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Coordinated Sale of Differential units with promotional cost and units' price through different shops in Fuzzy Environment.

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Abstract

In this paper, coordinated sale of differential units of an item through different shops for maximizing the profit is considered. Here demand of the item is influenced by the unit's price and promotional cost. For illustration, the inventory problem of differential units sold from three different shops owned by a single management is formulated under fuzzy environment. Primary shop initially receives the lot of differential (perfect, less defective and more defective) units, which are continuously separated. Only the perfect units are sold from the primary shop with a profit and its demand depends upon price and stock level. The separated defective items are continuously transferred to the adjacent secondary shops according to their defectiveness and then sold at a less reduced price from secondary shop–1 and at a more reduced price from secondary shop–2. Here we consider retailer's promotional cost, which generates additional demand for good units. In this paper, targeted total profit and mark up of selling prices of defective units are fuzzy in nature. The impreciseness of the objective goal has been expressed by linear membership function and vagueness in mark-up by triangular fuzzy numbers. The inventory model is developed and several scenarios are presented. These fuzzy inventory problems are converted to corresponding crisp ones and solved by Genetic Algorithm (GA) formulated for this purpose. The models are illustrated with some numerical data. A set of near optimum solutions and a study on the effect of initial demand on promotional cost and average optimal profit are presented.

1. Introduction

In competitive marketing situation, price of a commodity is an important factor in creating demand in the society. This led many researchers to investigate inventory [1,2,3,4] models with a price dependent demand. Moreover many business people use showrooms and the attractive display of units in the showroom to influence the demand. Many researchers also focused on the analysis of inventory system, which describes the demand rate to be dependent on the displayed inventory level. This type of model was

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first developed by Baker and Urban [5] considering an inventory system in which the demand rate is a function of on-hand inventory. Mandal and Phaujder [6] then extended this model to the case of deteriorating items with a constant production rate. Since then, a lot of research works has been done with the extension of this model [7,8,9,10,11,12].

In a manufacturing system, all the finished products can not be produced exactly as per required specifications. The products, which are within the allowable limits of violation of the specifications, are termed as perfect (non-defective) units and others as defective units. In all big manufacturing firms, these two types of units are produced and sold to a retailer in a lot. After receiving the lot, a retailer continuously separates them into three categories according to their qualities, i.e.-(i) perfect(non-defective) (ii) less defective (iii) more defective. Non defective units are sold with a profit from the primary shop, defective units from the secondary shops at reduced prices, even incurring a loss in such a way that the management makes a profit out of the total sales from these shops. Kar et al[13,14] have considered some such inventory models of differential units in crisp environment.

Now-a-days, a retailer normally creates the market for his goods through advertisements in mass and electronic medias, etc and controlling the units' selling prices. In the literature, there are few research papers (Sung et al[15], Datta et al[16] etc) which considered the effect of price and promotional cost on demand in inventory control system.

In this paper, co-ordinated sale of non-defective, less defective and more defective units purchased in a lot and sold separately from three different shops is considered with promotional cost and price dependent demand under a single management in fuzzy environment. In the primary shop, non-defective units are sold with a profit and the defective units are continuously transferred to the adjacent secondary shops. Demand of perfect units is assumed to increase with the increase of inventory level where as it decreases as price of the units goes up. Here, shortages are not allowed and the replenishment of units is instantaneous. From the secondary shop-1, less defective units are sold at a little less price than the primary shop and demand of these units are price dependent only. Similarly more defective units are sold at a more reduced price from secondary shop-2 having price dependent demand. These reduced prices are fuzzy in nature. Following business norms, there is a target for over-all profit that is also imprecise in nature. This inventory control system provides nine scenarios, and the analysis of one scenario is presented here in details.

In this system, one decision factor has been introduced for a retailer. A retailer can invest for promotion of sale generating additional demand.

Finally, the problem has been formulated as a fuzzy optimization problem associating fuzziness to the mark-up rate of selling prices of the defective units and the targeted profit goal. The fuzziness to the mark-up rate is introduced through triangular fuzzy numbers and the objective goal through linear membership function. The fuzzy inventory model is converted to the corresponding crisp one following Bellman and Zadeh[17] and solved by Genetic Algorithm developed for this purpose. There exist several scenarios for the model that are illustrated with some numerical data. A set of near-optimum solutions is evaluated to offer some alternatives to the decision maker for his/her choice according to the existing situation of his/her business. Also a study for maximizing the profit co-ordinations with the promotional sale and initial demand is presented.

2. Assumptions and notations:

A fuzzy inventory model of differential units for primary, secondary–1, secondary–2 shops under a single management has been developed with the following assumptions and notations.

2.1 For Primary shop:

(i) Rate of replenishment of differential units is infinite.

(ii) t_1 represents the total time period(time cycle) and the lead time is negligible (cf. Fig.1).

(iii) θ_1, θ_2 are the constant rates of defectiveness of less and more defective units out of on–hand inventory at time t, $0 < \theta_1, \theta_2 << 1$.

(iv) The defective units are available only when the units are in stock.

(v) The defective units can not be repaired but these units are sold at a reduced price from the adjacent shops.

(vi) $q(t)$ is the stock level at time t, from which less or more defective units are detected and transferred continuously to the secondary–1 and secondary–2 shops respectively. From primary shop, only perfect units are sold .The demand rate $D(p, q(t), d_1)$ is deterministic function of selling price p, stock level $q(t)$ and d_1 such as $D(p, q(t), d_1) = d_1 + d_{11}p^{-k}q(t)$, $d_1, d_{11}, k > 0$.

(vii) Shortages are not allowed.

(viii) The selling price for the perfect units is p per unit. A lot of differential units are purchased at the rate of c per unit. Therefore, $p = mc$, where $m > 1$.

(ix) The inventory holding cost per unit item per unit time is C_{11}, and the replenishment (ordering) cost is C_{31} per replenishment period.

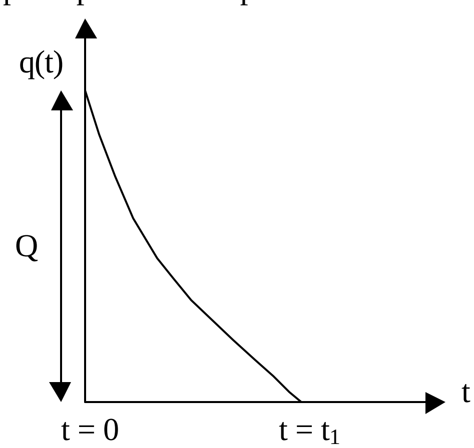


Fig. 1: Graphical representation of primary shop.

2.2 For Secondary shop–1:

(i) In this shop , only less defective units are received continuously from the primary shop at a variable rate $\theta_1\, q(t)$ per unit time and sold.
(ii) No shortages are allowed.
(ii) Lead time is zero.
(iii) The selling price of these defective units is $\tilde{p}_2 = \tilde{m}_2 c$ per unit, where $\tilde{m}_2$ = mark up of selling price.
(iv) The demand rate $\tilde{\lambda}(\tilde{p}_2) = a - b\tilde{p}_2$, a,b > 0 and $\tilde{p}_2 < p$.
(v) C_{12} and C_{32} are the holding cost per unit item per unit time and the set up cost per replenishment period respectively.
(vi) t_2 is the total time period for this shop.
(vii) C_{321} is the set up cost, when $t_2 < t_1$.
(viii) There are three scenarios depending upon the time periods of shops.

Scenario 1. $t_1 = t_2$. i.e., less defective units are exhausted just at the time when the next lot of differential units have arrived (cf. Fig.2).

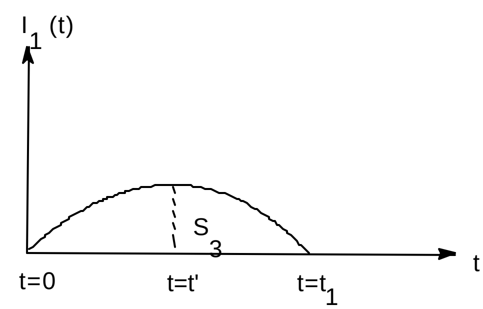


Fig. 2: Graphical representation of secondary-1 shop for secnario -1.

Scenario 2. $t_1 > t_2$. i.e., the stock of less defective units are exhausted before the arrival of new lot of differential items. In this case it is assumed that shop will sell only the less defective units immediately transferred from the primary shop for the remaining period till the next lot of items arrives (cf. Fig.3) incurring loss of customers.

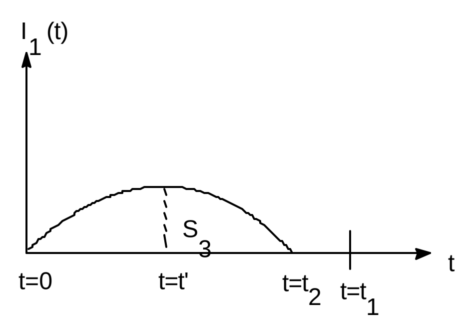


Fig. 3: Graphical representation of secondary-1 shop for secnario -2.

Scenario 3. $t_1 < t_2$. i.e., there will be some less defective units left to be sold at the end of the time period at the primary shop. It is assumed that to run along with the primary shop , all the remaining less defective units are sold at a throw-away price at the end of the time period t_1 and there is an unlimited market of less defective units at much reduced price (cf. Fig.4).

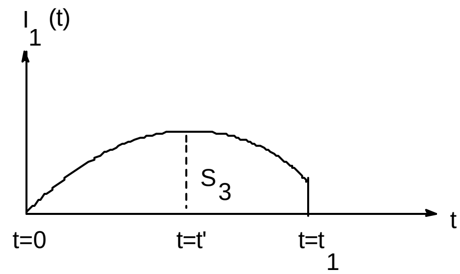


Fig. 4: Graphical representation of secondary-1 shop for secnario-3.

2.3 For Secondary shop–2:

(i) In this shop, only more defective units are received continuously from primary shop at a variable rate $\theta_2 q(t)$ per unit time and sold.

(ii) No shortages are allowed.

(iii) Lead time is zero.

(iv) The selling price of more defective units per unit is $\tilde{p}_3 = \tilde{m}_3 c$, where $\tilde{m}_3$ = mark up of selling price.

(v) The demand rate $\tilde{\lambda}'(\tilde{p}_3) = c - d\tilde{p}_3$, c,d > 0 and $\tilde{p}_3 < p$.

(vi) c_{13} and c_{33} are the holding cost per unit per unit time and the set up cost per replenishment period respectively.

(vii) t_3 is the total time period for this shop.

(viii) C_{331} is the set up cost, when $t_3 < t_1$.

(ix) There are nine scenarios depending upon time periods of shops.

Scenario 1. $t_1 = t_2 = t_3$ i.e., more defective units are exhausted just at the time when the next lot of differential items have arrived (cf. Fig.5).

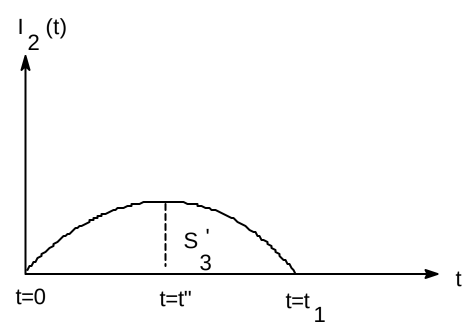


Fig. 5 Graphical representation of secondary-2 shop for secnario-1

Scenario 2. $t_1 = t_2 > t_3$ i.e., all more defective units are sold before the arrival of new lot of differential items. In this case , it is assumed that shop will sell only the more defective units immediately transferred from the primary shop for the remaining period till the next lot of items arrives (cf. Fig.6) incurring loss of customers.

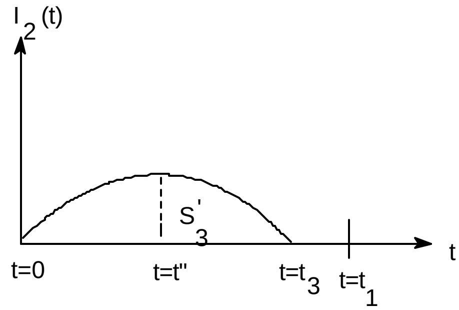


Fig.6: Graphical representation of secondary-2 shop for secnario-2.

Scenario 3. $t_1 = t_2 < t_3$ i.e., there will be some more defective units left to be sold at the end of the time period at the primary shop. It is assumed that to run along with the primary shop, all the remaining more defective units are sold at a throw-away price at the end of the time period t_1 and there is an unlimited market of more defective units at much reduced price (cf. Fig.7).

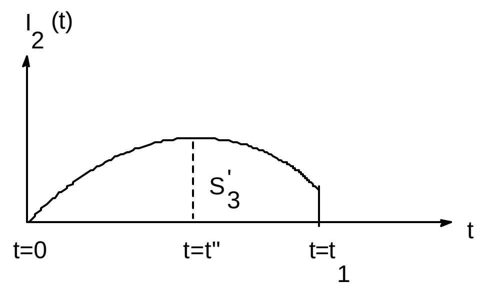


Fig. 7: Graphical representation of secondary-2 shop for secnario-3.

Scenario 4. $t_1 < t_2 = t_3.$ This is same as Scenario 3.
Scenario 5. $t_1 < t_2 < t_3.$ This is same as Scenario 3.
Scenario 6. $t_1 < t_2$ & $t_2 > t_3.$ There are two cases between t_1 & t_3
Case 1 : $t_1 < t_3.$ Case 2 : $t_1 > t_3.$
For case 1, relation among t_1, t_2, t_3 are $t_1 < t_3 < t_2.$ So this is same as scenario 3.
For case 2, relation among t_1, t_2, t_3 are $t_3 < t_1 < t_2.$ So this is same as scenario 2.
Scenario 7. $t_1 > t_2$ & $t_2 = t_3. \Rightarrow t_1 > t_2 = t_3.$ This is same as scenario 2.
Scenario 8. $t_1 > t_2$ & $t_2 < t_3.$ There are two cases between t_1 & t_3
Case 1 : $t_1 < t_3.$ Case 2 : $t_1 > t_3.$
For case 1, relation among t_1, t_2, t_3 are $t_2 < t_1 < t_3.$ So this is same as scenario 3.
For case 2, relation among t_1, t_2, t_3 are $t_2 < t_3 < t_1.$ So this is same as scenario 2.
Scenario 9. $t_1 > t_2$ & $t_2 > t_3. \Rightarrow t_1 > t_2 > t_3.$ This is same as scenario 2.

3. Model description and analysis:

3.1. Primary shop :

In the present model, the on-hand inventory level for differential units is Q at $t = 0$ and up to $t = t_1$, it gradually declines mainly to meet up demand and partly due to defective units which are continuously transferred to other shops for sale. By this process, the stock level reaches at zero at $t = t_1$. The pictorial representation of the system is given in Fig. 1. The differential equation describing the inventory level q(t) in the interval $0 \le t \le t_1$ is given by

$$\frac{dq(t)}{dt} = -\left\{d_1 + d_{11}p^{-k}q(t)\right\} - \theta_1 q(t) - \theta_2 q(t) \,, \qquad 0 \le t \le t_1 \tag{1}$$

subject to the conditions that

$$q(t) = 0 \ \text{ at } \ t = t_1 \tag{2}$$

$$\text{Also}, \quad q(t) = Q \ \text{ at } \ t = 0 \tag{3}$$

The solution of the equation (1) is given by

$$q(t) = \frac{d_1}{\beta}\{ e^{\beta(t_1 - t)} - 1\} \,, \qquad 0 \le t \le t_1 \tag{4}$$

where $\beta = d_{11}p^{-k} + \theta$ and $\theta = \theta_1 + \theta_2$

From (2) and (3) , we have

$$Q = \frac{d_1}{\beta}\left\{e^{\beta t_1} - 1\right\} \tag{5}$$

$$\text{and} \quad t_1 = \frac{1}{\beta}\ln\left|\frac{Q\beta}{d_1} + 1\right| \tag{6}$$

The total number of defective items during the entire cycle is

$$S_d = \theta\int_0^{t_1} q(t)dt$$

$$= \frac{\theta d_1}{\beta}\left\{\frac{e^{\beta t_1} - 1}{\beta} - t_1\right\} \tag{7}$$

The holding cost of the inventory for the entire cycle is

$$C_{HOL} = C_{11}\int_0^{t_1} q(t)dt$$

$$= C_{11}\left\{\frac{-d_1 t_1}{\beta} - \frac{d_1}{\beta^2}(1 - e^{\beta t_1})\right\} \tag{8}$$

Here, we introduce the distributor's promotion policy to increase the demand. Promotional efforts can be considered as investment for promotion to bring some change in demand. Promotional cost can be considered as an increase of d_1 over the constant demand, d_0 as $R_p(d_1)=s\left\{e^{(d_1 - d_0)} - 1\right\}$, where $d_0(<d_1)$ and s are given constants.

Hence the profit from the primary shop during the interval $0 \le t \le t_1$ is given by

$$Z_1(d_1, t_1) = (p-c)\frac{d_1}{\beta}\left\{e^{\beta t_1} - 1\right\} - C_{HOL} - c_{31} - pS_d - s\left\{e^{(d_1 - d_0)} - 1\right\} \tag{9}$$

where $d_1 > d_0$.

Now, substituting the values of C_{HOL}, S_d in (9) from (7) – (8), the following expression for Z_1 is obtained as :

$$Z_1 = (p-c)\frac{d_1}{\beta}\left\{e^{\beta t_1} - 1\right\} - C_{31} + (p\theta + C_{11})\left\{\frac{d_1}{\beta^2}(1 - e^{\beta t_1}) + \frac{d_1 t_1}{\beta}\right\} - s\left\{e^{d_1 - d_0} - 1\right\} \tag{10}$$

3.2. Secondary shop-1:

According to our assumptions, the less defective units are sold in secondary shop–1. In this shop the amount of stock is zero initially. Due to the defectiveness of the units at the primary shop, just after t = 0, the inventory level is raised at a rate, $\theta_1 q(t) - \lambda$ up to $t = t'$ (till $\theta_1 q(t) > \lambda$) and the stock attains a level S_3 at $t = t'$. After $t = t'$, the demand is greater than the rate of less defective units received from primary shop and is met up partly from the current supplied less defective units, and partly from the accumulated stock.

In this situation, the following three separate scenarios may arise.

(i) Scenario-1 for $t_1 = t_2$ (ii) Scenario-2 for $t_1 > t_2$ (iii) Scenario-3 for $t_1 < t_2$.

The pictorial representations of the system for three different scenarios are given in Fig.2, Fig.3 and Fig.4 respectively. Now we discuss only one scenario of this secondary-1 shop in details. Other scenarios can be analyzed similarly.

Scenario 1:

In this case, the cycle lengths of both primary and secondary-1 shops are same. The differential equations describing the instantaneous states of inventory, $I_1(t)$ in the interval $0 \le t \le t_2$ ($=t_1$) are given by

$$\frac{dI_1(t)}{dt} = \begin{cases} \theta_1 q(t) - \lambda, & 0 \le t \le t' \\ \theta_1 q(t) - \lambda, & t' \le t \le t_1 \end{cases} \tag{11}$$

with the boundary conditions $I_1(t) = 0$ at $t = 0$, t_1. Also, $I_1(t)$ is continuous at and $I_1(t) = S_3$ at $t = t'$.

The solutions of the differential equations (11) are

$$I_1(t) = \begin{cases} \theta_1\left\{-\frac{d_1 t}{\beta} - \frac{d_1 e^{\beta(t_1 - t)}}{\beta^2} + \frac{d_1 e^{\beta t_1}}{\beta^2}\right\} - \lambda t, & 0 \le t \le t' \\ \frac{\theta_1 d_1 (t_1 - t)}{\beta} - \frac{\theta_1 d_1}{\beta^2}\left\{e^{\beta(t_1 - t)} - 1\right\} + \lambda(t_1 - t), & t' \le t \le t_1 \end{cases} \tag{12}$$

The holding cost of the inventory in the secondary shop-1 for the entire cycle is,

$$C'_{HOL} = C_{12}\left[\theta_1\left\{\frac{-d_1t'^2}{2\beta}+\frac{d_1}{\beta^3}(e^{\beta(t_1-t')}-e^{\beta t_1})+\frac{d_1}{\beta^2}t'e^{\beta t_1}\right\}- \right.$$
$$\left. \frac{\lambda}{2}t'^2+\frac{\theta_1 d_1}{2\beta}(t_1-t')^2+\frac{\theta_1 d_1}{\beta^2}\left\{\frac{1-e^{\beta(t_1-t')}}{\beta}+(t_1-t')\right\}+\frac{\lambda}{2}(t_1-t')^2\right] \quad (13)$$

Now $I_1(t)=S_3$ at $t=t'$ gives,

$$S_3=\frac{\theta_1 d_1}{\beta}(t_1-t')-\frac{\theta_1 d_1}{\beta^2}\left\{e^{\beta(t_1-t')}-1\right\}+\lambda(t_1-t') \quad (14)$$

and $$t'=t_1-\frac{1}{\beta}\ln\left|1+\frac{\lambda\beta}{\theta_1 d_1}\right| \quad (15)$$

In this case the return from the secondary-1 shop is given by

$$Z_2=S'_d p_2-C'_{HOL}-C_{32} \quad (16)$$

where S'_d is given by

$$S'_d=\theta_1\int_0^{t_1}q(t)dt = \frac{\theta_1 d_1}{\beta}\left\{\frac{e^{\beta t_1}-1}{\beta}-t_1\right\}.$$

3.3 Secondary shop-2:

According to our assumptions, the 'more defective units' are sold in secondary shop–2 .In this shop, the amount of stock is zero initially. Like secondary shop-1, just after t = 0, the inventory level is raised at a rate, $\theta_2 q(t)-\lambda'$ up to $t=t''$ (till $\theta_2 q(t)>\lambda'$) and the stock attains a level S'_3 at $t=t''$. After $t=t''$, the demand is greater than the rate of more defective items and is met up partly from the current more defective items transferred from primary shop and partly from stock. In this situation, the following nine scenarios may arise. The pictorial representations of the system for three different scenarios are given in Fig.5, Fig.6, Fig.7 respectively. Now we discuss only one scenario of this secondary-2 shop in details and other scenarios can be dealt with similarly.

Scenario 1:

In this case, the cycle lengths of primary, secondary-1 and secondary-2 shops are same. The differential equations describing the instantaneous states of inventory, $I_2(t)$ in the interval $0\le t\le t_3$ $(=t_1=t_2)$ are given by

$$\frac{dI_2(t)}{dt}=\begin{cases}\theta_2 q(t)-\lambda', & 0\le t\le t'' \\ \theta_2 q(t)-\lambda', & t''\le t\le t_1\end{cases} \quad (17)$$

with the boundary conditions $I_2(t)$ = 0 at t = 0 , t_1. Also, $I_2(t)$ is continuous at $t=t''$ and $I_2(t)=S'_3$ at $t=t''$. The solutions of the differential equations (11) are

$$I_2(t)=\begin{cases}\theta_2\left\{-\dfrac{d_1 t}{\beta}-\dfrac{d_1 e^{\beta(t_1-t)}}{\beta^2}+\dfrac{d_1 e^{\beta t_1}}{\beta^2}\right\}-\lambda' t, & 0\le t\le t''\\[2ex] \dfrac{\theta_2 d_1(t_1-t)}{\beta}-\dfrac{\theta_2 d_1}{\beta^2}\left\{e^{\beta(t_1-t)}-1\right\}+\lambda'(t_1-t), & t''\le t\le t_1\end{cases} \tag{18}$$

The holding cost of the inventory in the secondary shop-2 for the entire cycle is ,

$$C''_{HOL}=C_{13}\left[\theta_2\left\{\frac{-d_1 t''^2}{2\beta}+\frac{d_1}{\beta^3}(e^{\beta(t_1-t'')}-e^{\beta t_1})+\frac{d_1}{\beta^2}t'' e^{\beta t_1}\right\}-\right.$$
$$\left.\frac{\lambda}{2}t''^2+\frac{\theta_2 d_1}{2\beta}(t_1-t'')^2+\frac{\theta_2 d_1}{\beta^2}\left\{\frac{1-e^{\beta(t_1-t'')}}{\beta}+(t_1-t'')\right\}+\frac{\lambda}{2}(t_1-t'')^2\right] \tag{19}$$

Now $I_2(t)=S'_3$ at $t=t''$ gives,

$$S'_3=\frac{\theta_2 d_1}{\beta}(t_1-t'')-\frac{\theta_2 d_1}{\beta^2}\left\{e^{\beta(t_1-t'')}-1\right\}+\lambda'(t_1-t'') \tag{20}$$

and $\quad t''=t_1-\dfrac{1}{\beta}\ln\left|1+\dfrac{\lambda'\beta}{\theta_2 d_1}\right|$ (21)

In this case the return from the secondary-2 shop is given by

$$Z_3=S''_d p_3-C''_{HOL}-C_{33} \tag{22}$$

where S''_d is given by

$$S''_d=\theta_2\int_0^{t_1}q(t)dt=\frac{\theta_2 d_1}{\beta}\left\{\frac{e^{\beta t_1}-1}{\beta}-t_1\right\}.$$

Fuzzy Model (objective goal and some parameters are fuzzy):

When objective (i.e, average profit) and mark-ups of the selling prices for defective units are fuzzy, the problem reduces to, Max $\tilde{Z}$ with appropriate expressions of $\tilde{Z}$ for different scenarios in which m_2, m_3 are fuzzy numbers. i,e, they are represented by $\tilde{m}_2$ and $\tilde{m}_3$.Under the above assumptions, the mathematical formulation of the fuzzy model is given by

$$\tilde{Z}(d_1,t_1,\tilde{m}_2,\tilde{m}_3)=Z_1(d_1,t_1)+\tilde{Z}_2(d_1,t_1,\tilde{m}_2,\tilde{m}_3)+\tilde{Z}_3(d_1,t_1,\tilde{m}_2,\tilde{m}_3) \tag{23}$$

where, $\quad Z_1=(p-c)\dfrac{d_1}{\beta}\left\{e^{\beta t_1}-1\right\}-C_{31}+(p\theta+C_{11})\left\{\dfrac{d_1}{\beta^2}(1-e^{\beta t_1})+\dfrac{d_1 t_1}{\beta}\right\}-s\left\{e^{d_1-d_0}-1\right\}$

$$Z_2=\tilde{p}_2\frac{\theta_1 d_1}{\beta}\left\{\frac{e^{\beta t_1}-1}{\beta}-t_1\right\}-C_{12}\left[\theta_1\left\{\frac{-d_1 t'^2}{2\beta}+\frac{d_1}{\beta^3}(e^{\beta(t_1-t')}-e^{\beta t_1})+\frac{d_1}{\beta^2}t' e^{\beta t_1}\right\}\right.$$
$$\left.-\frac{\lambda}{2}t'^2+\frac{\theta_1 d_1}{2\beta}(t_1-t')^2+\frac{\theta_1 d_1}{\beta^2}\left\{\frac{1-e^{\beta(t_1-t')}}{\beta}+(t_1-t')\right\}+\frac{\lambda}{2}(t_1-t')^2\right]-C_{32}.$$

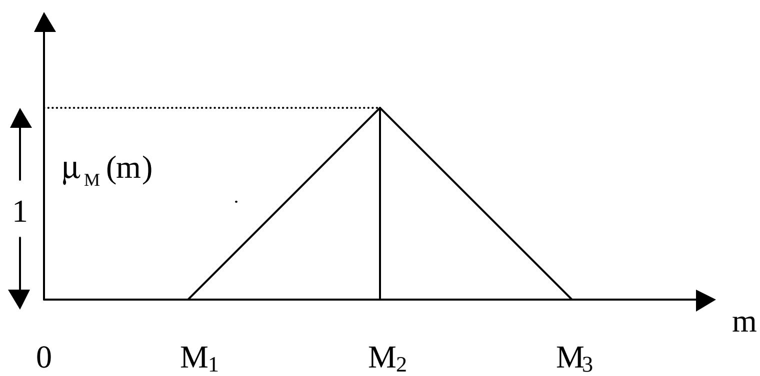
$$Z_3 = \tilde{p}_3 \frac{\theta_2 d_1}{\beta}\left\{\frac{e^{\beta t_1}-1}{\beta}-t_1\right\} - C_{13}\left[\theta_2\left\{\frac{-d_1 t''^2}{2\beta}+\frac{d_1}{\beta^3}(e^{\beta(t_1-t'')}-e^{\beta t_1})+\frac{d_1}{\beta^2}t'' e^{\beta t_1}\right\}\right.$$
$$\left.-\frac{\lambda}{2}t''^2+\frac{\theta_2 d_1}{2\beta}(t_1-t'')^2+\frac{\theta_2 d_1}{\beta^2}\left\{\frac{1-e^{\beta(t_1-t'')}}{\beta}+(t_1-t'')\right\}+\frac{\lambda}{2}(t_1-t'')^2\right] - C_{33}.$$

Fuzzy membership functions:

According to the fuzzy set theory, fuzzy objective may be represented by a linear and/or non-linear membership function and fuzzy parameters are taken as fuzzy numbers, namely trapezoidal fuzzy number, triangular fuzzy numbers, L-R fuzzy numbers, etc. Here, for the parameter m , these may be assumed as triangular fuzzy numbers $\tilde{M}$, which can be specified by the triplet (M_1, M_2, M_3) with membership function :

$$\mu_M(m) = \begin{cases} 0 & \text{if } m < M_1 \\ \dfrac{m - M_1}{M_2 - M_1} & \text{if } M_1 \le m \le M_2 \\ \dfrac{M_3 - m}{M_3 - M_2} & \text{if } M_2 \le m \le M_3 \\ 0 & \text{if } m > M_3 \end{cases}$$

So, β-cut of $\tilde{M}$ can be expressed by the following interval($\tilde{M}$)=(M^L_β, M^R_β)=[$M_1 + (M_2 - M_1)\beta$, $M_3 - (M_3 - M_2)\beta$] and $M=e_{11}M^L_\beta + (1-e_{11})\, M^R_\beta$ where $0<e_{11}<1$.

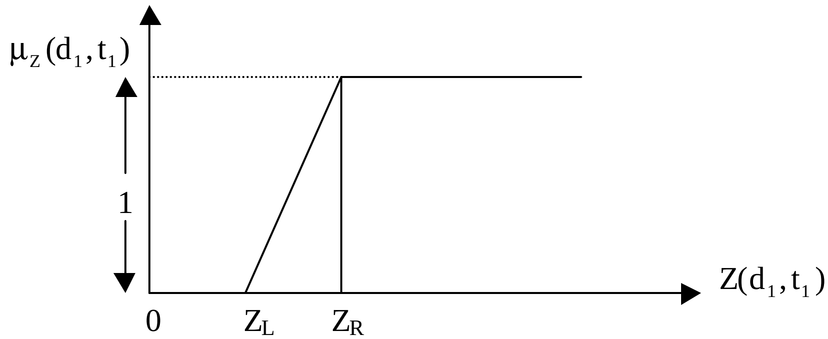


Pictorial representation of triangular fuzzy number

The fuzzy objective is defined by its membership function, which may be linear and / or non-linear. Here,

$$\mu_Z(d_1,t_1) = \begin{cases} 0 & \text{if} \quad Z(d_1,t_1) < Z_L \\ 1 + \dfrac{Z(d_1,t_1) - Z_R}{Z_R - Z_L} & \text{if} \quad Z_L \le Z(d_1,t_1) \le Z_R \\ 1 & \text{if} \quad Z(d_1,t_1) > Z_R \end{cases}$$

where Z_R the upper profit level and $(Z_R - Z_L)$ is the tolerance limit.



Membership function of fuzzy objective

Now using Bellman and Zadeh[15], the fuzzy problem(23) is converted to a crisp one as:

$$\text{Max } \lambda$$

subject to , $\lambda \le \alpha$, $\lambda \le \beta_1$, $\lambda \le \beta_2$

$$Z > Z_R-(1-\alpha)(Z_R-Z_L)$$

$$m_2 = e_{11}m_2^L + (1-e_{11})m_2^R$$

$$m_3 = e_{22}m_3^L + (1-e_{22})m_3^R$$

where $\left(m_{2,3}^L, m_{2,3}^R\right) = \left\{a_{2,3} + \left(b_{2,3} - a_{2,3}\right)\beta_{1,2}\,,\ c_{2,3} - \left(c_{2,3} - b_{2,3}\right)\beta_{1,2}\right\}$ and $\left(a_{2,3}\,, b_{2,3}\,, c_{2,3}\right)$ are being the fuzzy triangular numbers.

where e_{11}, e_{22} ($0 < e_{11}$, $e_{22} < 1$) are unknowns. α, β_1, and β_2 are the aspiration levels of profit goal and fuzzy numbers m_2, m_3 respectively. Here α, β_1, β_2 and e_{11}, e_{22} are decision variables and determined for optimum average profit.

4. Genetic Algorithm:

Genetic Algorithm is a class of adaptive search technique based on the principle of population genetics. The Algorithm is an example of a search procedure that uses random choice as a tool to guide a highly exploitative search through a coding of parameter space. Genetic Algorithm work according to the principles of natural genetics on a population of string structures representing the problem variables. All these features make Genetic Algorithm search robust, allowing them to be applied to a wide variety of problems.

Implementing GA:

The followings are adopted in the proposed GA to solve the problem:

(1) Parameters
(2) Chromosome representation
(3) Initial population production
(4) Evaluation
(5) Selection
(6) Crossover
(7) Mutation
(8) Termination

Parameters

Firstly, we set the different parameters on which this GA depends. All these are the number of generation (MAXGEN), population size (POPSIZE), probability of crossover (PCROS), probability of mutation (PMUTE).

Chromosome representation

An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chromosome are not effective in many non-linear problems. Since the proposed problem is highly non-linear, hence to overcome the difficulty, a real-number representation is used. In this representation, each chromosome V_i is a string of n number of genes G_{ij}, (j=1,2,………,n) where these n number of genes respectively denote n number of decision variables(X_i,i=1,2,…….,n).

Initial population production

The population generation technique proposed in the present GA is illustrated by the following procedure: For each chromosome V_i , every gene G_{ij} is randomly generated between its boundary (LB_j,UB_j) where LB_j and UB_j are the lower and upper bounds of the variables X_j, j=1,2,…………,n and i=1,2,………….,POPSIZE.

Evaluation

Evaluation function plays the same role in GA as that which the environment plays in natural evolution. Now, evolution function(EVAL) for the chromosome V_i is equivalent to the objective function PF(X). These are the following steps of evaluation.

step 1: find EVAL(V_i) by $EVAL(V_i)=f(X_1,X_2,……….,X_n)$
Where the genes G_{ij} represent the decision variable X_j , j=1,2,………..,POPSIZE and f is the objective function.

step 2: find total fitness of the population : $F=\sum_{i=1}^{POPSIZE} EVAL(V_i)$

step 3: calculate the probability p_i of selection for each chromosome V_i as: $Y_i=\sum_{j=1}^{i} p_j$

Selection

The selection scheme in GA determines which solutions in the current population are to be selected for recombination. Many selection schemes, such as Stochastic random sampling, Roulette wheel selection have been proposed for various problems. In this paper, we adopt the roulette wheel selection process.
This roulette wheel selection process is based on spinning the roulette wheel POPSIZE times, each time we select a single chromosome for the new population in the following way :

(a) generate a random(float) number r between 0 to 1.
(b) If $r < Y_1$ then the first chromosome is V_1 otherwise select the i-th chromosome V_i ($2 \le i \le POPSIZE$) such that $Y_{i-1} \le r \le Y_i$.

Crossover

Crossover operator is mainly responsible for the search of new strings. The exploration and exploitation of the solution space is made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generates offspring solutions by recombining both parent solution features. After selection chromosomes for new population, the crossover operator is applied. Here, the whole arithmetic crossover operation is used. It is defined as a linear combination of two consecutive selected chromosomes V_m and V_n and resulting offspring's V'_m and V'_n are calculated as:

$$V'_m = c.V_m + (1-c).V_n$$
$$V'_n = c.V_n + (1-c).V_m$$

where c is a random number between 0 and 1.

Mutation

Mutation operator is used to prevent the search process from converging to local optima rapidly. It is applied to a single chromosome V_i. The selection of a chromosome for mutation is performed in the following way:

step 1. Set $i \leftarrow 1$
step 2. Generate a random number u from the range [0,1].
step 3. If u < PMUTE, then we select the chromosome V_i .
step 4. Set $i \leftarrow i+1$
step 5. If $i \le$ POPSIZE, then go to step 2.

Then the particular gene G_{ij} of the chromosome V_i selected by the above-mentioned steps is randomly selected. In this problem, the mutation is defined as

G_{ij}^{mut} = random number from the range (0,1).

Termination

If the number of iteration is less than or equal to MAXGEN then the process is going on, otherwise it terminates.

The GAs procedure is given below:

```
begin
   do{
        t ← 0
        while(all constraints are not satisfied)
        {
        initialize Population(t)
        }
        evaluate Population(t)
```

```
while(not terminate-condition)
{
t ← t +1
select Population(t) from Population(t-1)
crossover and mutate Population(t)
evaluate Population(t)
}
Print Optimum Result
}
end.
```

Solution Procedure:

In our experiment, GA consists of the parameters, POPSIZE=50, PCROS=0.2, PMUTE=.2 and MAXGEN=50. A real-number presentation is used here. In this representation, each chromosome X is a string of n number of genes which respectively denote (here, n = 7) the decision variables $d_1, t_1, \alpha, \beta_1, \beta_2, e_{11}$ and e_{22}. To initialize the population, we first identify the independent and dependent variables and then their boundaries. Here, α, β_1, β_2, e_{11}, e_{22} and d_1 are independent variables and t_1 is the dependent variable. All independent variable except d_1 lie on the interval (0,1) whereas limits for d_1 is (d_1^L, d_1^R). For each chromosome X, every gene, which represents the independent variables, are randomly generated between their boundaries until it is feasible.
In this problem, arithmetic crossover and random mutation are applied to generate new offspring's.

5. Numerical Example

To illustrate the inventory models, we consider the following numerical data.
Let c_{31}=\$100, c_{11}=\$1.2, p=\$8, c=\$5, θ_1=0.1, θ_2=0.05, c_{12}=\$0.8, c_{13}=\$0.3, c_{32}=\$30, c_{33}=\$15, d_0=100, d_{11}=5, s=0.1, k=2, p_{21}=\$2.4, p_{31}=\$1.6, c_{321}=\$26, c_{331}=\$12, $\tilde{m}_2$ = (.6, .8, .9), $\tilde{m}_3$=(.5, .6, .7).
The optimal values of Q, d_1, t_1 along with the maximum average profit and other variables have been calculated for different scenarios and results are displayed in Table 1.

Table-1
Optimum results for different cases

Case	**(1,1,1)**	**(1,1,2)**	**(1,1,3)**	**(1,2,1)**	**(1,2,2)**	**(1,2,3)**	**(1,3,1)**	**(1,3,2)**	**(1,3,3)**
Q	256.21	257.17	276.75	222.82	225.24	222.86	255.96	258.76	245.39
d_1	103.46	103.34	104.39	103.45	103.99	103.44	104.62	104.62	103.62
Z	178.68	180.04	177.57	181.71	184.40	180.52	178.58	192.34	178.18
M_2	0.8283	0.82257	0.8414	0.778	0.7825	0.785	0.8176	0.7692	0.87823
M_3	0.628	0.6156	0.6270	0.6229	0.6092	0.6257	0.6026	0.6005	0.63137
t_1	1.9632	1.9719	2.0733	1.7521	1.76	1.7524	1.944	1.9611	1.90885
t_2	—	—	—	1.7296	1.75	1.7385	—	—	—
t_3	—	1.90	—	—	1.75	—	—	1.889	—

where (1,i,j) represents the combination of primary shop(represented by 1), i-th scenario for secondary-1 shop and j-th scenario for secondary-2 shop (i,j = 1,2,3).

Table-2
A set of near-optimum solution for (1, 1, 1) model

No. of solutions	1	2	3	4	5	6	7	8	9	10
Q	255.96	255.94	255.96	255.97	255.95	255.97	255.96	255.969	255.94	255.97
D_1	103.25	103.43	103.12	103.46	103.672	103.47	104.22	103.05	103.48	103.279
Z	178.59	178.4	178.18	178.16	178.06	177.97	177.15	176.78	176.684	176.322
M_2	0.84	0.8181	0.823	0.8175	0.8244	0.8423	0.8219	0.8198	0.8116	0.8218
M_3	0.6158	0.6383	0.631	0.636	0.6264	0.6002	0.6329	0.6184	0.6254	0.6076
T_1	1.9648	1.96	1.967	1.961	1.958	1.9615	1.95	1.967	1.962	1.96

6. Sensitivity analysis:

Using the numerical example mentioned earlier, a sensitivity analysis is performed to study the effect of change of promotional cost and average optimal profit for (1,1,1) model.

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Table-3
Coordinated study for initial demand, promotional cost and optimal profit

Parameter d_0	% change in d_0's value	average optimal profit	% change in average optimal profit	promotional cost	% change in promotional cost
96	-4	169.9	-4.913	3.126	+1.5
97	-3	172.11	-3.676	3.115	+1.136
98	-2	174.31	-2.446	3.1	+0.65
99	-1	176.5073	-1.216	3.09	+0.325
100	0	178.68	0.000	3.08	0.000
101	+1	180.85	+1.214	3.07	-0.325
102	+2	183.02	+2.429	3.06	-0.65
103	+3	185.17	+3.632	3.05	-0.974
104	+4	187.31	+4.829	3.04	-1.3
105	+5	189.45	+6.027	3.032	-1.56

From the above study, it is revealed that with the increase of initial demand in the market the optimum promotional cost to augment demand (unknown parameter) gradually decreases whereas average optimal profit increases. The percentage change in the optimal profit with respect to the value of initial demand, d_0 = 100 is –ve initially and +ve later. In the case of promotional cost, it is reversed.

7. Conclusion:

The present paper proposes a solution procedure for inventory model of differential units sold from three different shops owned by a single management. Nine different scenarios are considered depending upon the times of exhaustness of units at the shops and for each case, optimum order quantities are evaluated to maximize the corresponding average profit.

Since the proposed model has been formulated with vague parameters and imprecise informations, the decision maker may choose that solution which suits him/her best respect to resources, which will have to be augmented with difficulties. In this region, GA is most suitable method for the decision-maker.

This methodology can be extended to perishable units that deteriorate with time. It will more realistic if the problem is considered for a fixed time horizon including shortages. Though the problem has been presented in fuzzy environment, it will be also formulated in fuzzy-stochastic environment.

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