.81%

12 CG-hDYZ Algorithm. Function:Pressure Distribution Problem  
hDYZ: Hybrid Dai and Yuan zero. max(0,min(HS,DY)). stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
---	------	-----	-------	-------	---------	-------	-------

```

40000 1232 355 1940 707 2408 -.2828928692524E+00 .2865598941648E-04
-----
TOTAL 1232 355 1940 707 24.08 (seconds) proc= 28.81%


13 CG-GN Algorithm. Function:Pressure Distribution Problem
    Gilbert and Nocedal. stoptest= 1

    n iter irls fgcnt lscnt time(c) fxnew gnorm
-----
40000 823 234 1310 486 1599 -.2828928199235E+00 .2805383021583E-04
-----
TOTAL 823 234 1310 486 15.99 (seconds) proc= 28.43%


14 CG-HuS Algorithm. Function:Pressure Distribution Problem
    PRP-FR: Hu and Storey. max(0,min(PRP,FR)). stoptest= 1

    n iter irls fgcnt lscnt time(c) fxnew gnorm
-----
40000 823 234 1310 486 1602 -.2828928199235E+00 .2805383021583E-04
-----
TOTAL 823 234 1310 486 16.02 (seconds) proc= 28.43%


15 CG-TAS Algorithm. Function:Pressure Distribution Problem
    Touat-Ahmed and Storey. (PRP <= FR). stoptest= 1

    n iter irls fgcnt lscnt time(c) fxnew gnorm
-----
40000 823 234 1310 486 1600 -.2828928199235E+00 .2805383021583E-04
-----
TOTAL 823 234 1310 486 16.00 (seconds) proc= 28.43%


16 LS-CD Algorithm. Function:Pressure Distribution Problem
    Hybrid Liu&Storey-Conjugate Descent. max(0,min(LS,CD)) stoptest= 1

    n iter irls fgcnt lscnt time(c) fxnew gnorm
-----
40000 918 274 1459 540 1791 -.2828922215603E+00 .5257096171368E-04
-----
TOTAL 918 274 1459 540 17.91 (seconds) proc= 29.85%


17 CG-BM Algorithm. Function:Pressure Distribution Problem
    Birgin and Martinez (scaled Perry). Theta spectral. stoptest= 1

    n iter irls fgcnt lscnt time(c) fxnew gnorm
-----
40000 1109 300 1754 644 2179 -.2828928096628E+00 .2432824766449E-04
-----
TOTAL 1109 300 1754 644 21.79 (seconds) proc= 27.05%


18 CG-BM+ Algorithm. Function:Pressure Distribution Problem
    Birgin and Martinez plus (scaled Perry). Theta spectral. stoptest= 1

    n iter irls fgcnt lscnt time(c) fxnew gnorm
-----
40000 821 229 1288 466 1586 -.2828927939461E+00 .3216140948582E-04
-----
TOTAL 821 229 1288 466 15.86 (seconds) proc= 27.89%


19 CG-sPRP Algorithm. Function:Pressure Distribution Problem
    Scaled Polak-Ribiere and Polyak. Theta spectral. stoptest= 1

    n iter irls fgcnt lscnt time(c) fxnew gnorm
-----
40000 802 236 1292 489 1582 -.2828928385060E+00 .2466694383593E-04
-----
TOTAL 802 236 1292 489 15.82 (seconds) proc= 29.43%


20 CG-sFR Algorithm. Function:Pressure Distribution Problem
    Scaled Fletcher and Revees. Theta spectral. stoptest= 1

```

```

n    iter    irs    fgcnt lscnt    time(c)      fxnew          gnorm
-----
40000    846    248    1334    487        1637 -.2828928021693E+00 .2980700549320E-04
-----
TOTAL    846    248    1334    487        16.37 (seconds) proc= 29.31%
-----
```

21 New CG Algorithm. Function:Pressure Distribution Problem  
Andrei New Sufficient Descent from PRP. stoptest= 1  
PRP-SDC: beta = (ytg-yty\*stg/gtgprev)/yts  
n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 1206 386 1956 748 2426 -.2828929492914E+00 .7433452643742E-05
-----
TOTAL 1206 386 1956 748 24.26 (seconds) proc= 32.01%

22 New CG Algorithm. Function:Pressure Distribution Problem  
Andrei New Sufficient Descent from DY. stoptest= 1  
ACGSD: beta=ytg/yts-ytg\*stg/(yts\*\*2)  
n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 1063 283 1685 621 2082 -.2828927318698E+00 .3190897658234E-04
-----
TOTAL 1063 283 1685 621 20.82 (seconds) proc= 26.62%

23 New CG Algorithm. Function:Pressure Distribution Problem  
Andrei Sufficient Descent from DY (max). stoptest= 1  
ACGSDz: beta=max(0,ytg/yts)\*(1-stg/yts)  
n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 1055 294 1662 606 2054 -.2828928354122E+00 .3065409312930E-04
-----
TOTAL 1055 294 1662 606 20.54 (seconds) proc= 27.87%

24 New CG Algorithm. Function:Pressure Distribution Problem  
PRP - DY with Newton direction. (NDOMB) stoptest= 1  
n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 1035 281 1619 583 2002 -.2828929493123E+00 .1164462556719E-04
-----
TOTAL 1035 281 1619 583 20.02 (seconds) proc= 27.15%

25 New CG Algorithm. Function:Pressure Distribution Problem  
PRP - DY with Conjugacy Condition. (CCOMB) stoptest= 1  
n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 968 271 1520 551 1876 -.2828929338199E+00 .4271620184242E-04
-----
TOTAL 968 271 1520 551 18.76 (seconds) proc= 28.00%

## 2. Accelerated Conjugate Gradient Algorithms

```
*-----  
| Fastest Conjugate Gradient Algorithm Project  
| April 1, 2009  
| stoptest = 1  
| maxiter = 10000  
| Accelerated Conjugate Gradient with Powell restart.  
*-----  
  
1 CG-HS Algorithm. Function:Pressure Distribution Problem  
Hestenes and Stiefel. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 634 0 1303 34 1595 -.2828929487185E+00 .2267804724615E-04  
-----  
TOTAL 634 0 1303 34 15.95 (seconds) proc= .00%  
  
2 CG-FR Algorithm. Function:Pressure Distribution Problem  
Fletcher and Reeves. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 631 0 1297 34 1543 -.2828929485411E+00 .2457336785793E-04  
-----  
TOTAL 631 0 1297 34 15.43 (seconds) proc= .00%  
  
3 CG-PRP Algorithm. Function:Pressure Distribution Problem  
Polak-Ribiere and Polyak. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 631 0 1297 34 1559 -.2828929485421E+00 .2454677340183E-04  
-----  
TOTAL 631 0 1297 34 15.59 (seconds) proc= .00%  
  
4 CG-PRP+ Algorithm. Function:Pressure Distribution Problem  
Polak-Ribiere and Polyak plus. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 631 0 1297 34 1555 -.2828929485421E+00 .2454677340183E-04  
-----  
TOTAL 631 0 1297 34 15.55 (seconds) proc= .00%  
  
5 CG-CD Algorithm. Function:Pressure Distribution Problem  
Conjugate Descent - Fletcher. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 631 0 1297 34 1543 -.2828929485416E+00 .2455850712383E-04  
-----  
TOTAL 631 0 1297 34 15.43 (seconds) proc= .00%  
  
6 CG-LS Algorithm. Function:Pressure Distribution Problem  
Liu and Storey. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 631 0 1297 34 1553 -.2828929485424E+00 .2453431572966E-04  
-----  
TOTAL 631 0 1297 34 15.53 (seconds) proc= .00%  
  
7 CG-DY Algorithm. Function:Pressure Distribution Problem  
Dai and Yuan. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----
```

```

40000   631      0    1297     34      1556 -.2828929485425E+00 .2452443036839E-04
-----
TOTAL   631      0    1297     34      15.56 (seconds) proc= .00%

8   CG-DL Algorithm. Function:Pressure Distribution Problem
Dai and Liao (t=1). stoptest= 1

n  iter   irs   fgcnt lscnt   time(c)      fxnew       gnorm
-----
40000   631      0    1297     34      1559 -.2828929485420E+00 .2454429030950E-04
-----
TOTAL   631      0    1297     34      15.59 (seconds) proc= .00%

9   CG-DL+ Algorithm. Function:Pressure Distribution Problem
Dai and Liao plus (t=1). stoptest= 1

n  iter   irs   fgcnt lscnt   time(c)      fxnew       gnorm
-----
40000   631      0    1297     34      1562 -.2828929485434E+00 .2450352142472E-04
-----
TOTAL   631      0    1297     34      15.62 (seconds) proc= .00%

10  CG-SDC Algorithm. Function:Pressure Distribution Problem
Andrei - Sufficient Descent Condition. stoptest= 1
aCGSD: beta = gtg/yts - ytg*stg/(yts*yts)
n  iter   irs   fgcnt lscnt   time(c)      fxnew       gnorm
-----
40000   631      0    1297     34      1563 -.2828929485445E+00 .2448290684214E-04
-----
TOTAL   631      0    1297     34      15.63 (seconds) proc= .00%

11  CG-hDY Algorithm. Function:Pressure Distribution Problem
hDY: Hybrid Dai and Yuan. max(c*DY,min(HS,DY)). stoptest= 1

n  iter   irs   fgcnt lscnt   time(c)      fxnew       gnorm
-----
40000  1058    188   2682    565      3235 -.2828927256667E+00 .4062853423683E-04
-----
TOTAL  1058    188   2682    565      32.35 (seconds) proc= 17.77%

12  CG-hDYz Algorithm. Function:Pressure Distribution Problem
hDYz: Hybrid Dai and Yuan zero. max(0,min(HS,DY)). stoptest= 1

n  iter   irs   fgcnt lscnt   time(c)      fxnew       gnorm
-----
40000  1058    188   2682    565      3238 -.2828927256667E+00 .4062853423683E-04
-----
TOTAL  1058    188   2682    565      32.38 (seconds) proc= 17.77%

13  CG-GN Algorithm. Function:Pressure Distribution Problem
Gilbert and Nocedal. stoptest= 1

n  iter   irs   fgcnt lscnt   time(c)      fxnew       gnorm
-----
40000   631      0    1297     34      1552 -.2828929485422E+00 .2454290905047E-04
-----
TOTAL   631      0    1297     34      15.52 (seconds) proc= .00%

14  CG-HuS Algorithm. Function:Pressure Distribution Problem
PRP-FR: Hu and Storey. max(0,min(PRP,FR)). stoptest= 1

n  iter   irs   fgcnt lscnt   time(c)      fxnew       gnorm
-----
40000   631      0    1297     34      1554 -.2828929485422E+00 .2454290905047E-04
-----
TOTAL   631      0    1297     34      15.54 (seconds) proc= .00%

15  CG-TAS Algorithm. Function:Pressure Distribution Problem
Touat-Ahmed and Storey. (PRP <= FR). stoptest= 1

```

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	631	0	1297	34	1553	-.2828929485422E+00	.2454290905047E-04
TOTAL	631	0	1297	34	15.53 (seconds)	proc=	.00%
16	LS-CD Algorithm. Function:Pressure Distribution Problem Hybrid Liu&Storey-Conjugate Descent. max(0,min(LS,CD)) stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	631	0	1297	34	1554	-.2828929485425E+00	.2453418661740E-04
TOTAL	631	0	1297	34	15.54 (seconds)	proc=	.00%
17	CG-BM Algorithm. Function:Pressure Distribution Problem Birgin and Martinez (scaled Perry). Theta spectral. stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	631	0	1297	34	1565	-.2828929485433E+00	.2450378986066E-04
TOTAL	631	0	1297	34	15.65 (seconds)	proc=	.00%
18	CG-BM+ Algorithm. Function:Pressure Distribution Problem Birgin and Martinez plus (scaled Perry). Theta spectral. stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	631	0	1297	34	1567	-.2828929485436E+00	.2449702546283E-04
TOTAL	631	0	1297	34	15.67 (seconds)	proc=	.00%
19	CG-sPRP Algorithm. Function:Pressure Distribution Problem Scaled Polak-Ribiere and Polyak. Theta spectral. stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	1059	205	2688	569	3246	-.2828928849703E+00	.2006996590907E-04
TOTAL	1059	205	2688	569	32.46 (seconds)	proc=	19.36%
20	CG-sFR Algorithm. Function:Pressure Distribution Problem Scaled Fletcher and Revees. Theta spectral. stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	845	176	2117	426	2536	-.2828929012097E+00	.3449419575132E-04
TOTAL	845	176	2117	426	25.36 (seconds)	proc=	20.83%
21	New CG Algorithm. Function:Pressure Distribution Problem Andrei New Sufficient Descent from PRP. stoptest= 1 PRP-SDC: beta = (ytg-yty*stg/gtgprev)/yts						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	979	142	2591	631	3115	-.2828928394355E+00	.2949356181982E-04
TOTAL	979	142	2591	631	31.15 (seconds)	proc=	14.50%
22	New CG Algorithm. Function:Pressure Distribution Problem Andrei New Sufficient Descent from DY. stoptest= 1 ACGSD: beta=ytg/yts-ytg*stg/(yts**2)						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	631	0	1297	34	1562	-.2828929485441E+00	.2448880474119E-04
TOTAL	631	0	1297	34	15.62 (seconds)	proc=	.00%

```

23  New CG Algorithm. Function:Pressure Distribution Problem
    Andrei Sufficient Descent from DY (max). stoptest= 1
    ACGSDz: beta=max(0,ytg/yts)*(1-stg/yts)

    n   iter   irs   fgcnt lscnt   time(c)       fxnew           gnorm
-----
40000   631     0   1297     34      1561 -.2828929485432E+00 .2450919878429E-04
-----
TOTAL   631     0   1297     34      15.61 (seconds)  proc=    .00%
-----
```

  

```

24  New CG Algorithm. Function:Pressure Distribution Problem
    PRP - DY with Newton direction. (NDOMB) stoptest= 1
    n   iter   irs   fgcnt lscnt   time(c)       fxnew           gnorm
-----
40000   631     0   1297     34      1560 -.2828929485422E+00 .2453919907835E-04
-----
TOTAL   631     0   1297     34      15.60 (seconds)  proc=    .00%
-----
```

  

```

25  New CG Algorithm. Function:Pressure Distribution Problem
    PRP - DY with Conjugacy Condition. (CCOMB) stoptest= 1
    n   iter   irs   fgcnt lscnt   time(c)       fxnew           gnorm
-----
40000   631     0   1297     34      1561 -.2828929485425E+00 .2452443140802E-04
-----
TOTAL   631     0   1297     34      15.61 (seconds)  proc=    .00%
-----
```

### Remarks.

This problem belongs to the MINPACK2 collection.

These results have been obtained with APPL2.FOR package which implements 25 conjugate gradient algorithms in classical and accelerated versions. To use the accelerated version of these algorithms the value of the logical parameter accel must be set to .true.

Observe the differences between cpu time corresponding to the conjugate gradient algorithms and their accelerated versions.

### References

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- Andrei, N., Acceleration of conjugate gradient algorithms for unconstrained optimization.** Applied Mathematics and Computation, 2009. DOI information: 10.1016/j.amc.2009.03.020.
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