

Elastic-Plastic Torsion Problem

25 Conjugate Gradient Algorithms Comparison between conjugate gradient algorithms and their accelerated variants

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1. Conjugate Gradient Algorithms

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*-----  
| Fastest Conjugate Gradient Algorithm Project  
| April 1, 2009  
| stoptest = 1  
| maxiter = 10000  
| Conjugate Gradient with Powell restart.  
*-----  
  
1 CG-HS Algorithm. Function:Elastic-Plastic Torsion Problem  
Hestenes and Stiefel. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 368 97 582 213 715 -.4392668610228E+00 .5436589407442E-04  
-----  
TOTAL 368 97 582 213 7.15 (seconds) proc= 26.36%  
  
2 CG-FR Algorithm. Function:Elastic-Plastic Torsion Problem  
Fletcher and Reeves. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 407 118 633 225 753 -.4392655894982E+00 .6693023574976E-04  
-----  
TOTAL 407 118 633 225 7.53 (seconds) proc= 28.99%  
  
3 CG-PRP Algorithm. Function:Elastic-Plastic Torsion Problem  
Polak-Ribiere and Polyak. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 388 118 615 226 731 -.4392661058352E+00 .5025911855389E-04  
-----  
TOTAL 388 118 615 226 7.31 (seconds) proc= 30.41%  
  
4 CG-PRP+ Algorithm. Function:Elastic-Plastic Torsion Problem  
Polak-Ribiere and Polyak plus. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 388 118 615 226 732 -.4392661058352E+00 .5025911855389E-04  
-----  
TOTAL 388 118 615 226 7.32 (seconds) proc= 30.41%
```

5 CG-CD Algorithm. Function:Elastic-Plastic Torsion Problem
Conjugate Descent - Fletcher. stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	442	135	689	246	811	-.4392660177220E+00	.6894213365544E-04
TOTAL	442	135	689	246	8.11 (seconds)	proc=	30.54%

6 CG-LS Algorithm. Function:Elastic-Plastic Torsion Problem
Liu and Storey. stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	441	124	704	262	841	-.4392667629587E+00	.5686798908306E-04
TOTAL	441	124	704	262	8.41 (seconds)	proc=	28.12%

7 CG-DY Algorithm. Function:Elastic-Plastic Torsion Problem
Dai and Yuan. stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	396	103	615	218	735	-.4392668806583E+00	.5539177164620E-04
TOTAL	396	103	615	218	7.35 (seconds)	proc=	26.01%

8 CG-DL Algorithm. Function:Elastic-Plastic Torsion Problem
Dai and Liao (t=1). stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	406	110	620	213	738	-.4392677432931E+00	.4643440652585E-04
TOTAL	406	110	620	213	7.38 (seconds)	proc=	27.09%

9 CG-DL+ Algorithm. Function:Elastic-Plastic Torsion Problem
Dai and Liao plus (t=1). stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	435	121	679	243	810	-.4392659416678E+00	.6802904252760E-04
TOTAL	435	121	679	243	8.10 (seconds)	proc=	27.82%

10 CG-SDC Algorithm. Function:Elastic-Plastic Torsion Problem
Andrei - Sufficient Descent Condition. stoptest= 1
aCGSD: beta = gtg/yts - ytg*stg/(yts*yts)

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	10001	9934	10046	45	12893	-.4343571798784E+00	.9029803144368E-02
TOTAL	10001	9934	10046	45	128.93 (seconds)	proc=	99.33%

11 CG-hDY Algorithm. Function:Elastic-Plastic Torsion Problem
hDY:Hybrid Dai and Yuan. max(c*DY,min(HS,DY)). stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	441	128	704	262	838	-.4392666064706E+00	.6071848374273E-04
TOTAL	441	128	704	262	8.38 (seconds)	proc=	29.02%

12 CG-hDYz Algorithm. Function:Elastic-Plastic Torsion Problem
hDYz: Hybrid Dai and Yuan zero. max(0,min(HS,DY)). stoptest= 1

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
---	------	-----	-------	-------	---------	-------	-------

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40000 441 128 704 262 842 -.4392666064706E+00 .6071848374273E-04
-----
TOTAL 441 128 704 262 8.42 (seconds) proc= 29.02%

13 CG-GN Algorithm. Function:Elastic-Plastic Torsion Problem
Gilbert and Nocedal. stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 360 118 567 206 674 -.4392653353816E+00 .6340222266705E-04
-----
TOTAL 360 118 567 206 6.74 (seconds) proc= 32.78%

14 CG-HuS Algorithm. Function:Elastic-Plastic Torsion Problem
PRP-FR: Hu and Storey. max(0,min(PRP,FR)). stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 360 118 567 206 672 -.4392653353816E+00 .6340222266705E-04
-----
TOTAL 360 118 567 206 6.72 (seconds) proc= 32.78%

15 CG-TAS Algorithm. Function:Elastic-Plastic Torsion Problem
Touat-Ahmed and Storey. (PRP <= FR). stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 360 118 567 206 675 -.4392653353816E+00 .6340222266705E-04
-----
TOTAL 360 118 567 206 6.75 (seconds) proc= 32.78%

16 LS-CD Algorithm. Function:Elastic-Plastic Torsion Problem
Hybrid Liu&Storey-Conjugate Descent. max(0,min(LS,CD)) stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 461 128 735 273 875 -.4392657108720E+00 .6453101786929E-04
-----
TOTAL 461 128 735 273 8.75 (seconds) proc= 27.77%

17 CG-BM Algorithm. Function:Elastic-Plastic Torsion Problem
Birgin and Martinez (scaled Perry). Theta spectral. stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 393 114 607 213 727 -.4392662851099E+00 .6548988353399E-04
-----
TOTAL 393 114 607 213 7.27 (seconds) proc= 29.01%

18 CG-BM+ Algorithm. Function:Elastic-Plastic Torsion Problem
Birgin and Martinez plus (scaled Perry). Theta spectral. stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 369 100 573 203 686 -.4392667077490E+00 .6669242382365E-04
-----
TOTAL 369 100 573 203 6.86 (seconds) proc= 27.10%

19 CG-sPRP Algorithm. Function:Elastic-Plastic Torsion Problem
Scaled Polak-Ribiere and Polyak. Theta spectral. stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 413 116 663 249 793 -.4392663680385E+00 .6193243390135E-04
-----
TOTAL 413 116 663 249 7.93 (seconds) proc= 28.09%

20 CG-sFR Algorithm. Function:Elastic-Plastic Torsion Problem
Scaled Fletcher and Revees. Theta spectral. stoptest= 1

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```

n    iter    irs    fgcnt lscnt    time(c)      fxnew          gnorm
-----
40000    467    138    738    270        883 -.4392664993328E+00 .5341923652523E-04
-----
TOTAL    467    138    738    270        8.83 (seconds) proc= 29.55%


21  New CG Algorithm. Function:Elastic-Plastic Torsion Problem
    Andrei New Sufficient Descent from PRP. stoptest= 1
    PRP-SDC: beta = (ytg-yty*stg/gtgprev)/yts
    n    iter    irs    fgcnt lscnt    time(c)      fxnew          gnorm
-----
40000    435    127    696    260        838 -.4392664453296E+00 .5306522750304E-04
-----
TOTAL    435    127    696    260        8.38 (seconds) proc= 29.20%


22  New CG Algorithm. Function:Elastic-Plastic Torsion Problem
    Andrei New Sufficient Descent from DY. stoptest= 1
    ACGSD: beta=ytg/yts-ytg*stg/(yts**2)
    n    iter    irs    fgcnt lscnt    time(c)      fxnew          gnorm
-----
40000    376     99    578    201        694 -.4392673575219E+00 .4909018905049E-04
-----
TOTAL    376     99    578    201        6.94 (seconds) proc= 26.33%


23  New CG Algorithm. Function:Elastic-Plastic Torsion Problem
    Andrei Sufficient Descent from DY (max). stoptest= 1
    ACGSDz: beta=max(0,ytg/yts)*(1-stg/yts)
    n    iter    irs    fgcnt lscnt    time(c)      fxnew          gnorm
-----
40000    360     99    561    200        669 -.4392676068443E+00 .6107996580066E-04
-----
TOTAL    360     99    561    200        6.69 (seconds) proc= 27.50%


24  New CG Algorithm. Function:Elastic-Plastic Torsion Problem
    PRP - DY with Newton direction. (NDOMB) stoptest= 1
    n    iter    irs    fgcnt lscnt    time(c)      fxnew          gnorm
-----
40000    406    114    627    220        749 -.4392656198067E+00 .6655180118190E-04
-----
TOTAL    406    114    627    220        7.49 (seconds) proc= 28.08%


25  New CG Algorithm. Function:Elastic-Plastic Torsion Problem
    PRP - DY with Conjugacy Condition. (CCOMB) stoptest= 1
    n    iter    irs    fgcnt lscnt    time(c)      fxnew          gnorm
-----
40000    403    115    628    224        753 -.4392676684103E+00 .5504297139146E-04
-----
TOTAL    403    115    628    224        7.53 (seconds) proc= 28.54%

```

2. Accelerated Conjugate Gradient Algorithms

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*-----  
| Fastest Conjugate Gradient Algorithm Project  
| April 1, 2009  
| stoptest = 1  
| maxiter = 10000  
| Accelerated Conjugate Gradient with Powell restart.  
*-----  
  
1 CG-HS Algorithm. Function:Elastic-Plastic Torsion Problem  
Hestenes and Stiefel. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 241 0 510 27 609 -.4392678188055E+00 .3175820769772E-04  
-----  
TOTAL 241 0 510 27 6.09 (seconds) proc= .00%  
  
2 CG-FR Algorithm. Function:Elastic-Plastic Torsion Problem  
Fletcher and Reeves. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 241 0 510 27 597 -.4392678188055E+00 .3175915504546E-04  
-----  
TOTAL 241 0 510 27 5.97 (seconds) proc= .00%  
  
3 CG-PRP Algorithm. Function:Elastic-Plastic Torsion Problem  
Polak-Ribiere and Polyak. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 241 0 510 27 592 -.4392678188056E+00 .3175864821436E-04  
-----  
TOTAL 241 0 510 27 5.92 (seconds) proc= .00%  
  
4 CG-PRP+ Algorithm. Function:Elastic-Plastic Torsion Problem  
Polak-Ribiere and Polyak plus. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 241 0 510 27 588 -.4392678188056E+00 .3175864821436E-04  
-----  
TOTAL 241 0 510 27 5.88 (seconds) proc= .00%  
  
5 CG-CD Algorithm. Function:Elastic-Plastic Torsion Problem  
Conjugate Descent - Fletcher. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 241 0 510 27 582 -.4392678188045E+00 .3180171595878E-04  
-----  
TOTAL 241 0 510 27 5.82 (seconds) proc= .00%  
  
6 CG-LS Algorithm. Function:Elastic-Plastic Torsion Problem  
Liu and Storey. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----  
40000 241 0 510 27 588 -.4392678188055E+00 .3175850148118E-04  
-----  
TOTAL 241 0 510 27 5.88 (seconds) proc= .00%  
  
7 CG-DY Algorithm. Function:Elastic-Plastic Torsion Problem  
Dai and Yuan. stoptest= 1  
  
n iter irs fgcnt lscnt time(c) fxnew gnorm  
-----
```

```

40000 241 0 510 27 596 -.4392678188043E+00 .3180653373005E-04
-----
TOTAL 241 0 510 27 5.96 (seconds) proc= .00%

8 CG-DL Algorithm. Function:Elastic-Plastic Torsion Problem
Dai and Liao (t=1). stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 241 0 510 27 593 -.4392678188055E+00 .3175800592310E-04
-----
TOTAL 241 0 510 27 5.93 (seconds) proc= .00%

9 CG-DL+ Algorithm. Function:Elastic-Plastic Torsion Problem
Dai and Liao plus (t=1). stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 241 0 510 27 592 -.4392678188055E+00 .3175904049287E-04
-----
TOTAL 241 0 510 27 5.92 (seconds) proc= .00%

10 CG-SDC Algorithm. Function:Elastic-Plastic Torsion Problem
Andrei - Sufficient Descent Condition. stoptest= 1
aCGSD: beta = gtg/yts - ytg*stg/(yts*yts)
n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 241 0 510 27 591 -.4392678188056E+00 .3175752837159E-04
-----
TOTAL 241 0 510 27 5.91 (seconds) proc= .00%

11 CG-hDY Algorithm. Function:Elastic-Plastic Torsion Problem
hDY: Hybrid Dai and Yuan. max(c*DY,min(HS,DY)). stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 428 83 1092 235 1271 -.4392672102975E+00 .4043265483200E-04
-----
TOTAL 428 83 1092 235 12.71 (seconds) proc= 19.39%

12 CG-hDYz Algorithm. Function:Elastic-Plastic Torsion Problem
hDYz: Hybrid Dai and Yuan zero. max(0,min(HS,DY)). stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 428 83 1092 235 1270 -.4392672102975E+00 .4043265483200E-04
-----
TOTAL 428 83 1092 235 12.70 (seconds) proc= 19.39%

13 CG-GN Algorithm. Function:Elastic-Plastic Torsion Problem
Gilbert and Nocedal. stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 241 0 510 27 588 -.4392678188054E+00 .3176498414142E-04
-----
TOTAL 241 0 510 27 5.88 (seconds) proc= .00%

14 CG-HuS Algorithm. Function:Elastic-Plastic Torsion Problem
PRP-FR: Hu and Storey. max(0,min(PRP,FR)). stoptest= 1

n iter irs fgcnt lscnt time(c) fxnew gnorm
-----
40000 241 0 510 27 590 -.4392678188054E+00 .3176498414142E-04
-----
TOTAL 241 0 510 27 5.90 (seconds) proc= .00%

15 CG-TAS Algorithm. Function:Elastic-Plastic Torsion Problem
Touat-Ahmed and Storey. (PRP <= FR). stoptest= 1

```

n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	241	0	510	27	588	-.4392678188054E+00	.3176498414142E-04
TOTAL	241	0	510	27	5.88 (seconds)	proc=	.00%
16	LS-CD Algorithm. Function:Elastic-Plastic Torsion Problem Hybrid Liu&Storey-Conjugate Descent. max(0,min(LS,CD)) stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	241	0	510	27	591	-.4392678188056E+00	.3175812838347E-04
TOTAL	241	0	510	27	5.91 (seconds)	proc=	.00%
17	CG-BM Algorithm. Function:Elastic-Plastic Torsion Problem Birgin and Martinez (scaled Perry). Theta spectral. stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	241	0	510	27	593	-.4392678188056E+00	.3175774391693E-04
TOTAL	241	0	510	27	5.93 (seconds)	proc=	.00%
18	CG-BM+ Algorithm. Function:Elastic-Plastic Torsion Problem Birgin and Martinez plus (scaled Perry). Theta spectral. stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	241	0	510	27	594	-.4392678188055E+00	.3175819224527E-04
TOTAL	241	0	510	27	5.94 (seconds)	proc=	.00%
19	CG-sPRP Algorithm. Function:Elastic-Plastic Torsion Problem Scaled Polak-Ribiere and Polyak. Theta spectral. stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	423	83	1071	224	1247	-.4392659302976E+00	.6410421638038E-04
TOTAL	423	83	1071	224	12.47 (seconds)	proc=	19.62%
20	CG-sFR Algorithm. Function:Elastic-Plastic Torsion Problem Scaled Fletcher and Revees. Theta spectral. stoptest= 1						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	469	106	1184	245	1384	-.4392675687609E+00	.6679284527663E-04
TOTAL	469	106	1184	245	13.84 (seconds)	proc=	22.60%
21	New CG Algorithm. Function:Elastic-Plastic Torsion Problem Andrei New Sufficient Descent from PRP. stoptest= 1 PRP-SDC: beta = (ytg-yty*stg/gtgprev)/yts						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	463	87	1202	275	1400	-.4392677974666E+00	.3766831541914E-04
TOTAL	463	87	1202	275	14.00 (seconds)	proc=	18.79%
22	New CG Algorithm. Function:Elastic-Plastic Torsion Problem Andrei New Sufficient Descent from DY. stoptest= 1 ACGSD: beta=ytg/yts-ytg*stg/(yts**2)						
n	iter	irs	fgcnt	lscnt	time(c)	fxnew	gnorm
40000	241	0	510	27	594	-.4392678188056E+00	.3175789503236E-04
TOTAL	241	0	510	27	5.94 (seconds)	proc=	.00%

```

23  New CG Algorithm. Function:Elastic-Plastic Torsion Problem
    Andrei Sufficient Descent from DY (max). stoptest= 1
    ACGSDz: beta=max(0,ytg/yts)*(1-stg/yts)

    n   iter   irs   fgcnt lscnt   time(c)       fxnew           gnorm
-----
40000   241     0     510     27      594 -.4392678188056E+00 .3175762053098E-04
-----
TOTAL   241     0     510     27      5.94 (seconds) proc=.00%

24  New CG Algorithm. Function:Elastic-Plastic Torsion Problem
    PRP - DY with Newton direction. (NDOMB) stoptest= 1
    n   iter   irs   fgcnt lscnt   time(c)       fxnew           gnorm
-----
40000   241     0     510     27      591 -.4392678188056E+00 .3175782091383E-04
-----
TOTAL   241     0     510     27      5.91 (seconds) proc=.00%

25  New CG Algorithm. Function:Elastic-Plastic Torsion Problem
    PRP - DY with Conjugacy Condition. (CCOMB) stoptest= 1
    n   iter   irs   fgcnt lscnt   time(c)       fxnew           gnorm
-----
40000   241     0     510     27      591 -.4392678188056E+00 .3175802064054E-04
-----
TOTAL   241     0     510     27      5.91 (seconds) proc=.00%

```

Remarks.

This problem belongs to the MINPACK2 collection.

These results have been obtained with APPL1.FOR package which implements 25 conjugate gradient algorithms in classical and accelerated versions. To use the accelerated version of these algorithms the value of the logical parameter accel must be set to .true.

Observe the differences between cpu time corresponding to the conjugate gradient algorithms and their accelerated versions.

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