A collection of 75 unconstrained optimization test functions

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Abstract. In this work I present a collection of 75 unconstrained optimization test problems. For each problem, its algebraic expression and a standard initial point are provided. A large part of the test problems, in this collection, is taken from CUTE by Bongartz, Conn, Gould and Toint [7], others are from Andrei [1, 2, 3, 4, 5, 6], as well as from other publications.

1. Freudenstein & Roth FREUROTH (CUTE)

$$f(x)=\sum_{i=1}^{n/2}\left(-13+x_{2i-1}+((5-x_{2i})x_{2i}-2)x_{2i}\right)^2 + \left(-29+x_{2i-1}+((x_{2i}+1)x_{2i}-14)x_{2i}\right)^2,$$

$$x_0=[0.5,-2,0.5,-2,\ldots,0.5,-2].$$

2. Extended Trigonometric ET1

$$f(x)=\sum_{i=1}^{n}\left(\left(n-\sum_{j=1}^{n}\cos x_j\right)+i(1-\cos x_i)-\sin x_i\right)^2,$$

$$x_0=[0.2,0.2,\ldots,0.2].$$

3. Extended Rosenbrock SROSENBR (CUTE)

$$f(x)=\sum_{i=1}^{n/2}c\left(x_{2i}-x_{2i-1}^2\right)^2+\left(1-x_{2i-1}\right)^2,\quad x_0=[-1.2,1,\ldots,-1.2,1].\quad c=1000.$$

4. Extended White & Holst

$$f(x)=\sum_{i=1}^{n/2}c\left(x_{2i}-x_{2i-1}^3\right)^2+\left(1-x_{2i-1}\right)^2,\quad x_0=[-1.2,1,\ldots,-1.2,1],\quad c=1.$$

5. Extended Beale BEALE (CUTE)

$$f(x)=\sum_{i=1}^{n/2}\left(1.5-x_{2i-1}\left(1-x_{2i}\right)\right)^2+\left(2.25-x_{2i-1}\left(1-x_{2i}^2\right)\right)^2+\left(2.625-x_{2i-1}\left(1-x_{2i}^3\right)\right)^2,$$

$$x_0=[1,0.8,\ldots,1,0.8].$$

6. Extended Penalty

$$f(x)=\sum_{i=1}^{n-1}\left(x_i-1\right)^2+\left(\sum_{j=1}^{n}x_j^2-0.25\right)^2,\quad x_0=[1,2,\ldots,n].$$

7. Perturbed Quadratic

$$f(x)=\sum_{i=1}^{n}ix_i^2+\frac{1}{100}\left(\sum_{i=1}^{n}x_i\right)^2,\quad x_0=[0.5,0.5,\ldots,0.5]$$

8. Raydan 1

$$f(x)=\sum_{i=1}^{n}\frac{i}{10}\left(\exp(x_i)-x_i\right),\quad x_0=[1,1,\ldots,1].$$

9. Raydan 2

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-x_i\right),\quad x_0=[1,1,\ldots,1].$$

10. TR-SUMM of quadratics

$$f(x)=\sum_{i=1}^{n-1}x_i^2+c(x_{i+1}+x_i^2)^2\quad x_0=[1,1,\ldots,1].\quad c=100000.$$

11. Diagonal 1

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-\frac{x_i}{i}\right),\quad x_0=[1/n,1/n,\ldots,1/n].$$

12. Diagonal 2

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-i\sin(x_i)\right),\quad x_0=[1/1,1/2,\ldots,1/n].$$

13. Hager

$$f(x)=\sum_{i=1}^{n}\left(\exp(x_i)-\sqrt{i}x_i\right),\quad x_0=[1,1,\ldots,1].$$

14. Generalized Tridiagonal 1

$$f(x)=\sum_{i=1}^{n-1}\left(x_i+x_{i+1}-3\right)^2+\left(x_i-x_{i+1}+1\right)^4,\quad x_0=[2,2,\ldots,2].$$

15. Extended Tridiagonal 1

$$f(x)=\sum_{i=1}^{n/2}\left(x_{2i-1}+x_{2i}-3\right)^2+\left(x_{2i-1}-x_{2i}+1\right)^4,\quad x_0=[2,2,\ldots,2].$$

16. Extended Three Expo Terms

$$f(x)=\sum_{i=1}^{n/2}\Big(\exp(x_{2i-1}+3x_{2i}-0.1)+\exp(x_{2i-1}-3x_{2i}-0.1)+\exp(-x_{2i-1}-0.1)\Big),$$
$$x_0=[0.1,0.1,\ldots,0.1].$$

17. Generalized Tridiagonal 2 (Pentadiagonal)

$$f(x)=\Big((5-3x_1-x_1^2)x_1-3x_2+1\Big)^2+$$
$$\sum_{i=1}^{n-1}\Big((5-3x_i-x_i^2)x_i-x_{i-1}-3x_{i+1}+1\Big)^2+\Big((5-3x_n-x_n^2)x_n-x_{n-1}+1\Big)^2,$$
$$x_0=[-1,-1,\ldots,-1].$$

18. Diagonal 3

$$f(x)=\frac{1}{2}\sum_{i=1}^{n/2}\Big(x_{2i-1}^2+cx_{2i}^2\Big),\qquad x_0=[1,1,\ldots,1].\quad c=10000.$$

19. Diagonal Full Borded

$$f(x)=(x_1-1)^4+(x_n^2-x_1^2)^2+\sum_{i=1}^{n-2}(\sin(x_{i+1}-x_n)-x_1^2-x_{i+1}^2)^2,\quad x_0=[0.1,0.1,\ldots,0.1].$$

20. Extended Himmelblau HIMMELBC (CUTE)

$$f(x)=\sum_{i=1}^{n/2}\Big(x_{2i-1}^2+x_{2i}-11\Big)^2+\Big(x_{2i-1}+x_{2i}^2-7\Big)^2,\qquad x_0=[1,1,\ldots,1].$$

21. Extended Powell

$$f(x)=\sum_{i=1}^{n/4}\Big(x_{4i-3}+10x_{4i-2}\Big)^2+5\Big(x_{4i-1}-x_{4i}\Big)^2+$$
$$\Big(x_{4i-2}-2x_{4i-1}\Big)^4+10\Big(x_{4i-3}-x_{4i}\Big)^4,$$
$$x_0=[3,-1,0,1,\ldots,3,-1,0.1].$$

22. Tridiagonal Double Borded Arrow Up

$$f(x)=(x_1-1)^2+\sum_{i=1}^{n-1}(x_1-0.5x_i^2-05x_{i+1}^2)^2,\qquad x_0=[-1,-1,\ldots,-1].$$

23. Extended PSC1

$$f(x)=\sum_{i=1}^{n/2}\Big(x_{2i-1}^2+x_{2i}^2+x_{2i-1}x_{2i}\Big)^2+\sin^2(x_{2i-1})+\cos^2(x_{2i}),$$
$$x_0=[3,0.1,\ldots,3,0.1].$$

24. Extended Block-Diagonal BD1

$$f(x)=\sum_{i=1}^{n/2}\Big(x_{2i-1}^2+x_{2i}^2-2\Big)^2+\Big(\exp(x_{2i-1})-x_{2i}\Big)^2,\quad x_0=[0.1,0.1,\ldots,0.1].$$

25. Extended Maratos

$$f(x)=\sum_{i=1}^{n/2}x_{2i-1}+c\Big(x_{2i-1}^2+x_{2i}^2-1\Big)^2,\quad x_0=[1.1,0.1,\ldots,1.1,0.1],\qquad c=1.$$

26. Full Hessian FH1 (Sum of quadratics, quadratic inside)

$$f(x)=\sum_{i=1}^{m}\left(\sum_{j=1}^{n} ijx_j^2-1\right)^2, \quad m=50,$$
$$x_0=[0.01,0.01,\ldots,0.01].$$

27. Extended Cliff

$$f(x)=\sum_{i=1}^{n/2}\left(\frac{x_{2i-1}-3}{100}\right)^2-\left(x_{2i-1}-x_{2i}\right)+\exp\left(2(x_{2i-1}-x_{2i})\right),$$
$$x_0=[0,-0.1,\ldots,0,-0.1].$$

28. Quadratic Diagonal Perturbed

$$f(x)=\left(\sum_{i=1}^{n} x_i\right)^2+\sum_{i=1}^{n}\frac{i}{100}x_i^2, \qquad x_0=[0.5,0.5,\ldots,0.5].$$

29. Full Hessian FH2

$$f(x)=\left(\sum_{i=1}^{n} x_i\right)^2+\sum_{i=1}^{n} i(\sin(x_i)+\cos(x_i))/1000, \qquad x_0=[1,1,\ldots,1].$$

30. Full Hessian FH3

$$f(x)=\left(\sum_{i=1}^{n} x_i^2\right)^2+\sum_{i=1}^{n} i(\sin(x_i)+\cos(x_i))/1000, \qquad x_0=[1,1,\ldots,1].$$

31. Tridiagonal Double Borded – NONDQUAR

$$f(x)=(x_1-x_2)^2+\sum_{i=1}^{n-2}\left(x_i+x_{i+1}+x_n\right)^4+(x_{n-1}+x_n)^2, \quad x_0=[1.,-1.,\ldots,1.,-1.,].$$

32. Tridiagonal White & Holst (c=4)

$$f(x)=\sum_{i=1}^{n-1} c(x_{i+1}-x_i^3)^2+(1-x_i)^2, \quad x_0=[-1.2,1,\ldots,-1.2,1]. \qquad c=4.$$

33. Diagonal Double Borded Arrow Up

$$f(x)=\sum_{i=1}^{n} 4(x_i^2-x_1)^2+(x_i-1)^2, \qquad x_0=[4,0,\ldots,4,0].$$

34. TRIDIA (CUTE)

$$f(x)=\gamma\left(\delta x_1-1\right)^2+\sum_{i=2}^{n} i\left(\alpha x_i-\beta x_{i-1}\right)^2,$$
$$\alpha=2, \quad \beta=1, \quad \gamma=1, \quad \delta=1, \quad x_0=[1,1,\ldots,1].$$

35. ARWHEAD (CUTE)

$$f(x)=\sum_{i=1}^{n-1}\left(-4x_i+3\right)+\sum_{i=1}^{n-1}\left(x_i^2+x_n^2\right)^2, \qquad x_0=[1.,1.,\ldots,1.].$$

36. NONDIA (CUTE) (Shanno)

$$f(x) = (x_1 - 1)^2 + c(x_1 - x_1^2)^2 + \sum_{i=2}^{n} c\left(x_1 - x_i^2\right)^2, \quad x_0 = [-1.,-1.,\ldots,-1.], \quad c = 100.$$

37. Extended WOODS (CUTE)

$$f(x) = \sum_{i=1}^{n/4} 100\left(x_{4i-3}^2 - x_{4i-2}\right)^2 + \left(x_{4i-3} - 1\right)^2 + 90\left(x_{4i-1}^2 - x_{4i}\right)^2 +$$
$$\left(1 - x_{4i-1}\right)^2 + 10.1\left\{\left(x_{4i-2} - 1\right)^2 + \left(x_{4i} - 1\right)^2\right\} + 19.8\left(x_{4i-2} - 1\right)\left(x_{4i} - 1\right),$$
$$x_0 = [-3,-1,-3,-1,\ldots,-3,-1,-3,-1].$$

38. Extended Hiebert

$$f(x) = \sum_{i=1}^{n/2} \left(x_{2i-1} - 10\right)^2 + \left(x_{2i-1}x_{2i} - 500\right)^2, \quad x_0 = [0,0,\ldots,0].$$

39. BDQRTIC (CUTE)

$$f(x) = \sum_{i=1}^{n-4} \left(-4x_i + 3\right)^2 + \left(x_i^2 + 2x_{i+1}^2 + 3x_{i+2}^2 + 4x_{i+3}^2 + 5x_n^2\right)^2,$$
$$x_0 = [1.,1.,\ldots,1.].$$

40. DQDRTIC (CUTE)

$$f(x) = \sum_{i=1}^{n-2} \left(x_i^2 + cx_{i+1}^2 + dx_{i+2}^2\right), \quad c = 1000, \quad d = 1000, \quad x_0 = [3.,3.,\ldots,3.].$$

41. EG2 (CUTE)

$$f(x) = \sum_{i=1}^{n-1} \sin(x_1 + x_i^2 - 1) + \frac{1}{2}\sin(x_n^2), \quad x_0 = [1.,1.,\ldots,1.].$$

42. EDENSCH (CUTE)

$$f(x) = 16 + \sum_{i=1}^{n-1} \left[(x_i - 2)^4 + (x_i x_{i+1} - 2x_{i+1})^2 + (x_{i+1} + 1)^2\right], \quad x_0 = [0,0,\ldots,0].$$

43. Broyden Pentadiagonal (CUTE)

$$f(x) = (3x_1 - 2x_1^2)^2 + \sum (3x_i - 2x_i^2 - x_{i-1} - 2x_{i+1} + 1)^2 + (3x_n + 2x_n^2 - x_{n-1} + 1)^2,$$
$$x_0 = [-1,-1,\ldots,-1].$$

44. Almost Perturbed Quadratic

$$f(x) = (x_1 + x_n)^2 / 100 + \sum_{i=1}^{n} i x_i^2, \quad x_0 = [0.5,0.5,\ldots,0.5].$$

45. Almost Perturbed Quartic

$$f(x) = (x_1 + x_n)^2 / 100 + \sum_{i=1}^{n} i x_i^4, \quad x_0 = [0.5,0.5,\ldots,0.5].$$

46. FLETCHCR (CUTE)

$$f(x)=\sum_{i=1}^{n-1} c\left(x_{i+1}-x_i+1-x_i^2\right)^2, \quad x_0=[0.5,0.5,\ldots,0.5], \qquad c=100.$$

47. ENGVAL1 (CUTE)

$$f(x)=\sum_{i=1}^{n-1}(x_i^2+x_{i+1}^2)^2+\sum_{i=1}^{n-1}(-4x_i+3), \qquad x_0=[2,2,\ldots,2].$$

48. DENSCHNA (CUTE)

$$f(x)=\sum_{i=1}^{n/2} x_{2i-1}^4+(x_{2i-1}+x_{2i})^2+(-1+\exp(x_{2i}))^2, \qquad x_0=[1.,1.,\ldots,1.].$$

49. DENSCHNB (CUTE)

$$f(x)=\sum_{i=1}^{n/2}\left(x_{2i-1}-2\right)^2+\left(x_{2i-1}-2\right)^2 x_{2i}^2+\left(x_{2i}+1\right)^2, \quad x_0=[1,1,\ldots,1].$$

50. DENSCHNC (CUTE)

$$f(x)=\sum_{i=1}^{n/2}(-2+x_{2i-1}^2+x_{2i}^2)^2+(-2+\exp(x_{2i-1}-1)+x_{2i}^3)^2, \qquad x_0=[0.1,0.1,\ldots,0.1].$$

51. DENSCHNF (CUTE)

$$f(x)=\sum_{i=1}^{n/2}\left(2(x_{2i-1}+x_{2i})^2+(x_{2i-1}-x_{2i})^2-8\right)^2+\left(5x_{2i-1}^2+(x_{2i}-3)^2-9\right)^2,$$
$$x_0=[2.,0.,2.,0.,\ldots,2.,0.].$$

52. SINQUAD (CUTE)

$$f(x)=(x_1-1)^4+(x_n^2-x_1^2)^2+\sum_{i=1}^{n-2}(\sin(x_{i+1}-x_n)-x_1^2+x_{i+1}^2)^2, \quad x_0=[0.1,0.1,\ldots,0.1].$$

53. HIMMELBG (CUTE)

$$f(x)=\sum_{i=1}^{n/2}\left(2x_{2i-1}^2+3x_{2i}^2\right)\exp\left(-x_{2i-1}-x_{2i}\right), \quad x_0=[1.5,1.5,\ldots,1.5].$$

54. HIMMELBH (CUTE)

$$f(x)=\sum_{i=1}^{n/2}(-3x_{2i-1}-2x_{2i}+2+x_{2i-1}^3+x_{2i}^2), \quad x_0=[1.5,1.5,\ldots,1.5].$$

55. DIXON3DQ (CUTE)

$$f(x)=(x_1-1)^2+\sum_{i=1}^{n-1}(x_i-x_{i+1})^2+(x_n-1)^2, \qquad x_0=[-1.,-1.,\ldots,-1].$$

56. BIGGSB1 (CUTE)

$$f(x)=(x_1-1)^2+(1-x_n)^2+\sum_{i=2}^{n}(x_i-x_{i-1})^2, \quad x_0=[0,0,\ldots,0].$$

57. Perturbed Quadratic

$$f(x)=\left(\sum_{i=1}^{n} i x_i\right)^2+\sum_{i=1}^{n} i x_i^2, \qquad x_0=[0.5,0.5,\ldots,0.5],$$

58. GENROSNB (CUTE)

$$f(x)=(x_1-1)^2+\sum_{i=2}^{n} 100(x_i-x_{i-1}^2)^2, \qquad x_0=[-1.2,1,\ldots,-1.2,1].$$

59. QP1 Extended Quadratic Penalty

$$f(x)=\left(\sum_{i=1}^{n} x_i^2-0.5\right)^2+\sum_{i=1}^{n-1}(x_i^2-2)^2, \qquad x_0=[1.,1.,\ldots,1.].$$

60. QP2 Extended Quadratic Penalty

$$f(x)=\left(\sum_{i=1}^{n} x_i^2-100\right)^2+\sum_{i=1}^{n-1}(x_i^2-\sin(x_i))^2, \qquad x_0=[1.,1.,\ldots,1.].$$

61. Tridiagonal TS1 (Staircase S1)

$$f(x)=\sum_{i=1}^{n-1}(x_i+x_{i+1}-i)^2, \qquad x_0=[1.,1.,\ldots,1.].$$

62. Tridiagonal TS2 (Staircase S2)

$$f(x)=\sum_{i=2}^{n}(x_{i-1}+x_i-i)^2, \qquad x_0=[1.,1.,\ldots,1.].$$

63. Tridiagonal TS3 (Staircase S3)

$$f(x)=\sum_{i=2}^{n}(x_{i-1}+x_i+i)^2, \qquad x_0=[2.,2.,\ldots,2.].$$

64. Extended Trigonometric ET2

$$f(x)=\sum_{i=1}^{n}\left(\left(n-\sum_{i=1}^{n}\sin(x_i)\right)+i(1-\sin(x_i))-\sin(x_i)\right)^2, \qquad x_0=[0.2,0.2,\ldots,0.2].$$

65. QP3 Extended Quadratic Penalty

$$f(x)=\left(\sum_{i=1}^{n} x_i^2-0.25\right)^2-\sum_{i=1}^{n-1}(x_i^2-1)^2, \qquad x_0=[1.,1.,\ldots,1.].$$

66. EG1

$$f(x)=\frac{1}{2}\cos(x_n^2)+\sum_{i=1}^{n-1}\cos(x_1+x_i^2-1), \qquad x_0=[1.,1.,\ldots,1.].$$

67. GENROSEN-2

$$f(x)=(x_1-1)^2+\sum_{i=2}^{n} c(x_{i-1}^2-x_i)^2, \qquad x_0=[-1.2,1,\ldots,-1.2,1]. \qquad c=100.$$

68. PRODsin

$$f(x)=\left(\sum_{i=1}^{m}x_i^2\right)\left(\sum_{i=1}^{n}\sin(x_i)\right),\quad x_0=[1.,1.,\ldots,1.].\quad m=n-1.$$

69. PROD1 (m=n)

$$f(x)=\left(\sum_{i=1}^{m}x_i\right)\left(\sum_{i=1}^{n}x_i\right),\quad x_0=[1.,1.,\ldots,1.].\quad m=n.$$

70. PRODcos

$$f(x)=\left(\sum_{i=1}^{m}x_i^2\right)\left(\sum_{i=1}^{n}\cos(x_i)\right),\quad x_0=[1.,1.,\ldots,1.].\quad m=n-1.$$

71. PROD2 (m=1)

$$f(x)=\left(\sum_{i=1}^{m}x_i^4\right)\left(\sum_{i=1}^{n}ix_i\right),\quad x_0=[15,15,\ldots,15].\quad m=1.$$

72. ARGLINB (m=5)

$$f(x)=\sum_{i=1}^{m}\left(\sum_{j=1}^{n}ijx_j-1\right)^2,\quad x_0=[0.01,0.001,\ldots,0.001].\quad m=5.$$

73. VARDIM (CUTE)

$$f(x)=\sum_{i=1}^{n}(x_i-1)^2+\left(\sum_{i=1}^{n}ix_i-\frac{n(n+1)}{2}\right)^2+\left(\sum_{i=1}^{n}ix_i-\frac{n(n+1)}{2}\right)^4,$$

$$x_0=\left[1-\frac{1}{n},1-\frac{2}{n},\ldots,1-\frac{n}{n}\right].$$

74. DIAG-AUP1

$$f(x)=\sum_{i=1}^{n}4(x_i^2-x_1)^2+(x_i^2-1)^2,\quad x_0=[4,4,\ldots,4].$$

75. ENGVAL8

$$f(x)=\sum_{i=1}^{n-1}(x_i^2+x_{i+1}^2)^2-(7-8x_i),\qquad x_0=[2,2,\ldots,2].$$

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