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Multi-objective inventory models of multi-items with quality and stock-dependent demand and stochastic deterioration

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Abstract: In affluent countries, people are more interested about the quality of an item irrespective its price structure and motivated by the decorative display of the commodity. Taking this fact into consideration, multi-objective and single objective inventory models of stochastically deteriorating items are developed in which demand is a function of inventory level and quality of the commodity. Production rate depends upon the quality level of the items produced and unit production cost is a function of production rate. Deterioration depends upon both the quality of the item and duration of time for storage. The time related function for deterioration follows a two parameter Weibull distribution in time. In these models, results are derived without shortages and for partially backlogged shortages. Here, objectives for profit maximization for each item are separately formulated and compromise solutions of the multi-objective non-linear optimization problems are obtained by two different fuzzy optimization methods. The models are illustrated with numerical examples and results from different methods are compared. The results for the models assuming them to be single house integrated business are also obtained and compared with those from the respective non-integrated models.

Keywords: Inventory, Multi-objective, Modelling, Optimization, Production, Stochastic, quality.

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1. INTRODUCTION

In real life, the demand of a particular type of commodity depends upon many factors such as time, selling price, stock level at the showroom, quality of the item, etc. It is a common practice that the higher selling price of an item negates the demand of that item whereas less price has the reverse effect. Several researchers-Abad[1], Urban[2] and others investigated the dependence on pricing for non-deteriorating items whereas Cohen[3], Mukharjee[4], etc. did the same for deteriorating commodities.

Again, according to Levin at al.[5], "it is a common belief that large piles of goods displayed in a super market will lead the customers to buy more". For this reason, several authors- Gupta and Vrat[6], Baker and Urban[7], Mandal and Maiti[8] and others have studied inventory models with stock-dependent demand.

Moreover, in a competitive society, the quality level of the commodities plays a vital role to stimulate the demand for the items. Very few researchers (cf. Chen[9] etc.) have considered the quality of items in their analysis. But till now, no inventory model has been formulated taking demand to be dependent jointly on inventory and quality of the commodities.

In conventional studies of inventory models, it is normally assumed that the lifetime of an item is infinite while in storage. In reality, it is not always true. Due to the ill preservation conditions, etc., some portion of items like food grains, vegetables, fruits, drugs etc, are damaged or decayed due to dryness, spoilage, vaporization, etc. and are not in a condition to satisfy the future demand of customers. Some authors- Mandal and Phoujder[10], Ting and Chung[11], Mandal and Maiti[12], and others considered the inventory models for deteriorated / damaged items assuming deterioration to be constant or linearly dependent on time or stock level. A very few scientists have taken deterioration to be stochastically dependent on time. But deterioration rate also depends upon the physical condition of the commodities i.e., it is quality level dependent too. An item of high quality will decay much less than the low quality ones. This phenomenon has been overlooked by the researchers. Till now, none has related deterioration jointly with quality level of the products and duration of storage.

Again, in classical inventory problems, the production rate of a machine is assumed to be predetermined and inflexible. Production rate gets affected to maintain the quality of the items. If the quality of the products is very high, the production suffers. Hence, rate of production is inversely related with the quality.This relationship too has been ignored by most of researchers.

The production cost per unit item is partly related to the rate of production. If the production is more, the wages of the workers and fixed over-head expenditures are spread over more units and as a result, average production cost comes down. Hence, unit cost and production rate are partly inversely related. Khauja[13], etc. have taken production dependent unit cost but none has related this with the quality of the commodities.

In the important market places, space problem is a big hindrance and takes a vital role to expand the business. Taking space limitation as constraints, several workers- Roy and Maiti[14], Mandal and Maiti[15], etc. have considered multi-item inventory models in crisp and fuzzy environments.

Now-a-days, almost every important real world problem involves more than one objective. More so than ever before, decision makers find it imperative to evaluate solution alternatives according to multiple conflicting criteria. These problems are modelled as multi-criteria decision making (MCDM) problems identifying the types of measures that might be said as 'criteria'. The thrust of these models is to design the 'best'alternative by considering the various interaction within the objectives and constraints which best satisfies decision makers (DM) by way of attaining some acceptable levels of a set of some quantifiable objectives. Recently various new methods (cf. Gabriele and Ragsdell[16], Tiwari, et al.[17], Zimmermann[18]) have been outlined to find the compromise solution of MCDM problems.

In this paper, under limited storage space, multi-objective inventory models for stochastically deteriorating items under a single management is formulated. Here, demand is influenced jointly by quality level of goods and on-hand inventory level. Deterioration depends upon both the quality level of the item and time duration for storage. The time related function for deterioration follows a two parameter Weibull distribution in time, t. Production rate changes inversely with the quality level of items and the unit production cost is partly inversely dependent upon the production rate. Moreover, the set-up cost is also quality dependent and the selling price is assumed to be marker over the production cost. It is assumed that the items are produced separately in different production firms under a single ownership. The profit maximization objectives are derived for each item and hence multi-objective inventory problems are formulated and solved for a compromise solution by recently developed methods-(i) Fuzzy Non-linear Programming Technique (FNLP) and (ii) Fuzzy Additive Goal Programming Technique (FAGP). For the models, results are derived without shortages and for partially backlogged shortages. The models are illustrated numerically and the best possible solutions from different methods are compared. The problems have been solved also formulating them as a single objective.

2. ASSUMPTIONS AND NOTATIONS

Multi-objective and single objective economic lot size problems for stochastically deteriorating items are developed under the following notations and assumptions :

2.1. Notations

V	= available storage space.
PF	=total average profit.
n	= number of items in the inventory system.
For i-th item,	
$q_i(t)$	=inventory level at any time t.

K_i = rate of production.
d_i = rate of demand.
q_{ui} = quality level of the item
p_i = per unit production cost
s_i = selling price
c_{2i} =shortage cost per unit per unit time (when it is allowed).
h_i = holding cost per unit per unit time.
u_i = setup cost in a time cycle.
S_{hi} = total shortage cost in a time cycle.
g_i = deteriorated units in a time cycle
H_i = total holding cost in a time cycle
TP_i = total production cost in a time cycle
t_{4i} = one time cycle
v_i = space required to store a unit.
b_i =the fraction of demand to be backlogged.
Q_{1i} = maximum inventory level at time t_{1i}.
Q_{2i} = maximum shortage level at time t_{2i} (when it is allowed).
PF_i = total average profit.

2.2. Assumptions

(i) The inventory system involves n items.

(ii) The planning horizon is infinite.

(iii) Lead time is negligible.

(iv) There is no repair or replacement of the deteriorated units.

(v) Deterioration is a function of both quality level and time where time related function follows a two parameter Weibull distribution: $\theta_i(t, q_{ui}) = \theta_{1i}(q_{ui})\theta_{2i}(t)$ where $\theta_{1i}(q_{ui}) = \alpha_i^{'} q_{ui}^{-\delta_i}$ and $\theta_{2i}(t) = \alpha_i^{''}\beta_i t^{\beta_i - 1}$; $t \geq 0$.
Here $\alpha_i^{'}$ is a positive real constant, $\alpha_i^{''}$ and β_i are scale and shape parameters respectively.

(vi) The rate of production is a function of quality level of the product: $K_i = a_i q_{ui}^{-\phi_{1i}}$, Here a_i is positive and $0 < \phi_{1i} < 1$.

(vii) The setup cost consists of two parts: one part is constant, a_{3i} and another changes with the quality level of the product. Thus $u_i = u_{1i} + u_{2i}q_{ui}^{\phi_{3i}}$, here u_{1i}, u_{2i} are positive and $0 \leq \phi_{3i} \leq 1$.

(viii) Unit production cost is a function of production rate as $p_i = x_i + y_i K_i^{-n_i}$, where x_i, y_i are positive and n_i, $(i = 1, 2, ..., n)$ are integers.

(ix) Rate of demand is a function of quality level of the product and inventory level at any time t, i.e., $d_i = d_{0i} + \lambda_i q_{ui}^{\phi_{2i}} q_i(t)$ for $q_i(t) > 0$ where d_{0i} and λ_i are positive and $0 < \phi_{2i} < 1$. Let, $d_{1i} = \lambda_i q_{ui}^{\phi_{2i}}$, then d_i can be written as $d_i = d_{0i} + d_{1i} q_i(t)$. During the shortage period, demand is assumed to be constant.

(x) unit selling price depends on per unit production cost i.e. $s_i = m_i p_i$, where $m_i (> 1)$ is the mark-up.

3. MATHEMATICAL FORMULATION

3.1 Model $-1a$ [Non-integrated model with shortages]

The production starts at time $t = 0$. Hence, for the $i-$ th item, initially the stock is zero and later, reaches maximum inventory level, Q_{1i} after time, t_{1i}. During this time, produced units are partly exhausted against the demand and deterioration and excess units are stored. Then the production is stopped, the stock level declines continuously due to both demand and deterioration and inventory level becomes zero at time $t = t_{2i}$. Now shortages are allowed up to maximum shortage level, Q_{2i}, at time $t = t_{3i}$. At this instant of time, fresh production starts to meet the current demand and to clear the accumulated backlogged shortages partially by the time $t = t_{4i}$(cf. Fig.-1). Here, it is assumed that due to the non-availability of the item, some customers back away. Now, the objective is to find out the optimal values of t_{1i}, t_{2i}, t_{3i} and t_{4i}, that maximize the profit over the time horizon $[0, t_{4i}]$ following the restriction on the storage space.

If $q_i(t)$ be the inventory level of the i-th item at time t at the production center, then the differential equations governing the stock status during the period $[0, t_{4i}]$ can be written as

$$\begin{aligned}
\frac{dq_i(t)}{dt} &= K_i - (d_{0i} + d_{1i} q_i(t)) - \theta_i(t, q_{ui}) q_i(t), \qquad 0 \le t \le t_{1i} && (1)\\
&= -(d_{0i} + d_{1i} q_i(t)) - \theta_i(t, q_{ui}) q_i(t), \qquad t_{1i} \le t \le t_{2i} && (2)\\
&= -b_i d_{0i}, \qquad t_{2i} \le t \le t_{3i} && (3)\\
&= K_i - d_{0i}, \qquad t_{3i} \le t \le t_{4i} && (4)
\end{aligned}$$

with boundary condition $q_i(t) = 0$ at $t = 0,\ t_{2i},\ t_{4i}$.

Following equations (1) – (4), we get (see Appendix)

$$\begin{aligned}
g_i &= \int_0^{t_{1i}} \theta_i(t, q_{ui}) q_i(t) dt + \int_{t_{1i}}^{t_{2i}} \theta_i(t, q_{ui}) q_i(t) dt\\
H_i &= h_i \Big\{ \int_0^{t_{1i}} q_i(t) dt + \int_{t_{1i}}^{t_{2i}} q_i(t) dt \Big\}\\
S_{hi} &= c_{2i} \Big\{ \int_{t_{2i}}^{t_{3i}} q_i(t) dt + \int_{t_{3i}}^{t_{4i}} q_i(t) dt \Big\}
\end{aligned}$$

$$
\begin{aligned}
TP_i &= p_i\Big\{\int_0^{t_{1i}} K_i dt + \int_{t_{3i}}^{t_{4i}} K_i dt\Big\} \\
PF_i &= \frac{1}{t_{4i}}\Big\{s_i\{K_i(t_{1i}+t_{4i}-t_{3i})-g_i\} \\
&- \{K_i p_i(t_{1i}+t_{4i}-t_{3i})+H_i+u_i+S_{hi}\}\Big\} \\
& i=1,2,......,n.
\end{aligned}
$$

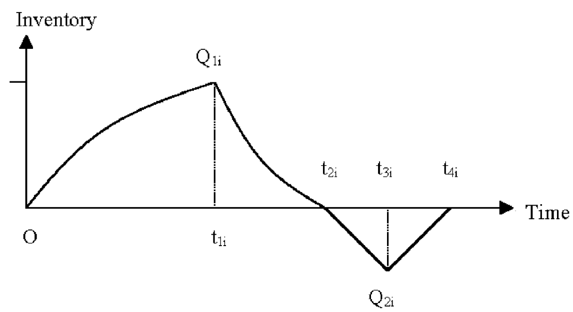

Figure-1: The pattern of inventory when shortages allowed

Now, the model can be represented as the following multi-objective problem:

$$
\begin{aligned}
&\text{Maximize} \quad \{PF_1, PF_2, ..., PF_n\} \qquad (5)\\
&\text{subject to} \quad \sum_{i=1}^{n} v_i Q_{1i} \leq V; [\text{ due to storage space limitation }]
\end{aligned}
$$

where t_{1i}, t_{2i}, t_{3i} and t_{4i} are related by the equations(20) and (23)[See Appendix].

3.2 Model-1b [Integrated model with shortages]

Assuming that the items are dealt with collectively as a single integrated business process, the corresponding single objective model is

$$
\begin{aligned}
&\text{Maximize} \quad PF = \sum_{i=1}^{n} PF_i \qquad (6)\\
&\text{subject to} \quad \sum_{i=1}^{n} v_i Q_{1i} \leq V;
\end{aligned}
$$

3.3 Model-2a [Non-integrated model without shortages]

When shortages are not allowed, the inventory model is represented as Fig.-2

Here problem is to find out the optimal values of t_{1i} and t_{2i} that maximize the profit over the time horizon $[0, t_{2i}]$.

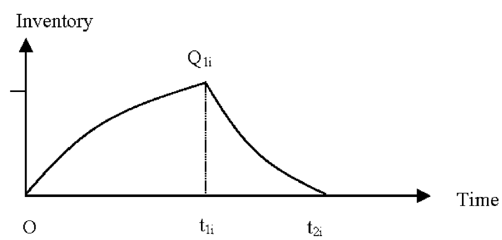

Figure-2 : The pattern of inventory when shortages not allowed

In this case,for the $i-$th item, the differential equations and other expressions are

$$\begin{aligned}\frac{dq_i(t)}{dt} &= K_i - (d_{0i} + d_{1i}q_i(t)) - \theta_i(t, q_{ui})q_i(t), \qquad 0 \le t \le t_{1i} &(7)\\ &= -(d_{0i} + d_{1i}q_i(t)) - \theta_i(t, q_{ui})q_i(t), \qquad t_{1i} \le t \le t_{2i} &(8)\end{aligned}$$

with boundary condition $q_i(t) = 0,$ at $t = 0,\ t_{2i}$.

$$\begin{aligned} g_i &= \int_0^{t_{1i}} \theta_i(t, q_{ui})q_i(t)dt + \int_{t_{1i}}^{t_{2i}} \theta_i(t, q_{ui})q_i(t)dt \\ H_i &= h_i\Big\{ \int_0^{t_{1i}} q_i(t)dt + \int_{t_{1i}}^{t_{2i}} q_i(t)dt \Big\} \\ TP_i &= p_i \int_0^{t_{1i}} K_i dt = K_i p_i t_{1i} \\ PF_i &= \frac{1}{t_{2i}}\Big\{ s_i\{K_i t_{1i} - g_i\} - \{K_i p_i t_{1i} + H_i + u_i)\}\Big\} \\ & \quad i = 1, 2, ..., n. \end{aligned}$$

Hence the model reduces to a multi-objective maximization problem as

$$\begin{aligned} &\text{Maximize} \qquad \{PF_1, PF_2, ..., PF_n\} \qquad (9)\\ &\text{subject to} \qquad \sum_{i=1}^{n} v_i Q_{1i} \le V; \end{aligned}$$

where t_{1i} and t_{2i} are related by the equation(20)[See Appendix].

3.4 Model-2b [Integrated model without shortages

The corresponding single objective model is

$$\begin{aligned} &\text{Maximize} \qquad PF = \sum_{i=1}^{n} PF_i \qquad (10)\\ &\text{subject to} \qquad \sum_{i=1}^{n} v_i Q_{1i} \le V; \text{[due to storage space limitation]} \end{aligned}$$

4. MATHEMATICAL ANALYSIS OF MULTI-OBJECTIVE PROBLEMS

4.1 Multi-objective Non-linear Programming (MONLP) Problem

A MONLP problem can be stated as:

$$\begin{aligned} \text{Find} \quad & x = (x_1, x_2, ..., x_n)^T \\ \text{which maximizes} \quad & F(x) = (f_1(x), f_2(x), ..., f_k(x))^T \\ \text{subject to} \quad & g_j(x) \le a_j, \ \ (j = 1, 2, ..., l_1) \\ & h_t(x) = b_t, \ \ (t = 1, 2, ..., l_2) \\ & x_i \ge 0 \ \ (i = 1, 2, ..., n) \end{aligned} \tag{11}$$

It is noted that, if the objectives of the original problem are to maximize $f_i(x)$ for $i = 1, 2, ..., k_0$ and minimize $f_i(x)$ for $i = k_0+1, k_0+2, ..., k$, then the objective in the mathematical formulation will be

$$F(x) = (f_1(x), f_2(x)...., f_{k_0}(x), -f_{k_0+1}(x), -f_{k_0+2}(x), ..., -f_k(x))^T.$$

The functions $f_i(x)$, $g_j(x)$ and $h_t(x)$ may be linear or non-linear.

Solution of MONLP problems

4.2 Fuzzy Programming Techniques

A crisp multi-objective non-linear programming problem (11) can be solved by fuzzy programming method. The steps of the method are as follows:

Step-1: Solve the Multi-Objective Non-linear Programming (MONLP) as a single objective non-linear programming problem by using any gradient based non-linear programming algorithm, considering only one of the objectives at a time and ignoring all others. Repeat the process k times for k different objective functions. Let $x^1, x^2, x^3, ..., x^k$ be the ideal solutions for the respective objective functions where $x^r = (x_1^r, x_2^r, ..., x_n^r)$

Step-2: Using all the above ideal solutions in step-1, construct a pay-off matrix of size $(k \times k)$ as follows:

$$\begin{array}{cc} & \begin{array}{cccc} f_1(x^r) & f_2(x^r) & ... & f_k(x^r) \end{array} \\ \begin{array}{c} x^1 \\ x^2 \\ ... \\ x^k \end{array} & \begin{pmatrix} f_1(x^1) & f_2(x^1) & ... & f_k(x^1) \\ f_1(x^2) & f_2(x^2) & ... & f_k(x^2) \\ ... & ... & ... & ... \\ f_1(x^k) & f_2(x^k) & ... & f_k(x^k) \end{pmatrix} \end{array}$$

From the pay-off matrix, estimate the lower bound L_r and the upper bound U_r for the r-th objective as

$$L_r \le f_r \le U_r, \ \ (r = 1, 2, ..., k)$$

$$\text{where} \quad L_r = \min\,[f_r(x^1), f_r(x^2), ..., f_r(x^k)]$$
$$\text{and} \quad U_r = \max\,[f_r(x^1), f_r(x^2), ..., f_r(x^k)]$$

Step-3. Define a fuzzy linear or non-linear membership function $\mu_{f_r}(x)$ for the r-th objective function, $f_r(x)$, $(r = 1, 2, 3, ..., k.)$. Here for simplicity a linear membership for r-th objective function $f_r(x)$, $(r = 1, 2, 3, ..., k)$ is taken as

$$\mu_{f_r}(x) = \begin{cases} 0 & \text{for } L_r > f_r(x) \\ \dfrac{f_r(x) - L_r}{U_r - L_r} & \text{for } L_r \leq f_r(x) \leq U_r \\ 1 & \text{for } f_r(x) > U_r \end{cases} \tag{12}$$

Step-4. At this stage, either fuzzy additive goal programming method or fuzzy non-linear programming method can be used to formulate the corresponding objective optimization problem.

4.2.1 Fuzzy Additive Goal Programming (FAGP) Method

Use the above membership functions to formulate a crisp non-linear programming model (following Tiwari, et al.[22],) by adding the membership functions together as

$$\begin{aligned} \text{Max} \quad & A(\mu_{f_1}(x), \mu_{f_2}(x), ..., \mu_{f_k}(x)) = w_1\mu_{f_1}(x) + w_2\mu_{f_2}(x) + ... + w_k\mu_{f_k}(x) \\ \text{subject to} \quad & \mu_{f_r}(x) = 1 - \frac{f_r(x) - L_r}{U_r - L_r}, \ (r = 1, 2, 3, ..., k) \\ & g_j(x) \leq a_j, \ (j = 1, 2, ..., l_1) \\ & h_t(x) = b_t, \ (t = 1, 2, ..., l_2) \\ & x_i \geq 0, \ (i = 1, 2, ..., n) \end{aligned} \tag{13}$$

Here $A(\mu_{f_1}(x), \mu_{f_2}(x), ..., \mu_{f_k}(x))$ is called fuzzy achievement function or fuzzy decision function. w_i's are weights associated with the i-th objective function $f_i(x)$ such that $w_i > 0$ and $\sum_{i=1}^{k} w_i = 1$.

4.2.2 Fuzzy Non-linear Programming (FNLP) Method

According to Zimmermann[18], the crisp model (11) can be solved by FNLP method as

$$\begin{aligned} \text{Max} \quad & \alpha \\ \text{subject to} \quad & \alpha \leq \frac{f_r(x) - L_r}{U_r - L_r}, \ (r = 1, 2, 3, ..., k) \\ & g_j(x) \leq a_j, \ (j = 1, 2, ..., l_1) \\ & h_t(x) = b_t, \ (t = 1, 2, ..., l_2) \\ & x_i \geq 0 \ (i = 1, 2, ..., n) \end{aligned} \tag{14}$$

Here α is an aspiration level of the objective functions.

5. SOLUTION OF INVENTORY MODELS

Following the analysis in article-4, the multi-objective models (5) or (9) can be represented as:

(a) Fuzzy Non-linear Programming Method:

$$\text{Max } \alpha \tag{15}$$

$$\text{subject to} \quad \alpha \leq \frac{PF_i - L_{PF_i}}{U_{PF_i} - L_{PF_i}} \quad i = 1, 2, 3.$$

$$\sum_{i=1}^{3} v_i Q_{1i} \leq V$$

where U_{PF_i} and L_{PF_i} are respectively upper and lower limits of profit function PF_i.

(b) Fuzzy Additive Goal Programming Problem:

$$\text{Max } \sum_{i=1}^{3} w_i \mu_{PF_i} \tag{16}$$

$$\text{subject to} \quad \mu_{PF_i} = \frac{PF_i - L_{PF_i}}{U_{PF_i} - L_{PF_i}}, \; i = 1, 2, 3.$$

$$\sum_{i=1}^{3} v_i Q_{1i} \leq V$$

Here equal weights are used i.e. $w_i = \frac{1}{3}$, i=1, 2, 3.

6. NUMERICAL EXAMPLES

Here, the multi-objective models (i.e. model-1a and model-2a) are solved by the methods, namely, FNLP and FAGP and the single objective models (i.e. model $-1b$ and model $-2b$) by Generalized Reduced Gradient (GRG) method- as described in the earlier section.

To illustrate the above problems, we consider the following parameters as $n = 3$, $V =$ 500 units and the parameter values giver in Table-1.

Table-1: Input data table

items(i)	m_i	x_i	y_i	n_i	$\alpha_i^{'}$	δ_i	$\alpha_i^{''}$	β_i	$h_i(in\$)$	q_{ui}
1	1.35	30	4000	1	0.06	0.5	0.03	0.7	3.5	8
2	1.35	30	5000	1	0.06	0.6	0.03	0.7	3.0	9
3	1.30	30	4000	1	0.06	0.3	0.06	0.7	3.5	10
items(i)	a_i	d_{0i}	λ_{1i}	u_{1i}	u_{2i}	ϕ_{1i}	ϕ_{2i}	ϕ_{3i}	v_i	
1	850	180	0.01	300	50	0.6	0.6	0.7	2	
2	1000	300	0.01	350	100	0.5	0.6	0.7	3	
3	850	200	0.01	300	50	0.5	0.6	0.7	5	

Results of Models$-1a$ and $-1b$

Shortages are partially backlogged

Using the above numerical values for different parameters and taking $b_1 = 0.8,\ b_2 = 0.7,\ b_3 = 0.75,$ the results of the models with partially backlogged shortages are obtained. In this case, pay-off matrix is:

pay-off matrix

$$\begin{array}{c} \begin{array}{ccc} PF_1 & PF_2 & PF_3 \end{array} \\ \begin{pmatrix} 2560.07 & 3929.18 & 1843.87 \\ 2086.01 & 4394.26 & 1983.49 \\ 2051.98 & 13730.01 & 2289.13 \end{pmatrix} \end{array}$$

In different multi-objective optimization techniques, the Model$-1b$ is reduced to the following problems.

(a.) FNLP Problem

Following (15), the problem reduces to

$$\begin{aligned} & \text{Max } \alpha \\ \text{subject to} \quad & \alpha \leq \frac{PF_1 - 2051.98}{2560.07 - 2051.98} \\ & \alpha \leq \frac{PF_2 - 3730.01}{4394.26 - 3730.01} \\ & \alpha \leq \frac{PF_3 - 1843.87}{2289.13 - 1843.87} \\ & \sum_{i=1}^{3} v_i Q_{1i} \leq V \end{aligned}$$

(b.) FAGP Problem

Following (16), the problem reduces to

$$\begin{aligned} & \text{Max } \mu_{PF_1} + \mu_{PF_2} + \mu_{PF_3} \\ \text{subject to} \quad & \mu_{PF_1} = \frac{PF_1 - 2051.98}{2560.07 - 2051.98} \\ & \mu_{PF_2} = \frac{PF_2 - 3730.01}{4394.26 - 3730.01} \\ & \mu_{PF_3} = \frac{PF_3 - 1843.87}{2289.13 - 1843.87} \\ & \sum_{i=1}^{3} v_i Q_{1i} \leq V \end{aligned}$$

Here equal weights are used i.e. $w_1 = w_2 = w_3 = \frac{1}{3}$. The Results of Model $-1a$ and $-1b$ are given in Table-2.

Table-2

Optimal solutions for the model with shortage which are partially backlogged

Objective(s)	Method	items (i)	PF_i (in \$)	t_{1i}	t_{2i}	t_{3i}	t_{4i}	Super criteria
MONLP	FNLP	1	2443.05	0.7517228	1.014663	1.223186	1.691644	1147479
		2	4242.32	1.502845	1.664696	1.770896	2.439951	
		3	2187.28	0.7643590	1.021793	1.264537	1.793824	
	FAGP	1	2510.69	1.085992	1.462889	1.622990	1.982665	949429
		2	4268.97	1.677797	1.857844	1.952881	2.551615	
		3	2135.67	0.5910677	0.79107458	1.078084	1.703889	
SONLP	GRG	1	2511.16	1.089274	1.467281	1.627827	1.988501	
		2	4304.51	1.968693	2.178703	2.261602	2.783870	
		3	2106.02	0.5090839	0.6817338	0.9943830	1.676094	

Results of Models$-2a$ and $-2b$

When shortages are not allowed, using the above relevant numerical values for different parameters, we obtain the pay off matrix as follows:

pay-off matrix

$$\begin{array}{ccc} PF_1 & PF_2 & PF_3 \end{array}$$
$$\begin{pmatrix} 2292.65 & 2417.80 & 546.26 \\ 921.60 & 4389.25 & 0.0 \\ 0.0 & 0.0 & 2254.48 \end{pmatrix}$$

The optimal results for the models without shortage (i.e., Models $-2a$ and $-2b$) are presented in Table-3.

Table-3

Optimal results for the models without shortage

Objective(s)	Method	items (i)	PF_i (in \$)	t_{1i}	t_{2i}	*Super* criteria
MONLP	*FNLP*	1	2134.47	0.8322516	1.122809	7451101
		2	4086.42	1.249747	1.385043	
		3	2098.93	0.8063751	1.077652	
	FAGP	1	2204.03	1.032248	1.390943	5251002
		2	4110.91	1.309582	1.451180	
		3	2043.13	0.7148490	0.9559304	
SONLP	*GRG*	1	2183.11	0.9605577	1.294902	
		2	4206.24	1.619959	1.794005	
		3	1995.59	0.6535478	0.8743218	

7. DISCUSSION

Here, two types of formulations—one as multi-objective and other as single objective have been presented. In multi-objective formulation, different producers / manufacturers produces the items and perform other management operations separately but they share a common storing space for storing the items. Single objective is formulated when the items are produced and managed under a single management. It is observed from the Tables-2 and -3 that FNLP and FAGP gives different compromise solution for each of the model. Between two multi-objective solution methods, FAGP gives better result than FNLP in terms of super-criteria (cf. Dhingra and Rao[20]) for both the model. From the Tables-2 and -3, the expected results are reflected. For both the models, each method gives better results in terms of total profit for with-shortage model than that of without shortage model which is obvious. Actually, the optimum solutions for inventory control problems given by the various multi-objective optimization methods can be observed to be different from each other. Since the solutions are different, the set of active constraints and time periods will also be different in each case. This is an inherent characteristic of any multi-objective optimization problem. Hence a solution concept or procedure has to be defined on the basis of other attributes such as the mathematical basis of the method, its generality, its easiness for computation etc.

8. CONCLUSION

In this paper, some realistic inventory models with quality conscious customers are formulated and solved. Here, following new features have been introduced in the inventory models for the first time.

(i) Demand is a function of quality level and on-hand stock of the item.
(ii) Unit production and setup cost are quality level dependent.
(iii) Deterioration is dependent jointly with quality and duration of storage(stochastically).
(iv) Production cost is dependent on production rate i.e. it is quality level dependent.
(v) Lastly, two newly developed MCDM methods are illustrated.

The models have been formulated here in probabilistic environment taking stochastic deterioration and all other inventory parameters as deterministic. The models can be extended to include discount, salvage of deteriorated quantities, etc. This may also be formulated in fuzzy-stochastic environment taking the goal of the profit as fuzzy or some other inventory parameters like inventory cost, quality level, etc as imprecisely defined.

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APPENDIX

Solution of the differential equation (1)

$$\begin{aligned} q_i(t) &= (K_i - d_{0i})\Big\{t - \frac{1}{2}d_{1i}t^2 - \frac{\alpha_i''\theta_{1i}\beta_i t^{\beta_i+1}}{\beta_i+1} + \frac{1}{6}d_{1i}^2t^3 \\ &+ \frac{\alpha_i''\theta_{1i}d_{1i}\beta_i(\beta_i+3)t^{\beta_i+2}}{2(\beta_i+1)(\beta_i+2)} + \frac{\alpha_i''^2\theta_{1i}^2\beta_i^2t^{2\beta_i+1}}{(\beta_i+1)(2\beta_i+1)}\Big\} \end{aligned} \tag{17}$$

(taking upto second order terms only). Solving the differential equation (2), we get

$$\begin{aligned} q_i(t) &= -d_{0i}e^{-(d_{1i}t+\alpha_i''\theta_{1i}t^{\beta_i})}\int_{t_{1i}}^{t} e^{(d_{1i}t+\alpha_i''\theta_{1i}t^{\beta_i})}dt \\ &= -d_{0i}\Big\{t - \frac{1}{2}d_{1i}t^2 - \frac{\alpha_i''\theta_{1i}\beta_i t^{\beta_i+1}}{\beta_i+1} + \frac{1}{6}d_{1i}^2t^3 + \frac{\alpha_i''\theta_{1i}d_{1i}\beta_i(\beta_i+3)t^{\beta_i+2}}{2(\beta_i+1)(\beta_i+2)} \\ &+ \frac{\alpha_i''^2\theta_{1i}^2\beta_i^2t^{2\beta_i+1}}{2(\beta_i+1)(2\beta_i+1)}\Big\} + c_{0i}\Big\{1 - (d_{1i}t + \alpha_i''\theta_{1i}t^{\beta_i}) + \frac{1}{2}d_{1i}^2t^2 \\ &+ \alpha_i''\theta_{1i}d_{1i}t^{\beta_i+1} + \frac{1}{2}\alpha_i''^2\theta_{1i}^2t^{2\beta_i}\Big\} \end{aligned} \tag{18}$$

$$\begin{aligned} \text{where } c_{0i} &= d_{0i}e^{-(d_{1i}t_{2i}+\alpha_i''\theta_{1i}t_{2i}^{\beta_i})}\Big\{t_{2i} - \frac{1}{2}d_{1i}t_{2i}^2 - \frac{\alpha_i''\theta_{1i}\beta_i t_{2i}^{\beta_i+1}}{\beta_i+1} + \frac{1}{6}d_{1i}^2t_{2i}^3 \\ &+ \frac{\alpha_i''\theta_{1i}d_{1i}\beta_i(\beta_i+3)t_{2i}^{\beta_i+2}}{2(\beta_i+1)(\beta_i+2)} + \frac{\alpha_i''^2\theta_{1i}^2\beta_i^2t_{2i}^{2\beta_i+1}}{(\beta_i+1)(2\beta_i+1)}\Big\} \end{aligned} \tag{19}$$

As $q_i(t)$ is continuous at $t = t_{1i}, (i = 1, 2, \cdots, n)$, and $q_i(t_{1i}) = Q_{1i}$, we have from equation (17) and (18)

$$\begin{aligned} &K_it_{1i}\Big\{1 - \frac{1}{2}d_{1i}t_{1i}^1 - \frac{\alpha_i''\theta_{1i}\beta_i t_{1i}^{\beta_i}}{\beta_i+1} + \frac{1}{6}d_{1i}^2t_{1i}^2 + \frac{\alpha_i''\theta_{1i}d_{1i}\beta_i(\beta_i+3)t_{1i}^{\beta_i+1}}{2(\beta_i+1)(\beta_i+2)} + \frac{\alpha_i''^2\theta_{1i}^2\beta_i^2t_{1i}^{2\beta_i}}{(\beta_i+1)(2\beta_i+1)}\Big\} \\ = \; &c_{0i}\Big\{1 - (d_{1i}t_{1i} + \alpha_i''\theta_{1i}t_{1i}^{\beta_i}) + \frac{1}{2}d_{1i}^2t_{1i}^2 + \alpha_i''\theta_{1i}d_{1i}t_{1i}^{\beta_i+1} + \frac{1}{2}\alpha_i''^2\theta_{1i}^2t_{1i}^{2\beta_i}\Big\} \end{aligned} \tag{20}$$

Either sides of equation(20) is equal to Q_{1i}. The equation(20) gives the relation among t_{1i}, t_{2i} and t_{3i}. Solving the differential equation (3), we get

$$q_i(t) = -b_i d_{0i}(t - t_{2i}) \tag{21}$$

Similarly, Solutio of the differential equation (4) is

$$q_i(t) = -(K_i - d_{0i})(t_{4i} - t) \tag{22}$$

As $q_i(t)$ is continuous at $t = t_{3i}, (i = 1, 2, \cdots, n)$, and $q_i(t_{3i}) = Q_{2i}$ we have from (21) and (22)

$$b_i d_{0i}(t_{3i} - t_{2i}) = (K_i - d_{0i})(t_{4i} - t_{3i}) \tag{23}$$

Either sides of equation(23) is equal to Q_{2i}. The equation(23) gives the relation among t_{2i}, t_{3i} and t_{4i}. The deteriorated units during $(0, t_{4i})$ is

$$\begin{aligned}
g_i &= K_i\Big\{\frac{\alpha_i^{''}\theta_{1i}\beta_i t_{1i}^{\beta_i+1}}{\beta_i+1} - \frac{d_{1i}\alpha_i^{''}\theta_{1i}\beta_i t_{1i}^{\beta_i+2}}{2(\beta_i+2)} - \frac{{\alpha_i^{''}}^2\theta_{1i}^2\beta_i^2 t_{1i}^{2\beta_i+1}}{(\beta_i+1)(2\beta_i+1)}\Big\}\\
&- d_{0i}\Big\{\frac{\alpha_i^{''}\theta_{1i}\beta_i}{\beta_i+1}t_{2i}^{\beta_i+1} - \frac{d_{1i}\alpha_i^{''}\theta_{1i}\beta_i}{2(\beta_i+2)}t_{2i}^{\beta_i+2} - \frac{{\alpha_i^{''}}^2\theta_{1i}^2\beta_i^2}{(\beta_i+1)(2\beta_i+1)}t_{2i}^{2\beta_i+1}\Big\}\\
&+ c_{0i}\Big\{\alpha_i^{''}\theta_{1i}(t_{2i}^{\beta_i} - t_{1i}^{\beta_i}) - \frac{d_{1i}\alpha_i^{''}\theta_{1i}\beta_i}{\beta_i+1}(t_{2i}^{\beta_i+1} - t_{1i}^{\beta_i+1}) - \frac{{\alpha_i^{''}}^2\theta_{1i}^2}{2}(t_{2i}^{2\beta_i} - t_{1i}^{2\beta_i})\Big\}
\end{aligned}$$

The inventory carrying cost over the period $(0, t_{4i})$ is

$$\begin{aligned}
H_i &= h_i\Big\{K_i\Big\{\frac{t_{1i}^2}{2} - \frac{d_{1i}t_{1i}^3}{6} - \frac{\alpha_i^{''}\theta_{1i}\beta_i t_{1i}^{\beta_i+2}}{(\beta_i+1)(\beta_i+2)} + \frac{d_{1i}^2 t_{1i}^4}{24} + \frac{\alpha_i^{''}\theta_{1i}d_{1i}\beta_i t_{1i}^{\beta_i+3}}{2(\beta_i+1)(\beta_i+2)}\\
&+ \frac{{\alpha_i^{''}}^2\theta_{1i}^2\beta_i^2 t_{1i}^{2\beta_i+2}}{(\beta_i+1)(2\beta_i+1)(2\beta_i+2)}\Big\} - d_{0i}\Big\{\frac{1}{2}t_{2i}^2 - \frac{d_{1i}}{6}t_{2i}^3 - \frac{\alpha_i^{''}\theta_{1i}\beta_i}{(\beta_i+1)(\beta_i+2)}t_{2i}^{\beta_i+2}\\
&+ \frac{d_{1i}^2}{24}t_{2i}^4 + \frac{\alpha_i^{''}\theta_{1i}d_{1i}\beta_i}{2(\beta_i+1)(\beta_i+2)}t_{2i}^{\beta_i+3} + \frac{{\alpha_i^{''}}^2\theta_{1i}^2\beta_i^2}{(\beta_i+1)(2\beta_i+1)(2\beta_i+2)}t_{2i}^{2\beta_i+2}\Big\}\\
&+ c_{0i}\Big\{(t_{2i} - t_{1i})\frac{d_{1i}}{2}(t_{2i}^2 - t_{1i}^2) - \frac{\alpha_i^{''}\theta_{1i}}{(\beta_i+1)}(t_{2i}^{\beta_i+1} - t_{1i}^{\beta_i+1}) + \frac{d_{1i}^2}{6}(t_{2i}^3 - t_{1i}^3)\\
&+ \frac{\alpha_i^{''}\theta_{1i}d_{1i}}{(\beta_i+2)}(t_{2i}^{\beta_i+2} - t_{1i}^{\beta_i+2}) + \frac{{\alpha_i^{''}}^2\theta_{1i}^2}{2(2\beta_i+1)}(t_{2i}^{2\beta_i+1} - t_{1i}^{2\beta_i+1})\Big\}\Big\}
\end{aligned}$$

The shortage cost over the period $(0, t_{4i})$ is

$$S_{hi} = \frac{1}{2}c_{2i}b_i d_{0i}(t_{3i} - t_{2i})^2 + \frac{1}{2}c_{2i}(K_i - d_{0i})(t_{4i} - t_{3i})^2$$

Total production cost for i-th item is

$$TP_i = p_i\Big\{\int_0^{t_{1i}} K_i dt + \int_{t_{3i}}^{t_{4i}} K_i dt\Big\} = K_i p_i(t_{1i} + t_{4i} - t_{3i})$$