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Two-Species Fish Cultivation Policy with Price and Biomass Dependent Catch Rate

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Abstract:

For the first time, a cultivation policy of two living species is outlined via an evolutionary method when their demands are price and biomass dependent. Here, the species ameliorates and deteriorates with time and are of the type of prey – predator. Predator species do have some natural growth in addition to their growth depending on other species. The species are cultivated for one period only. Simulated Annealing (SA) algorithm has been developed and implemented to find the optimum values of initial stocks of the species and optimum time period are evaluated to have maximum possible profit out of the system. The system has been illustrated numerically and results for some particular cases are obtained.

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Keywords: Self inhibiting effect of growth, Prey-Predator, Simulated Annealing.

1. Introduction:

Since the development of classical EOQ model by Harris (1915), a lot of research work on inventory control system (c.f Naddor (1966), Whitin (1957), etc.) is available in the literature. These conventional inventories are related with non-living items in industry and business sector like raw materials, finished goods, vegetables, food-grains etc. But, presently genetic research has created a revolution and as a result, inventory of livestocks are built-up for business. As a part of it, now-a-days, high breed fish cultivation for a short period with different type of species in bhery, fields, ponds, etc are very popular in waterlogged countries like India, Bangladesh, etc. The cultivation of the species of the prey and predator types can be formed as a inventory control problem and solved easily for global optimum using the evolutionary methods such as SA.

In this area, the most of the authors have derived their models out of the work of Clark (1990) who found the optimal equilibrium policy for joint harvesting of two independent species. Clark (1990) assumed that each species follows a logistic growth law in absence of harvesting and its harvest rate is proportional to both its stock level and harvesting effort. This analysis has been extended by several authors such as Mesterton –Gibbson(1987,1988), Wilen and Brown (1986), etc. Kapur(1992) discussed simple Lotka-Volterra model in case of two species population, where logistic growth of one species in the absence of other is proportional to the amount of biomass, but every species has an self inhibiting effect in the growth rate and have an effect in the growth rate for the presence of other species. Till now, none has considered the cultivation of prey-predator fishes for finite time period and formulated it as a profit maximizing inventory model taking cost of amelioration and cultivation, rent of pond, etc. into consideration.

The inventory control of fish cultivation with the above realistic assumption is so complex and non-linear that it is very difficult to get the optimum solution via analytical approach and thus researchers are forced to apply numerical optimization techniques for approximate optimum solution. There are some inherent difficulties in the traditional nonlinear optimization methods used for solution of this type of problems. These methods are –

(i) initial solution dependent.
(ii) get struck to a sub optimal solution.
(iii) are not efficient in handling problems having discrete variables.
(v) cannot be efficiently used on parallel machines.
and (vi) are not universal rather specific problem dependent.

To overcome these difficulties recently SA algorithms have been used as optimization techniques for decision-making problems. Annealing is the physical process of heating up a solid until it melts followed by cooling it down slowly until crystallizes into a state with a perfect lattice. Following this physical phenomenon, SA has been developed to find the global optimum for a complicated complex cost surface. In the early 1980's, Kirkpatrick(1984), Vecchi & Kirkpatrick (1983) and Cerny (1985) introduced the concepts of annealing in combinatorial optimization problems. Aarts and Korst (1988) discussed the conditions under which asymptotic convergence of the SA process is guaranteed. Recently SA have been applied in different areas like Traveling Salesman Problem [Kirkpatrick. and Toulouse (1985)], Matching Problems [Lutton and Bonomi(1986)], etc.. But till now, none has applied this technique for decision making in inventory/ two species harvesting system.

In this paper, for the first time, a cultivation model for the joint cultivation of two prey-predator type of species in a bhery/ pond over a fixed period of time is formulated and for its solution, a non-traditional optimization method is proposed. Here, predator takes food out of the environment for its growth to some extent. The rates of growth and decay of prey and predator in the absence of other are assumed to be stochastic governed by two parameter Weibull distribution. Each species has some self-inhibiting effect in growth rate, which are directly proportional to the squares of the amount of the species and inversely proportional to the area of the pond. Prey fishes have a deterioration rate for the presence of predator, which is directly proportional to the amount of both the species. Predator fishes have a growth rate for the presence of prey, which is also directly proportional to the amount of both the species. Initially cultivation is commenced with some amount of prey and predator species which are to be determined. The demand i.e,, catch rates of the species are non-linearly proportional to the selling price and total biomass of the species at time t. To solve this system, the evolutionary method, SA has been developed and implemented to find out the optimum quantities of harvest and initial stock of species so that the total proceeds out of the system is maximum. The model has been illustrated through some numerical results. In particular, some results have been presented assuming withdrawal to be dependent/ independent of bio-masses and prices.

2.Simulated Annealing:

Consider an ensemble of molecules at a high temperature, which are moving around freely. Since physical systems tend towards lower energy states, the molecules are likely to move to positions that lower the energy of the ensemble as a whole as the system cools. However molecules actually move to positions that increase the energy of the system with a probability $e^{-\Delta E/T}$, where ΔE is the increase in the energy of the system and T is the current temperature. If the ensemble is allowed to cool down slowly it will eventually promote a regular crystal, which is the optimal state rather than flawed solid, the poor local minima.

In function optimization, a similar process can be defined. This process can be formulated as the problem of finding- among a potentially very large number of solutions- a solution with minimum cost. By establishing the correspondence between the cost function and the free energy and between the solutions and the physical states, a solution method was introduced by Kirkpatrick in the field of optimization based on a simulation of the physical annealing process. This method is called Simulated Annealing. The Simulated Annealing algorithm to solve such problems is given below:

1. Start with some state, S.
2. $T=T_0$
3. Repeat {
4. While (not at equilibrium){
5.   Perturb S to get a new state S_n
6. $\Delta E=E(S_n)-E(S)$
7.   If $\Delta E<0$
8.   Replace S with S_n
9.   Else with probability $e^{-\Delta E/T}$
10.   Replace S with S_n
11.   }
12.   T=C*T /* 0<C< 1 */
13.   } Until (frozen)

In this algorithm, the state, S becomes the state (approximate solution) of the problem in question rather than the ensemble of molecules, energy, E corresponds to the quality of S and is determined by a cost function used to assign a value to the state, temperature, T is a control parameter used to guide the process of finding a low cost state, T_0 is the initial value of T and C ($0< C <1$) is a constant used to decrease the value of T.

3. Assumptions and Notations:

The proposed mathematical model of harvesting the species in a bhery/pond over a fixed time period 'T' is developed under the following assumption and notations:

(i) The harvesting process involves two species- one is prey and other is predator.

(ii) Replenishment of species are instantaneous.

(iii) The deterioration and amelioration occur when the item is effectively in bhery.

(iv) S is the total initial biomass and KS is the initial biomass of predator where $0<K<1$.

(v) As growth rate of initial biomass (small size fishes) is very high at the beginning and gradually decreases with time, amelioration rate of each species in the absence of other is assumed to be follow two parameter Weibull distribution. Let α_1, β_1 be the parameters of the Weibull distribution that follow growth rate of prey fishes in the absence of predator and so it' s probability density function $f_1(t)$ is given by

$$f_1(t) = \alpha_1 \beta_1 t^{\beta_1 - 1} e^{-\alpha_1 t^{\beta_1}}$$

(vi) So the instantaneous rate of amelioration (growth rate) A_1 (t) of prey fishes is given by

$$A_1(t) = \frac{f_1(t)}{1 - F_1(t)} = \alpha_1 \beta_1 t^{\beta_1 - 1}$$

where F_1 (t) is distribution function of growth rate of prey.

(vii) Similarly, the instantaneous rate of amelioration (growth rate) A_2 (t) of predator fishes obey Weibull distribution with parameters α_2, β_2 and is given by

$$A_2(t) = \frac{f_2(t)}{1 - F_2(t)} = \alpha_2 \beta_2 t^{\beta_2 - 1}$$

where F_2 (t) is distribution function of growth rate of predator.

(viii) As growth rate decreases with amount of biomass in a pond due to environmental effect, self-inhibiting effect on growth rate is assumed as directly proportional to the squares of the amount of the species and inversely proportional to the area of the pond.

(ix) Initial deterioration of small size fishes is high and it decreases with time, it is assumed that deterioration of a species in the absence of other follows Weibull distribution. So instantaneous rate of deterioration of prey fishes is assumed to be follow two parameter Weibull distribution with parameters γ_1, δ_1 and so it' s probability density function g(t) is given by

$$g_1(t) = \gamma_1 \delta_1 t^{\delta_1 - 1} e^{-\gamma_1 t^{\delta_1}} .$$

(x) So the instantaneous rate of deterioration $B_1(t)$ of prey fish is given by

$$B_1(t) = \frac{g_1(t)}{1 - G_1(t)} = \gamma_1 \delta_1 t^{\delta_1 - 1}$$

where $G_1(t)$ is distribution function of deterioration of prey fish.

(xi) Similarly the instantaneous rate of deterioration $B_2(t)$ of predator fish follow two-parameter Weibull distribution with parameters γ_2, δ_2 and so it is given by $B_2(t) = \frac{g_2(t)}{1 - G_2(t)} = \gamma_2 \delta_2 t^{\delta_2 - 1}$,

where g_2 and G_2 are density function and distribution function of deterioration of predator fish.

(xii) a_{11} is the deterioration coefficient of prey fish due to the presence of predator fish.

(xiii) a_{21} is the growth coefficient of predator fish due to the presence of prey fish.

(xiv) a_{12}, a_{22} are the coefficient of self inhibiting effect in the growth for prey and predator fishes respectively.

(xv) A is the area of the pond.

(xvi) c_A is the rent of pond per unit area.

(xvii) T_0 is the time after which withdrawal of fishes occurs.

(xviii) Withdrawal of fishes is assumed to be continuous.

(xix) t_1 ,t_2 are the respective duration of time for which prey and predator fishes exists.

(xx)Withdrawal rate of prey fish is $K_{11} + K_{12} q_1^{\beta_{11}} - K_{13}^{\beta_{12}}$ for $T_0 < t \le t_1$, where two constant β_{11}, β_{12} are so chosen to best fit of the withdrawal rate of prey fish.

(xxi)Withdrawal rate of prey fish is $K_{21} + K_{22} q_2^{\beta_{21}} - K_{23}^{\beta_{22}}$ for $T_0 < t \le t_2$, where two constant β_{21}, β_{22} are so chosen to best fit of the withdrawal rate of predator fish.

(xxii)] Here initial amount of biomass (S) is the only decision variable.

(xxiii) C_a C_r, C_h are the cost of amelioration, cost of deterioration and cost of cultivation per unit biomass respectively
(xxiv) p_1,p_2 are the purchase costs per unit biomass of prey and predator fishes respectively.
(xxv) s_1,s_2 be the selling prices per unit biomass of prey and predator fishes respectively.

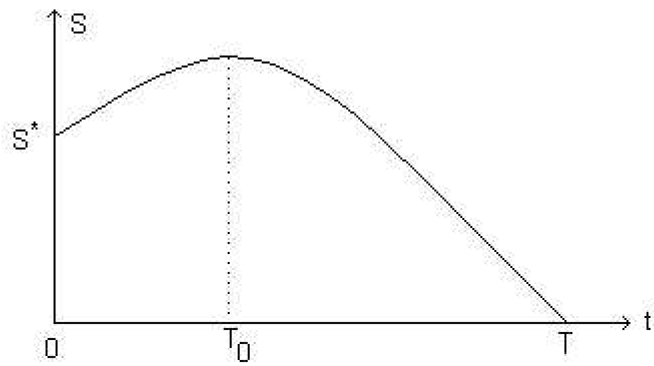

Fig.1: Biomass level at different time

4. Mathematical Formulation:

Cultivation starts with an amount S units of biomass, among which, an amount KS is predator fish and (1-K)S is prey fish. Withdrawal start at $t=T_0$ and withdrawn continuously until all the fishes are exhausted. Then next cycle starts. Let $q_1(t)$ and $q_2(t)$ denote the biomass of prey and predator fishes at time t respectively. Then the differential equations describing the instantaneous states of $q_1(t)$ and $q_2(t)$ are given by:

$$\frac{dq_1}{dt} = A_1 q_1 - a_{11} q_1 q_2 - \frac{a_{12}}{A} q_1^2 - B_1 q_1 \quad for \quad 0 < t \le T_0$$

$$= A_1 q_1 - a_{11} q_1 q_2 - \frac{a_{12}}{A} q_1^2 - B_1 q_1 - (K_{11} + K_{12} q_1^{\beta_{11}} - K_{13} s_1^{\beta_{12}}) \quad for \quad T_0 < t \le t_1 \text{ ....(1)}$$

$$\frac{dq_2}{dt} = A_2 q_2 + a_{21} q_1 q_2 - \frac{a_{22}}{A} q_2^2 - B_2 q_2 \quad for \quad 0 < t \le T_0$$

$$= A_2 q_2 + a_{21} q_1 q_2 - \frac{a_{22}}{A} q_2^2 - B_2 q_2 - (K_{21} + K_{22} q_2^{\beta_{21}} - K_{23} s_2^{\beta_{22}}) \quad for \quad T_0 < t \le t_1 \text{ ....(2)}$$

with initial conditions $q_1(0)=(1-K)S$, $q_2(0)=KS$, $q_1(t_1)=0$, $q_2(t_2)=0$ and S is the total initial biomass.

The system of non-linear differential equations (1) and (2) can not be solved analytically. For the conventional problems in bio-mathematics with infinite time, these equations are solved by perturbation technique. Here for finite time horizon, these are solved numerically using 4th order Runge-Kutta Method.

Time length of each cycle is $T=\max\{t_1, t_2\}$
Total amount of prey fish withdrawn during each cycle

$$R_1 = \int_{T_0}^{t_1} (K_{11} + K_{12} q_1^{\beta_{11}} - K_{13} s_1^{\beta_{21}}) dt$$

$$= (K_{11} - K_{13} s_1^{\beta_{21}})(t_1 - T_0) + \int_{T_0}^{t_1} q_1^{\beta_{11}} dt$$

Total amount of predator fish withdrawn during each cycle

$$R_2 = \int_{T_0}^{t_2} (K_{21} + K_{22} q_2^{\beta_{21}} - K_{23} s_2^{\beta_{22}}) dt$$

$$= (K_{21} - K_{23} s_2^{\beta_{22}})(t_2 - T_0) + \int_{T_0}^{t_2} q_2^{\beta_{21}} dt$$

If A_{m1}, A_{m2} be the ameliorated units of prey and predator fishes respectively for time cycle T then

$$A_{m1} = \int_0^{t_1} A_1 q_1 dt \ , \qquad A_{m2} = \int_0^{t_2} A_2 q_2 dt$$

If D_1 and D_2 be the deteriorated units of prey and predator fishes respectively for time length T, then

$$D_1 = \int_0^{t_1} B_1 q_1 dt \quad , \qquad D_2 = \int_0^{t_2} B_2 q_2 dt$$

Total cultivation cost=$C_h(H_1+H_2)$

$$where \qquad H_1 = \int_0^{t_1} q_1 dt \quad , \quad H_2 = \int_0^{t_2} q_2 dt$$

So average profit per unit time is

$$Z = \frac{[(s_1 R_1 + s_2 R_2) - \{C_a (Am_1 + Am_2) + C_r (D_1 + D_2) + C_h (H_1 + H_2) + (1-K) Sp_1 + KSp_2 + AC_A\}]}{T}$$

Now the problem is reduced to maximize the average profit Z(S) and to find the optimum value of S for which Z(S) is maximum.

5. Solution Methodology:

Average profit Z(S) is optimized by SA process. The process is discussed in Art. 5.2. To evaluate value of Z(S) for a fixed value of 'S', t_1 and t_2 are calculated by numerically solving the system of non-linear differential equations (1) and (2) (by 4th order R-K method). Different Integrals of 'Z' are calculated numerically by Trapezoidal rule. To evaluate the integrals of Z numerically, $q_1(t)$, $q_2(t)$ at different values of t are obtained by solving numerically (1) and (2) at the time of calculation of t_1 and t_2. The following algorithm is used for this purpose.

5.1. Algorithm:

System of differential equations (1) and (2) can be written as :

$$\frac{dq_1}{dt} = f_1(q_1, q_2, t) \ \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\frac{dq_2}{dt} = f_2(q_1, q_2, t) \ \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

with boundary conditions $q_1(0)=Q_1=(1-K)S$, $q_2(0)=Q_2=KS$. Now we have to determine t_1, t_2 such that $q_1(t_1)=0$ and $q_2(t_2)=0$. Now we have to determine t_1, t_2 such that $q_1(t_1)=0$ and $q_2(t_2)=0$ and to determine value of an integral $I = \int_0^T g(q_1, q_2, t)dt$, where T=max $\{t_1,t_2\}$ and I is any one of the integral of Z. The following algorithm can be used for this purpose. In the algorithm the system of differential equations (3) and (4) is solved numerically by 4th order R-K method and the integration is done numerically by Trpezoidal rule. In the algorithm RK1(q_{10}, q_{20}, T, H) represents the value of q_1 at t=T+H calculated numerically solving (3) and (4) by 4th order R-K method where q_{10}, q_{20} are the values of q_1,q_2 at t=T. Similarly RK2(q_{10}, q_{20}, T, H) represents the value of q_2 at t=T+H. RK10(q_{10}, q_{20}, T, H) represents the value of q_1 at t=T+H calculated numerically solving (3) by 4th order R-K method. RK02(q_{10}, q_{20}, T, H) represents the value of q_2 at t=T+H calculated numerically solving (4) by 4th order R-K method.

START

1. $q_{10} \leftarrow Q_1$, $q_{20} \leftarrow Q_2$
2. $I \leftarrow 0$, $T \leftarrow 0$
3. $H \leftarrow 0.0001$
4. $q_1 \leftarrow RK1(q_{10}, q_{20}, T, H)$
5. $q_2 \leftarrow RK2(q_{10}, q_{20}, T, H)$
6. if $q_1>0$ and $q_2>0$ then
7. $T \leftarrow T+H$
8. $I \leftarrow I+g(q_1,q_2,T)$
9. $q_{10} \leftarrow q_1$, $q_{20} \leftarrow q_2$
10.   goto step 4.
11. endif
12. $t_1 \leftarrow T$, $t_2 \leftarrow T$
13. while($q_1>0$) do
14. $q_{20} \leftarrow 0$
15. $q_1 \leftarrow RK10(q_{10}, q_{20}, T, H)$
16.   if $q_1>0$ then
17. $T \leftarrow T+H$
18. $t_1 \leftarrow T$
19. $I \leftarrow I+g(q_1,0,T)$
20. $q_{10} \leftarrow q_1$
21.   endif
22. endwhile
23. while($q_2>0$) do
24. $q_{10} \leftarrow 0$
25. $q_2 \leftarrow RK02(q_{10},q_{20}, T, H)$
26.   if $q_2>0$ then
27. $T \leftarrow T+H$
28. $t_2 \leftarrow T$
29. $I \leftarrow I+g(0,q_2,T)$
30. $q_{20} \leftarrow q_2$
31.   endif

32. endwhile
33. $I \leftarrow H[g(Q_1,Q_2,0)+2.I-g(q_{10},q_{20},T)]/2$
34. output: T, I
35. END

5.2 S.A. Functions for the model:

5.2.1 Representation and initialization:

A real variable S is used to represent the initial amount of biomass. A real constant S_0 is randomly generated in search space and taken as the initial guess of S.

5.2.2 Perturbation function:

A random number r between –0.25 and +0.25 is generated using random number generator. S+r is taken as neighbour solution of S if S+r satisfies the constraints of the problem.

5.2.3 Energy function:
Our problem is to find the optimum amount of initial biomass S such that average profit Z(S) is maximum. Here –Z(S) is taken as the energy function of the solution S.

5.2.4 Cooling Schedule:

Initial temperature T_0 is taken according to different parameter values of the energy function and reducing factor for T (temperature), C is taken as 0.999.

6. Numerical Results:

The proposed cultivation model is now illustrated for certain numerical data. The following values of parameters are assumed to calculate optimum value of profit function (Z) along with optimum initial biomass size (S), and time period (T) and results are given in table-1.
$\alpha_1=2.5$, $\beta_1=0.4$, $\delta_1=0.5$, $\gamma_1=0.075$, $\alpha_2=1.5$, $\beta_2=0.35$, $\delta_2=0.4$,$\gamma_2=0.05$,$a_{11}=0.075$, $a_{12}=0.1$, $a_{21}=0.05$, $a_{22}=0.1$, $K_{11}=200$, $K_{12}=0.5$, $K_{13}=0.5$, $K_{21}=75$, $K_{22}=1$, $K_{23}=0.5$, $\beta_{11}=0.5$, $\beta_{12}=0.5$, $\beta_{21}=0.5$, $\beta_{22}=0.5$, p_1=\$ 15, p_2=\$ 15, s_1=\$ 40, s_2=\$ 70, C_a=\$ 5, C_r=Rs. 1, C_h=\$ 1, C_A=\$ 200, K=0.005 , A=10,T_0=0.1
Results are calculated for price independent catch rate ($K_{13}=0$, $K_{23}=0$) and biomass independent catch rates ($K_{12}=0$, $K_{22}=0$) also and are given in table-1.

Table-1
Results for different catch rates

Catch rate	S	t_1	t_2	Z(\$)
Price and Biomass Dependent	66.500000	1.1318	0.1164	3484.21
Price Independent	66.823997	1.1183	0.1156	3543.49
Biomass Independent	64.376007	1.1376	0.1157	3396.30

In the above scenarios, it is observed that predator vanishes before the prey. It happens because initial biomass of predator is very small compared to prey. It is also observed that if demand is price independent, then profit is maximum, which agrees with reality. Again when catch rate is biomass independent then profit decreases. It happens because in this case, catch rate decreases, which increases cultivation cost, deterioration and hence deterioration cost, which in turn decreases profit.

6.1 Sensitivity Analysis:

6.1.1. Effect of pond area (A) on average profit and cycle time period(T):

Again for the above parametric values and a fixed ratio of prey and predator (K=0.02), results are obtained for different areas (A) of pond/bhery for general catch rates (price and biomass dependent) and presented in Table-3. It is observed that as area increases, profit initially increases, and attains a maximum limit and then it gradually decreases. As area increases, self-inhibiting effect in growth rates decreases, which increases resultant growth rates of the fishes. As growth rate increases, amount of initial biomass decreases which ultimately increases the profit. Also as area increases rent of pond increases. But for the assumed parametric values initially as 'A' increases, increase in profit due to 'increase in growth rates of fishes and decrease in initial biomass' is more compared to decrease in profit due to the increase in rent of pond. But after a certain limit, increase in the rent of pond exceeds the increase in profit due to increase in area and so from that limit area profit gradually decreases.

Table-3
Results for different size of Pond/Bhery

A	S	t_1	t_2	T	Z($)
7	100.70320	0.8459	0.8458	0.8459	6073.07
8	97.662697	0.8840	0.8840	0.8840	6146.29
9	95.315460	0.9187	0.9186	0.9187	6178.80
10	93.457001	0.9505	0.9505	0.9505	6183.40
11	91.955002	0.9799	0.9799	0.9799	6168.10
12	90.718399	1.0074	1.0074	1.0074	6138.03
13	89.685997	1.0332	1.0331	1.0332	6096.35

It is also observed that as area increases, time cycle (T) increases. It happens because as 'A' increases, resultant growth rates of fishes increase and as a result, amount of total biomass at the pond increases although initial biomass is less. This process increases catch time period and ultimately total cycle period increases as initial withdrawal time is fixed.

6.1.2 Effect of 'K' on average profit and cycle time period(T):

For the above parametric values and different values of 'K' optimum parameters are obtained for different type of catch rates and presented in Table-2. Here, for different catch rates it is observed that if amount of initial biomass of predator with respect to prey increases, i.e., 'K' increases then average profit gradually increases and attains a maximum limit and then gradually decreases. This change of profit may be due to different factors. In the present cultivation, though 'K' increases, its value is quite small and hence, amount of prey is always much more than predator and sufficient for the growth of predator. As predator increases, amount of prey decreases. Here the proceeds out of both prey and predator initially increases as the excess of prey which was causing loss due to inhibition was eaten by predator. At certain stage, though the inhibition was controlled, the amount of prey eaten by predator becomes so much that total proceeds slowly decreases instead of going up.

It is also observed in different situations that as ‘K’ increases, cycle time period ‘T’ decreases. As ‘K’ increases amount of predator with respect to prey increases, which results in the increase of deterioration of prey due to predator. So to have the maximum possible profit in this scenario, amount of initial biomass also decreases. As a result total biomass of pond decreases. So catch time of fishes decrease, as a result ‘T’ decreases.

Table-2
Results for different amount of predator with respect to prey

K	S	t_1	t_2	T	Z($)
General Catch Rate					
0.020	93.457001	0.9505	0.9504	0.9505	6183.40
0.022	90.320000	0.9378	0.9359	0.9378	6193.61
0.024	87.570000	0.9245	0.9245	0.9245	6205.90
0.026	85.095795	0.9130	0.9130	0.9130	6214.50
0.028	82.871292	0.9018	0.9018	0.9018	6215.28
0.030	80.848694	0.8912	0.8912	0.8912	6214.00
0.032	79.000000	0.8810	0.8810	0.8810	6209.18
Price Independent Catch Rate					
0.020	95.717438	0.9374	0.9374	0.9374	6385.68
0.022	92.536000	0.9245	0.9245	0.9245	6402.43
0.024	89.723282	0.9125	0.9125	0.9125	6413.61
0.026	87.210037	0.9011	0.9011	0.9011	6420.01
0.028	84.945839	0.8902	0.8902	0.8902	6422.43
0.030	82.891998	0.8797	0.8797	0.8797	6419.32
0.032	81.005997	0.8700	0.8700	0.8700	6417.58
Biomass Independent Catch Rate					
0.020	91.598999	0.9665	0.9665	0.9665	5975.76
0.022	88.484589	0.9529	0.9526	0.9529	5991.79
0.024	85.735001	0.9398	0.9398	0.9398	6002.71
0.026	83.279404	0.9276	0.9276	0.9276	6008.26
0.028	81.067596	0.9160	0.9160	0.9160	6010.26
0.030	79.059998	0.9048	0.9048	0.9048	6008.47
0.032	77.224998	0.8942	0.8942	0.8942	6004.22

7. Discussion:

In the above scenarios, it is observed that prey and predator vanishes together. It happens because growth of predator in the absence of prey is very less, so initial biomass is so chosen that both are vanishes almost together except for very small values of ‘K’. In that case predator vanishes before prey. It is also observed that for the assumed parametric values, cultivation of predator together with prey is a profitable one only when amount of predator is very less compared to prey. This scenario may be changed for other set of parameters. But the cultivation of both prey and predator species together is a real-life phenomenon. During the fish cultivation, always some predator fishes are mixed-up with the prey fishes in the initial biomass used for this purpose. Moreover now-a-days, willfully predator fishes are cultivated along with prey fishes for more economical benefits as these two types of fishes move at different water levels of the pond/ bhery. This type of mixed cultivation is preferred than the single species cultivation as in the later case, the growth rate of the species is much reduced. Again there is a demand of predator fish along with prey fish in the market. So to capture the market, it is required to cultivate some amount of predator fish along with prey. It is required for natural balance also.

8. Conclusion:

In this paper, a two species harvesting policy in a bhery for a fixed period of time has been presented. It is to be noted that the existing harvesting policy available in the literature has been formulated for the infinite time period. Now-a-days, with the availability of high-breed species of fish, these cultivations in the third world countries like India, Bangladesh, etc. are very popular, economically beneficial and cultivated for the fixed time interval. For the first time, a real world multi-fish cultivation problem for a finite time period is formulated and an evolutionary optimization method like SA algorithm has been developed and implemented to find the optimum solution of a realistic problem of fish cultivation with the help of numerical solution of a system of nonlinear differential equation. A methodology is proposed to solve numerically a system of nonlinear equations, where simultaneously different numerical integration can be done. As the method of solution presented here is quite general, it can be applied to solve the models taking other conditions like withdrawal at intervals, partial withdrawals, multi-species cultivation etc. into account.

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