

UOP

A collection of 80 unconstrained optimization test problems

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Technical Report No. 7/2018
November 17, 2018

The unconstrained optimization test problems selected in this set, we call *UOP collection*, have different structures and complexities. The name of these problems and the initial point are given in Table 1. In this collection some problems are quadratic and some of them are highly nonlinear. The problems are presented in extended (separable) or generalized (chained) form. The Hessian for the problems in extended form has a block-diagonal structure. On the other hand, the Hessian for the problems in generalized form has a banded structure with small bandwidth, often being tri- or penta-diagonal. For some other optimization problems, from this set, the corresponding Hessian has a sparse structure, or it is a dense (full) matrix. The vast majority of the optimization problems included in our collection is taken from CUTER (Bongartz, Conn, Gould & Toint, 1995) collection. Others are from Andrei [1, 2, 3, 4, 5, 6], as well as from other publications.

Table 1
UOP collection of unconstrained optimization test problems.

No.	Problem name	Initial point
1	Extended Freudenstein & Roth	[0.5,-2,...,0.5,2]
2	Extended Trigonometric ET1	[0.2,0.2,...,0.2]
3	Extended Rosenbrock	[-1.2,1,-1.2,1,...,-1.2,1]
4	Extended White & Holst	[-1.2,1,-1.2,1,...,-1.2,1]
5	Extended Beale	[1,0.8,...,1,0.8]
6	Extended penalty	[1/100,2/100,3/100,...,n/100]
7	Perturbed quadratic	[1,1,...,1]
8	Raydan 1	[1,1,...,1]
9	Raydan 2	[1,1,...,1]
10	TR-SUMM	[1,1,...,1]
11	Diagonal 1	[1/1,1/2,...,1/n]
12	Diagonal 2	[20,20,...,20]]
13	Hager	[1,1,...,1]
14	Generalized tridiagonal 1	[2,2,...,2]
15	Extended tridiagonal 1	[2,2,...,2]
16	Extended three exponential terms	[0.1,0.1,...,0.1]
17	Generalized Tridiagonal 2	[-1,-1,...,-1]
18	Diagonal 3	[1,1,...,1]
19	Diagonal full borded	[0.001,0.001,...,0.001]
20	Extended Himmelblau HIMMELBC (CUTE)	[1,...,1]
21	Extended Powell	[3,-1,0,1,...,3,-1,0,1]
22	Tridiagonal double borded arrow up	[-1,-1,...,-1]
23	Extended PCS1	[3,0.1,3,0.1,...,3,0.1]
24	Extended block-diagonal BD1	[1,1,...,1]
25	Extended Maratos	[1.1,0.1,...,1.1,0.1]

26	Full Hessian FH1	[1/n,2/n,...,n/n]
27	Extended Cliff	[0.001,-0.001,...,0.001,-0.001]
28	Quadratic diagonal perturbed	[0.5,0.5,...,0.5]
29	Full Hessian FH2	[1,1,...,1]
30	Full Hessian FH3	[1,1,...,1]
31	Tridiagonal double borded NONDQUAR (CUTE)	[1,-1,1,-1,...,1,-1]
32	Tridiagonal White & Holst	[-1.2,1,...,-1.2,1]
33	Diagonal double borded arrow up	[0.4,1,0.4,1,...,0.4,1]
34	TRIDIA (CUTE)	[1,1,...,1]
35	ARWHEAD (CUTE)	[1,1,...,1]
36	NONDIA (CUTE)	[-0.01,-0.01,...,-0.01]
37	Extended Woods (CUTE)	[-3,-1,-3,-1,...,-3,-1]
38	Extended Hiebert	[5.001,5.001,...,5.001]]
39	BDQRTIC (CUTE)	[1,1,...,1]
40	DQDRTIC (CUTE)	[3,3,...,3]
41	EG2 (CUTE)	[0.001,0.001,...,0.001]
42	EDENSCH (CUTE)	[0,0,...,0]
43	Broyden pentadiagonal (CUTE)	[-1,-1,...,-1]
44	Almoust perturbed quadratic	[0.5,0.5,...,0.5]
45	Almost perturbed quartic	[0.5,0.5,...,0.5]
46	FLETCHER (CUTE)	[0.5,0.5,...,0.5]
47	ENGVAL1 (CUTE)	[2,2,...,2]
48	DENSCHNA (CUTE)	[1,1,...,1]
49	DENSCHNB (CUTE)	[10,10,...,10]
50	DENSCHNC (CUTE)	[1,1,...,1]
51	DENSCHNF (CUTE)	[100,-100,...,100,-100]
52	SINQUAD (CUTE)	[0,0,...,0]
53	HIMMELBG (CUTE)	[1.5,1.5,...,1.5]
54	HIMMELBH (CUTE)	[0.8,0.8,...,0.8]
55	DIXON3DQ (CUTE)	[-0.1,-0.1,...,-0.1]
56	BIGGSB1 (CUTE)	[0.1,0.1,...,0.1]
57	Perturbed quadratic	[0.5,0.5,...,0.5]
58	GENROSNB (CUTE)	[-1.2,1.1,-1.2,1.1,...,-1.2,1.1]
59	Extended quadratic penalty QP1	[1,1,...,1]
60	Extended quadratic penalty QP2	[2,2,...,2]
61	Tridiagonal TS1	[1,1,...,1]
62	Tridiagonal TS2	[1,1,...,1]
63	Tridiagonal TS3	[2,2,...,2]
64	Extended Trigonometric ET2	[0.2,0.2,...,0.2]
65	Extended quadratic penalty QP3	[1,1,...,1]
66	EG1	[0.02,0.02,...,0.02]
67	GENROSEN-2	[-1.2,1,-1.2,1,...,-1.2,1]
68	PRODsin (m=n-1)	[0.00001,...,0.00001]
69	PROD1 (m=n)	[1,1,...,1]
70	PRODcos(m=n-1)	[1,0,0,...,0]
71	PROD2 (m=1)	[[0.00001,0.00001,...,0.00001,1]]
72	ARGLINB (m=5)	[0.01,0.001,0.01,0.001,...,0.01,0.001]
73	DIXMAANA (CUTE)	[2,2,...,2]
74	DIXMAANB (CUTE)	[2,2,...,2]
75	DIXMAANC (CUTE)	[2,2,...,2]
76	DIXMAAND (CUTE)	[2,2,...,2]
77	DIXMAANE (CUTE)	[2,2,...,2]
78	VARDIM (CUTE)	[1-1/n,1-2/n,...,1-n/n]
79	DIAG-AUP1	[4.4,...,4]
80	ENGVAL8 (CUTE)	[2,2,...,2]

The algebraic description of the problems is as follows:

1. Freudenstein & Roth FREUROTH (CUTE)

$$f(x) = \sum_{i=1}^{n/2} \left(-13 + x_{2i-1} + ((5 - x_{2i})x_{2i} - 2)x_{2i} \right)^2 + \left(-29 + x_{2i-1} + ((x_{2i} + 1)x_{2i} - 14)x_{2i} \right)^2,$$

$$x_0 = [0.5, -2, 0.5, -2, \dots, 0.5, -2].$$

2. Extended Trigonometric ET1

$$f(x) = \sum_{i=1}^n \left(\left(n - \sum_{j=1}^n \cos x_j \right) + i(1 - \cos x_i) - \sin x_i \right)^2,$$

$$x_0 = [0.2, 0.2, \dots, 0.2].$$

3. Extended Rosenbrock SROSENBR (CUTE)

$$f(x) = \sum_{i=1}^{n/2} c \left(x_{2i} - x_{2i-1}^2 \right)^2 + \left(1 - x_{2i-1} \right)^2, \quad x_0 = [-1.2, 1, \dots, -1.2, 1], \quad c = 1000.$$

4. Extended White & Holst

$$f(x) = \sum_{i=1}^{n/2} c \left(x_{2i} - x_{2i-1}^3 \right)^2 + \left(1 - x_{2i-1} \right)^2, \quad x_0 = [-1.2, 1, \dots, -1.2, 1], \quad c = 1.$$

5. Extended Beale BEALE (CUTE)

$$f(x) = \sum_{i=1}^{n/2} \left(1.5 - x_{2i-1} (1 - x_{2i}) \right)^2 + \left(2.25 - x_{2i-1} (1 - x_{2i}^2) \right)^2 + \left(2.625 - x_{2i-1} (1 - x_{2i}^3) \right)^2,$$

$$x_0 = [1, 0.8, \dots, 1, 0.8].$$

6. Extended Penalty

$$f(x) = \sum_{i=1}^{n-1} (x_i - 1)^2 + \left(\sum_{j=1}^n x_j^2 - 0.25 \right)^2, \quad x_0 = [1/100, \dots, n/100].$$

7. Perturbed Quadratic

$$f(x) = \sum_{i=1}^n i x_i^2 + \frac{1}{100} \left(\sum_{i=1}^n x_i \right)^2, \quad x_0 = [1, 1, \dots, 1]$$

8. Raydan 1

$$f(x) = \sum_{i=1}^n \frac{i}{10} (\exp(x_i) - x_i), \quad x_0 = [1, 1, \dots, 1].$$

9. Raydan 2

$$f(x) = \sum_{i=1}^n (\exp(x_i) - x_i), \quad x_0 = [1, 1, \dots, 1].$$

10. TR-SUMM of quadratics

$$f(x) = \sum_{i=1}^{n-1} x_i^2 + c(x_{i+1} + x_i^2)^2 \quad x_0 = [1, 1, \dots, 1]. \quad c = 100000.$$

11. Diagonal 1

$$f(x) = \sum_{i=1}^n \left(\exp(x_i) - \frac{x_i}{i} \right), \quad x_0 = [1/1, 1/2, \dots, 1/n].$$

12. Diagonal 2

$$f(x) = \sum_{i=1}^n (\exp(x_i) - i \sin(x_i)), \quad x_0 = [20, 20, \dots, 20].$$

13. Hager

$$f(x) = \sum_{i=1}^n (\exp(x_i) - \sqrt{i}x_i), \quad x_0 = [1, 1, \dots, 1].$$

14. Generalized Tridiagonal 1

$$f(x) = \sum_{i=1}^{n-1} (x_i + x_{i+1} - 3)^2 + (x_i - x_{i+1} + 1)^4, \quad x_0 = [2, 2, \dots, 2].$$

15. Extended Tridiagonal 1

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1} + x_{2i} - 3)^2 + (x_{2i-1} - x_{2i} + 1)^4, \quad x_0 = [2, 2, \dots, 2].$$

16. Extended Three Expo Terms

$$f(x) = \sum_{i=1}^{n/2} (\exp(x_{2i-1} + 3x_{2i} - 0.1) + \exp(x_{2i-1} - 3x_{2i} - 0.1) + \exp(-x_{2i-1} - 0.1)), \\ x_0 = [0.1, 0.1, \dots, 0.1].$$

17. Generalized Tridiagonal 2 (Pentadiagonal)

$$f(x) = ((5 - 3x_1 - x_1^2)x_1 - 3x_2 + 1)^2 + \\ \sum_{i=1}^{n-1} ((5 - 3x_i - x_i^2)x_i - x_{i-1} - 3x_{i+1} + 1)^2 + ((5 - 3x_n - x_n^2)x_n - x_{n-1} + 1)^2, \\ x_0 = [-1, -1, \dots, -1].$$

18. Diagonal 3

$$f(x) = \frac{1}{2} \sum_{i=1}^{n/2} (x_{2i-1}^2 + cx_{2i}^2), \quad x_0 = [1, 1, \dots, 1]. \quad c = 10000.$$

19. Diagonal Full Banded

$$f(x) = (x_1 - 1)^4 + (x_n^2 - x_1^2)^2 + \sum_{i=1}^{n-2} (\sin(x_{i+1} - x_n) - x_1^2 - x_{i+1}^2)^2, \quad x_0 = [0.001, 0.001, \dots, 0.001].$$

20. Extended Himmelblau HIMMELBC (CUTE)

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1}^2 + x_{2i} - 11)^2 + (x_{2i-1} + x_{2i}^2 - 7)^2, \quad x_0 = [1, 1, \dots, 1].$$

21. Extended Powell

$$\begin{aligned} f(x) = & \sum_{i=1}^{n/4} (x_{4i-3} + 10x_{4i-2})^2 + 5(x_{4i-1} - x_{4i})^2 + \\ & (x_{4i-2} - 2x_{4i-1})^4 + 10(x_{4i-3} - x_{4i})^4, \\ x_0 = & [3, -1, 0, 1, \dots, 3, -1, 0, 1]. \end{aligned}$$

22. Tridiagonal Double Banded Arrow Up

$$f(x) = (x_1 - 1)^2 + \sum_{i=1}^{n-1} (x_i - 0.5x_i^2 - 0.5x_{i+1}^2)^2, \quad x_0 = [-1, -1, \dots, -1].$$

23. Extended PSC1

$$\begin{aligned} f(x) = & \sum_{i=1}^{n/2} (x_{2i-1}^2 + x_{2i}^2 + x_{2i-1}x_{2i})^2 + \sin^2(x_{2i-1}) + \cos^2(x_{2i}), \\ x_0 = & [3, 0.1, \dots, 3, 0.1]. \end{aligned}$$

24. Extended Block-Diagonal BD1

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1}^2 + x_{2i}^2 - 2)^2 + (\exp(x_{2i-1}) - x_{2i})^2, \quad x_0 = [1, 1, \dots, 1].$$

25. Extended Maratos

$$f(x) = \sum_{i=1}^{n/2} x_{2i-1} + c(x_{2i-1}^2 + x_{2i}^2 - 1)^2, \quad x_0 = [1.1, 0.1, \dots, 1.1, 0.1], \quad c = 1.$$

26. Full Hessian FH1 (Sum of quadratics, quadratic inside)

$$\begin{aligned} f(x) = & \sum_{i=1}^m \left(\sum_{j=1}^n i j x_j^2 - 1 \right)^2, \quad m = 50, \\ x_0 = & [1/n, 2/n, \dots, n/n]. \end{aligned}$$

27. Extended Cliff

$$\begin{aligned} f(x) = & \sum_{i=1}^{n/2} \left(\frac{x_{2i-1} - 3}{100} \right)^2 - (x_{2i-1} - x_{2i}) + \exp(2(x_{2i-1} - x_{2i})), \\ x_0 = & [0.001, -0.001, \dots, 0.001, -0.001]. \end{aligned}$$

28. Quadratic Diagonal Perturbed

$$f(x) = \left(\sum_{i=1}^n x_i \right)^2 + \sum_{i=1}^n \frac{i}{100} x_i^2, \quad x_0 = [0.5, 0.5, \dots, 0.5].$$

29. Full Hessian FH2

$$f(x) = \left(\sum_{i=1}^n x_i \right)^2 + \sum_{i=1}^n i(\sin(x_i) + \cos(x_i)) / 1000, \quad x_0 = [1, 1, \dots, 1].$$

30. Full Hessian FH3

$$f(x) = \left(\sum_{i=1}^n x_i^2 \right)^2 + \sum_{i=1}^n i(\sin(x_i) + \cos(x_i))/1000, \quad x_0 = [1, 1, \dots, 1].$$

31. Tridiagonal Double Banded – NONDQUAR (CUTE)

$$f(x) = (x_1 - x_2)^2 + \sum_{i=1}^{n-2} (x_i + x_{i+1} + x_n)^4 + (x_{n-1} + x_n)^2, \quad x_0 = [1., -1., \dots, 1., -1.,].$$

32. Tridiagonal White & Holst (c=4)

$$f(x) = \sum_{i=1}^{n-1} c(x_{i+1} - x_i^3)^2 + (1 - x_i)^2, \quad x_0 = [-1.2, 1, \dots, -1.2, 1], \quad c = 4.$$

33. Diagonal Double Banded Arrow Up

$$f(x) = \sum_{i=1}^n 4(x_i^2 - x_1)^2 + (x_i - 1)^2, \quad x_0 = [0.4, 1, \dots, 0.4, 1].$$

34. TRIDIA (CUTE)

$$f(x) = \gamma(\delta x_1 - 1)^2 + \sum_{i=2}^n i(\alpha x_i - \beta x_{i-1})^2, \\ \alpha = 2, \quad \beta = 1, \quad \gamma = 1, \quad \delta = 1, \quad x_0 = [1, 1, \dots, 1].$$

35. ARWHEAD (CUTE)

$$f(x) = \sum_{i=1}^{n-1} (-4x_i + 3) + \sum_{i=1}^{n-1} (x_i^2 + x_n^2)^2, \quad x_0 = [1., 1., \dots, 1.].$$

36. NONDIA (CUTE) (Shanno)

$$f(x) = (x_1 - 1)^2 + c(x_1 - x_1^2)^2 + \sum_{i=2}^n c(x_1 - x_i^2)^2, \quad x_0 = [-0.01, -0.01, \dots, -0.01], \quad c = 100.$$

37. Extended WOODS (CUTE)

$$f(x) = \sum_{i=1}^{n/4} 100(x_{4i-3}^2 - x_{4i-2})^2 + (x_{4i-3} - 1)^2 + 90(x_{4i-1}^2 - x_{4i})^2 + \\ (1 - x_{4i-1})^2 + 10.1 \left\{ (x_{4i-2} - 1)^2 + (x_{4i} - 1)^2 \right\} + 19.8(x_{4i-2} - 1)(x_{4i} - 1), \\ x_0 = [-3, -1, -3, -1, \dots, -3, -1, -3, -1].$$

38. Extended Hiebert

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1} - 10)^2 + (x_{2i-1}x_{2i} - 500)^2, \quad x_0 = [5.001, 5.001, \dots, 5.001].$$

39. BDQRTIC (CUTE)

$$f(x) = \sum_{i=1}^{n-4} (-4x_i + 3)^2 + (x_i^2 + 2x_{i+1}^2 + 3x_{i+2}^2 + 4x_{i+3}^2 + 5x_n^2)^2, \\ x_0 = [1., 1., \dots, 1.].$$

40. DQDRTIC (CUTE)

$$f(x) = \sum_{i=1}^{n-2} (x_i^2 + cx_{i+1}^2 + dx_{i+2}^2), \quad c = 1000, \quad d = 1000, \quad x_0 = [3., 3., \dots, 3.]$$

41. EG2 (CUTE)

$$f(x) = \sum_{i=1}^{n-1} \sin(x_i + x_i^2 - 1) + \frac{1}{2} \sin(x_n^2), \quad x_0 = [0.001, 0.001, \dots, 0.001]$$

42. EDENSCH (CUTE)

$$f(x) = 16 + \sum_{i=1}^{n-1} [(x_i - 2)^4 + (x_i x_{i+1} - 2x_{i+1})^2 + (x_{i+1} + 1)^2], \quad x_0 = [0, 0, \dots, 0]$$

43. Broyden Pentadiagonal (CUTE)

$$f(x) = (3x_1 - 2x_1^2)^2 + \sum (3x_i - 2x_i^2 - x_{i-1} - 2x_{i+1} + 1)^2 + (3x_n + 2x_n^2 - x_{n-1} + 1)^2,$$

$$x_0 = [-1, -1, \dots, -1]$$

44. Almost Perturbed Quadratic

$$f(x) = (x_1 + x_n)^2 / 100 + \sum_{i=1}^n i x_i^2, \quad x_0 = [0.5, 0.5, \dots, 0.5]$$

45. Almost Perturbed Quartic

$$f(x) = (x_1 + x_n)^2 / 100 + \sum_{i=1}^n i x_i^4, \quad x_0 = [0.5, 0.5, \dots, 0.5]$$

46. FLETCHCR (CUTE)

$$f(x) = \sum_{i=1}^{n-1} c (x_{i+1} - x_i + 1 - x_i^2)^2, \quad x_0 = [0.5, 0.5, \dots, 0.5], \quad c = 100$$

47. ENGVAL1 (CUTE)

$$f(x) = \sum_{i=1}^{n-1} (x_i^2 + x_{i+1}^2)^2 + \sum_{i=1}^{n-1} (-4x_i + 3), \quad x_0 = [2, 2, \dots, 2]$$

48. DENSCHNA (CUTE)

$$f(x) = \sum_{i=1}^{n/2} x_{2i-1}^4 + (x_{2i-1} + x_{2i})^2 + (-1 + \exp(x_{2i}))^2, \quad x_0 = [1., 1., \dots, 1.]$$

49. DENSCHNB (CUTE)

$$f(x) = \sum_{i=1}^{n/2} (x_{2i-1} - 2)^2 + (x_{2i-1} - 2)^2 x_{2i}^2 + (x_{2i} + 1)^2, \quad x_0 = [10, 10, \dots, 10]$$

50. DENSCHNC (CUTE)

$$f(x) = \sum_{i=1}^{n/2} (-2 + x_{2i-1}^2 + x_{2i}^2)^2 + (-2 + \exp(x_{2i-1} - 1) + x_{2i}^3)^2, \quad x_0 = [1, 1, \dots, 1]$$

51. DENSCHNF (CUTE)

$$f(x) = \sum_{i=1}^{n/2} \left(2(x_{2i-1} + x_{2i})^2 + (x_{2i-1} - x_{2i})^2 - 8 \right)^2 + \left(5x_{2i-1}^2 + (x_{2i} - 3)^2 - 9 \right)^2,$$

$$x_0 = [100, -100, \dots, 100, -100].$$

52. SINQUAD (CUTE)

$$f(x) = (x_1 - 1)^4 + (x_n^2 - x_1^2)^2 + \sum_{i=1}^{n-2} (\sin(x_{i+1} - x_i) - x_1^2 + x_{i+1}^2)^2, \quad x_0 = [0, 0, \dots, 0].$$

53. HIMMELBG (CUTE)

$$f(x) = \sum_{i=1}^{n/2} \left(2x_{2i-1}^2 + 3x_{2i}^2 \right) \exp(-x_{2i-1} - x_{2i}), \quad x_0 = [1.5, 1.5, \dots, 1.5].$$

54. HIMMELBH (CUTE)

$$f(x) = \sum_{i=1}^{n/2} (-3x_{2i-1} - 2x_{2i} + 2 + x_{2i-1}^3 + x_{2i}^2), \quad x_0 = [0.8, 0.8, \dots, 0.8].$$

55. DIXON3DQ (CUTE)

$$f(x) = (x_1 - 1)^2 + \sum_{i=1}^{n-1} (x_i - x_{i+1})^2 + (x_n - 1)^2, \quad x_0 = [-0.1, -0.1, \dots, -0.1].$$

56. BIGGSB1 (CUTE)

$$f(x) = (x_1 - 1)^2 + (1 - x_n)^2 + \sum_{i=2}^n (x_i - x_{i-1})^2, \quad x_0 = [0.1, 0.1, \dots, 0.1].$$

57. Perturbed Quadratic

$$f(x) = \left(\sum_{i=1}^n ix_i \right)^2 + \sum_{i=1}^n ix_i^2, \quad x_0 = [0.5, 0.5, \dots, 0.5],$$

58. GENROSNB (CUTE)

$$f(x) = (x_1 - 1)^2 + \sum_{i=2}^n 100(x_i - x_{i-1}^2)^2, \quad x_0 = [-1.2, 1, \dots, -1.2, 1].$$

59. Extended Quadratic Penalty QP1

$$f(x) = \left(\sum_{i=1}^n x_i^2 - 0.5 \right)^2 + \sum_{i=1}^{n-1} (x_i^2 - 2)^2, \quad x_0 = [1., 1., \dots, 1.].$$

60. Extended Quadratic Penalty QP2

$$f(x) = \left(\sum_{i=1}^n x_i^2 - 100 \right)^2 + \sum_{i=1}^{n-1} (x_i^2 - \sin(x_i))^2, \quad x_0 = [2, 2, \dots, 2].$$

61. Tridiagonal TS1 (Staircase S1)

$$f(x) = \sum_{i=1}^{n-1} (x_i + x_{i+1} - i)^2, \quad x_0 = [1., 1., \dots, 1.].$$

62. Tridiagonal TS2 (Staircase S2)

$$f(x) = \sum_{i=2}^n (x_{i-1} + x_i - i)^2, \quad x_0 = [1., 1., \dots, 1.].$$

63. Tridiagonal TS3 (Staircase S3)

$$f(x) = \sum_{i=2}^n (x_{i-1} + x_i + i)^2, \quad x_0 = [2., 2., \dots, 2.].$$

64. Extended Trigonometric ET2

$$f(x) = \sum_{i=1}^n \left(\left(n - \sum_{i=1}^n \sin(x_i) \right) + i(1 - \sin(x_i)) - \sin(x_i) \right)^2, \quad x_0 = [0.2, 0.2, \dots, 0.2].$$

65. Extended Quadratic Penalty QP3

$$f(x) = \left(\sum_{i=1}^n x_i^2 - 0.25 \right)^2 - \sum_{i=1}^{n-1} (x_i^2 - 1)^2, \quad x_0 = [1., 1., \dots, 1.].$$

66. EG1

$$f(x) = \frac{1}{2} \cos(x_n^2) + \sum_{i=1}^{n-1} \cos(x_1 + x_i^2 - 1), \quad x_0 = [0.02, 0.02, \dots, 0.02].$$

67. GENROSEN-2

$$f(x) = (x_1 - 1)^2 + \sum_{i=2}^n c(x_{i-1}^2 - x_i)^2, \quad x_0 = [-1.2, 1, \dots, -1.2, 1]. \quad c = 100.$$

68. PRODsin

$$f(x) = \left(\sum_{i=1}^m x_i^2 \right) \left(\sum_{i=1}^n \sin(x_i) \right), \quad x_0 = [0.00001, \dots, 0.00001]. \quad m = n - 1.$$

69. PROD1 (m=n)

$$f(x) = \left(\sum_{i=1}^m x_i \right) \left(\sum_{i=1}^n x_i \right), \quad x_0 = [1., 1., \dots, 1.]. \quad m = n.$$

70. PRODcos

$$f(x) = \left(\sum_{i=1}^m x_i^2 \right) \left(\sum_{i=1}^n \cos(x_i) \right), \quad x_0 = [1, 0, \dots, 0]. \quad m = n - 1.$$

71. PROD2 (m=1)

$$f(x) = \left(\sum_{i=1}^m x_i^4 \right) \left(\sum_{i=1}^n ix_i \right), \quad x_0 = [0.00001, \dots, 0.00001, 1]. \quad m = 1.$$

72. ARGLINB (m=5)

$$f(x) = \sum_{i=1}^m \left(\sum_{j=1}^n ijx_j - 1 \right)^2, \quad x_0 = [0.01, 0.001, \dots, 0.01, 0.001]. \quad m = 5.$$

73. DIXMAANA (CUTE)

$$\begin{aligned}
f(x) = & 1 + \sum_{i=1}^n \alpha x_i^2 \left(\frac{i}{n} \right)^{k1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left(\frac{i}{n} \right)^{k2} \\
& + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left(\frac{i}{n} \right)^{k3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left(\frac{i}{n} \right)^{k4}, \\
m = & n/3, \\
x_0 = & [2, 0, \dots, 2, 0], \\
\alpha = & 1, \quad \beta = 0, \quad \gamma = 0.125, \quad \delta = 0.125, \quad k1 = 0, \quad k2 = 0, \quad k3 = 0, \quad k4 = 0.
\end{aligned}$$

74. DIXMAANB (CUTE)

$$\begin{aligned}
f(x) = & 1 + \sum_{i=1}^n \alpha x_i^2 \left(\frac{i}{n} \right)^{k1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left(\frac{i}{n} \right)^{k2} \\
& + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left(\frac{i}{n} \right)^{k3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left(\frac{i}{n} \right)^{k4}, \\
m = & n/3, \\
x_0 = & [2, 0, \dots, 2, 0], \\
\alpha = & 1, \quad \beta = 0.0625, \quad \gamma = 0.0625, \quad \delta = 0.0625, \quad k1 = 0, \quad k2 = 0, \quad k3 = 0, \quad k4 = 1.
\end{aligned}$$

75. DIXMAANC (CUTE)

$$\begin{aligned}
f(x) = & 1 + \sum_{i=1}^n \alpha x_i^2 \left(\frac{i}{n} \right)^{k1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left(\frac{i}{n} \right)^{k2} \\
& + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left(\frac{i}{n} \right)^{k3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left(\frac{i}{n} \right)^{k4}, \\
m = & n/3, \\
x_0 = & [2, 0, \dots, 2, 0], \\
\alpha = & 1, \quad \beta = 0.125, \quad \gamma = 0.125, \quad \delta = 0.125, \quad k1 = 0, \quad k2 = 0, \quad k3 = 0, \quad k4 = 0.
\end{aligned}$$

76. DIXMAAND (CUTE)

$$\begin{aligned}
f(x) = & 1 + \sum_{i=1}^n \alpha x_i^2 \left(\frac{i}{n} \right)^{k1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left(\frac{i}{n} \right)^{k2} \\
& + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left(\frac{i}{n} \right)^{k3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left(\frac{i}{n} \right)^{k4}, \\
m = & n/3, \\
x_0 = & [2, 0, \dots, 2, 0], \\
\alpha = & 1, \quad \beta = 0.26, \quad \gamma = 0.26, \quad \delta = 0.26, \quad k1 = 0, \quad k2 = 0, \quad k3 = 0, \quad k4 = 0.
\end{aligned}$$

77. DIXMAANE (CUTE)

$$f(x) = 1 + \sum_{i=1}^n \alpha x_i^2 \left(\frac{i}{n} \right)^{k1} + \sum_{i=1}^{n-1} \beta x_i^2 (x_{i+1} + x_{i+1}^2)^2 \left(\frac{i}{n} \right)^{k2}$$

$$\begin{aligned}
& + \sum_{i=1}^{2m} \gamma x_i^2 x_{i+m}^4 \left(\frac{i}{n} \right)^{k3} + \sum_{i=1}^m \delta x_i x_{i+2m} \left(\frac{i}{n} \right)^{k4}, \\
& m = n/3, \\
& x_0 = [2, 0, \dots, 2, 0], \\
& \alpha = 1, \quad \beta = 0, \quad \gamma = 0.125, \quad \delta = 0.125, \quad k1 = 1, \quad k2 = 0, \quad k3 = 0, \quad k4 = 1.
\end{aligned}$$

78. VARDIM (CUTE)

$$\begin{aligned}
f(x) &= \sum_{i=1}^n (x_i - 1)^2 + \left(\sum_{i=1}^n i x_i - \frac{n(n+1)}{2} \right)^2 + \left(\sum_{i=1}^n i x_i - \frac{n(n+1)}{2} \right)^4, \\
x_0 &= \left[1 - \frac{1}{n}, 1 - \frac{2}{n}, \dots, 1 - \frac{n}{n} \right].
\end{aligned}$$

79. DIAG-AUP1

$$f(x) = \sum_{i=1}^n 4(x_i^2 - x_i)^2 + (x_i^2 - 1)^2, \quad x_0 = [4, 4, \dots, 4].$$

80. ENGVAL8 (CUTE)

$$f(x) = \sum_{i=1}^{n-1} (x_i^2 + x_{i+1}^2)^2 - (7 - 8x_i), \quad x_0 = [2, 2, \dots, 2].$$

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