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Multi-item Stochastic and Fuzzy-Stochastic Inventory Models Under Imprecise Goal and Chance Constraints

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Abstract: Multi-item single period stochastic and fuzzy-stochastic inventory models are formulated under investment and floor-space constraints. Here, for the models, inventory costs and one decision parameter involved in the objective function and goal on one of the constraints are assumed to be random variables. In fuzzy-stochastic model, in addition to the above assumptions, goal on other constraint alongwith the objective goal is imprecise in nature. In both inventory models, demand is a function of cost price which alongwith investment goal are random variables. In one model, impreciseness is also introduced in the available storage area and permissible total average cost. Thus, assuming random variables to be independent and to follow normal distribution, the models have been formulated as stochastic non-linear and fuzzy-probabilistic programming problems. The fuzzy-probabilistic programming problem is first reduced to a corresponding equivalent fuzzy non-linear programming problem. Both the problems are solved by fuzzy non-linear programming technique. The models are illustrated numerically and the results of different models are compared.

AMS Mathematics Subject Classification: 90B05

Key words: Inventory model, price dependent demand, random cost parameters, chance constraint, fuzzy goal and constraint.

INTRODUCTION

In most of the existing inventory models, it is assumed that the inventory parameters, objective goals and constraint goals are deterministic and fixed. But, if we think of their practical meaning, they are uncertain, either random or imprecise. When some or all parameters of an optimization problem are described by random variables, the problem is called stochastic or probabilistic programming problem.

In a stochastic programming problem, the uncertainties in the parameters are represented by probability distributions. This distribution is estimated on the basis of the available observed random data. Here, the parameters are treated as random variables. For solution, the stochastic problem is first reduced to a crisp one and then solved by an optimization method.

As classified by Mohon (2000), " There are two main approaches for solving single-objective stochastic programming problem : the 'wait and see' (distribution problem) and 'here and now' approaches. The second approach is very efficient in solving real life application problems. The methods based on this approach may be conveniently classified by distinguishing the treatment of the stochastic constraints and that of the stochastic objective (Roubers and Teghem (1988)). For treatment of the stochastic constraints, there are two approaches : (i) the chance-constrained programming approach in which a minimum probability level for satisfying each of the constraints is specified and (ii) the stochastic programming with recourse which consists in penalizing the violation of the constraints. For treatment of the stochastic objective, there are several approaches such as (i) E-model (which optimizes the expected value of the stochastic objective), (ii) V-model (in which the deviation of the stochastic objective is to be minimized), (iii) P-model (which maximizes the probability that the value of the stochastic objective is better than a certain aspiration level specified by the DM) and (iv) E-V-model (which optimizes both expected value and the deviation of the stochastic objective) etc. (cf. Stancu-Minsian (1984)).

In 1965, the first publication in fuzzy set theory by Zadeh (1965) showed the intention to accommodate uncertainty in the non-stochastic sense rather than the presence of random variables. Bellman and Zadeh (1970) first introduced fuzzy set theory in decision-making processes. Later, Tanaka, et-al. (1974) considered the objectives as fuzzy goals over the α-cuts of a fuzzy constraint set and Zimmermann (1976) showed that the classical algorithms could be used to solve a fuzzy linear programming problem and fuzzy additive goal programming technique.

Fuzzy mathematical programming has been applied to several fields like project network, reliability optimization, transportation, media selection for advertising, air pollution regulation etc. problems formulated in a fuzzy environments have been solved by fuzzy programming method. Detail literature on fuzzy linear and non-linear programming with application is available in two well-known books of Lie and Hwang (1992, 1994). In inventory problem, fuzzy set theory has not been much used. Park (1987) examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with cost data. Roy and Maiti (1995, 1998) solved the classical EOQ models in fuzzy environment with fuzzy objective goal and constraint by fuzzy non-linear programming and fuzzy additive goal programming techniques.

In this paper, EOQ models are developed in random and fuzzy-random environments considering demand to be dependent on unit cost which is a decision variable. Here, for both models, unit-purchasing cost, inventory costs and the investment limit are random. In addition to these, total average cost goal and constraint goal for storage area is fuzzy in nature for the probabilistic model in fuzzy environment. The fuzzy parameters have been represented by linear membership functions. The random variables have been assume to be independent and normally distributed. The stochastic inventory model has been formulated as a stochastic non-linear programming problem and then reduced to the equivalent crisp E-model, V-model and combined E-V models using chance constraint programming (CCP) technique. Similarly, following CCP, the fuzzy-stochastic inventory problem is first converted to an equivalent fuzzy problem and then to equivalent crisp problem using membership functions. Fuzzy non-linear programming (FNLP) technique solves all these crisp problems. The models are illustrated with some numerical values for inventory parameters and the results of different models are compared.

2. Model and Assumptions

We use the following notations in proposed model:

n = number of items,
W = Floor or shelf-space available,
B = total investment cost for replenishment.

For i-th item (i = 1, 2, ……, n.),

$D_i = D_i(p_i)$ demand rate (function of cost price),
Q_i = lot size (a decision variable),
S_i = set-up cost per cycle,
H_i = inventory holding cost per unit item,
p_i = price per unit item (a decision variable),
TC(p, Q) = average annual total cost.
(where p, Q are the vectors of n decision variables p_i (i = 1, 2 …, n) and Q_i (i = 1, 2, ….n) respectively.)

The basic assumptions about the model are:

(i) replenishment is instantaneous,
(ii) no back-order is allowed,
(iii) lead time is zero,
(iv) demand $D_i(p_i)$ is related to the unit price as:
$D_i = A_i p_i^{-\beta_i}$ where $A_i(>0)$ and β_i ($0 < \beta_i < 1$) are constant and real numbers selected to provide the best fit of the estimated price function. While $A_i > 0$ is an obvious condition since both D_i and p_i must be non-negative. The reason for $0 < \beta_i < 1$ is given in appendix.

Our objective is to minimize the average total annual cost (i.e., Sum of the purchasing, set up and inventory holding costs) subject to limitations on capital investment and storage area. That is

$$\text{MinTC}(p, Q) = \sum_{i=1}^{n}\left[\frac{A_i}{p_i^{\,b_i}}\left(p_i + \frac{S_i}{Q_i}\right) + \frac{H_i Q_i}{2}\right] \tag{1}$$

Subject to

$$\sum_{i=1}^{n} w_i Q_i \le W,$$

$$\sum_{i=1}^{n} p_i Q_i \le B,$$

$p_i, Q_i > 0$; (i = 1, 2, …..n).

2. 1 Probabilistic model:

When p_i's are probabilistic decision variables, set up cost, investment cost, holding cost are random parameters, the said crisp model (1) is transformed to a probabilistic model as

$$\text{MinTC}(\hat{p}, Q) = \sum_{i=1}^{n}\left[\frac{A_i}{\hat{p}_i^{\,b_i}}\left(\hat{p}_i + \frac{\hat{S}_i}{Q_i}\right) + \frac{\hat{H}_i Q_i}{2}\right] \tag{2}$$

Subject to

$$\sum_{i=1}^{n} w_i Q_i \le W$$

$$\sum_{i=1}^{n} \hat{p}_i Q_i \le \hat{B},$$

$\hat{p}_i$, $Q_i > 0$; (i = 1, 2, …..n).

(here cap '$\wedge$' denotes the randomization of the parameters.)

2. 2 Probabilistic model in fuzzy environment

When p_i's are probabilistic decision variables, investment cost, holding costs are random parameters, but cost goal and constraint goals on storage area become fuzzy, the said crisp model (1) is transformed to a probabilistic model in fuzzy environment as

$$M\tilde{i}n\,\text{TC}(\hat{p}, Q) = \sum_{i=1}^{n}\left[\frac{A_i}{\hat{p}_i^{\,b_i}}\left(\hat{p}_i + \frac{\hat{S}_i}{Q_i}\right) + \frac{\hat{H}_i Q_i}{2}\right] \tag{3}$$

subject to

$$\sum_{i=1}^{n} w_i Q_i \le \tilde{W},$$

$$\sum_{i=1}^{n} \hat{p}_i Q_i \le \hat{B},$$

$\hat{p}_i$, $Q_i > 0$ (i = 1, 2, …..n.)

(here weavy bar '~' denotes the fuzzification of the parameters.)

3. Mathematical analysis

Stochastic non-linear programming (SNLP)

We consider a stochastic non-linear programming problem with resources and objectives as:

Min $g_0(X)$

subject to

$$g'_j(X) \le b_j, \quad (j = 1, 2, \ldots\ldots, m.)$$

and $X \ge 0$.

$$\text{i.e.}\quad \text{Min } g_0(X) \tag{4}$$

subject to

$$g_j(X) \le 0, \quad j = 1, 2, \ldots\ldots, m.$$

and $X \ge 0$,

where $g_j(X) = g'_j(X) - b_j$ and $X \ge 0$.

Here X is the vector of N random variables $y_1, y_2, \ldots., y_N$ and it includes the decision variables $x_1, x_2, \ldots.., x_n$. The problem stated as equation (4) can be converted into an equivalent deterministic non-linear programming problem by applying the chance constraint programming technique as follows:

3. i) Objective function

The objective function $g_0(X)$ can be expanded about the mean values of random variables y_i, $\bar{y}_i$ as

$$g_0(X) = g_0(\bar{X}) + \sum_{i=1}^{N} \left(\frac{\partial g_0}{\partial y_i} \bigg|_{\bar{X}} \right) (y_i - \bar{y}_i) + \text{higher order derivative terms} \tag{5}$$

If the standard deviations of random variate y_i, σ_{y_i} are small, $g_0(X)$ can be approximated by the first two terms of equation (5)

$$g_0(X) = g_0(\bar{X}) - \sum_{i=1}^{N} \left(\frac{\partial g_0}{\partial y_i} \bigg|_{\bar{X}} \right) \bar{y}_i + \sum_{i=1}^{N} \left(\frac{\partial g_0}{\partial y_i} \bigg|_{\bar{X}} \right) y_i = \psi(X) \text{ (say)} \tag{6}$$

If y_i ($i = 1, 2, \ldots..N.$) follow normal distribution, then $\psi(X)$, which is linear function of X, also follows the normal distribution. The mean and variance of $\psi(X)$ are given by

$$\bar{\psi} = \psi(\bar{X}), \tag{7}$$

$$\sigma_\psi^2 = \text{Var}(\psi) = \sum_{i=1}^{N} \left(\frac{\partial g_0}{\partial y_i} \bigg|_{\bar{X}} \right)^2 \sigma_{y_i}^2 \tag{8}$$

since all y_i's are independent.

For the purpose of optimization in stochastic programming, there are two simultaneous objectives - one is minimization of mean value and other minimization of the standard deviation.

So, here the stochastic objective of (4) is equivalent to deterministic objective(s) which is(are),

$$\text{Min } \bar{y}, \quad \text{(E-Model)} \tag{9a}$$

or

$$\text{Min } \sigma_{\psi}. \quad \text{(V-model)} \tag{9b}$$

or

$$\begin{cases} Min\ \bar{y} \\ Min\ s_y \end{cases} \quad \text{(E-V model)} \tag{9c}$$

3. ii) Constraints

As some parameters of the constraints are random in nature, the constraints will be probabilistic and one would like to have the probability of realising $g_j \leq 0$ must be greater than or equal to specified probability, say r_j (j = 1, 2, ….., m). So the constraints of (4) can be expressed as

$$P(g_j \leq 0) \geq r_j, \quad (j = 1, 2, \ldots, m). \tag{10}$$

i.e. $$\int_{-\infty}^{0} f_{g_j}(g_j)dg_j \geq r_j, \quad (j = 1, 2, \ldots, m). \tag{11}$$

where $f_{g_j}(g_j)$ is the probability density function of the random variable g_j (a function of several random variables is also a random variable) whose range is assumed to be $-\infty$ to ∞. The constraint function $g_j(X)$ can be expanded around the vector of mean values of the random variables, $\bar{X}$ as

$$g_j(X) = g_j(\bar{X}) + \sum_{i=1}^{N}\left(\frac{\partial g_j}{\partial y_i}\bigg|_{\bar{X}}\right)(y_i - \bar{y}_i) \tag{12}$$

From this equation, the mean value, $\bar{g}_j$, and the standard deviation, s_{g_j} of g_j can be obtained as:

$$\bar{g}_j = g_j(\bar{Y}), \tag{13}$$

$$s_{g_j} = \left[\sum_{i=1}^{N} \left(\frac{\P g_j}{\P y_i} \bigg| \overline{Y} \right)^2 s^2 y_i \right]^{1/2} \tag{14}$$

By introducing the new variable

$$\theta_j = \frac{g_j - \bar{g}_j}{s_{g_j}}, \ (j = 1, 2, 3, \ldots, m) \tag{15}$$

and noting that $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = 1,$ (16)

equation (11) can be expressed as

$$\int_{\frac{g_j}{\sigma_{g_j}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{\theta_j^2}{2}} d\theta_j \geq \int_{\phi_j(r_j)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt, \tag{17}$$

where $\phi_j(r_j)$ is the value of the standard normal variate corresponding to the probability r_j.

Thus $\frac{\bar{g}_j}{\sigma_{g_j}} \leq \phi_j(r_j)$

or $\bar{g}_j - \sigma_{g_j} \phi_j(r_j) \leq 0.$ (18)

Using equation (14), equation (18) can be rewritten as

$$\bar{g}_j - f_j(r_j) \left[\sum_{i=1}^{N} \left(\frac{\P g_j}{\P y_i} \bigg| \overline{X} \right)^2 s^2 y_i \right]^{1/2} \leq 0, \quad (j = 1, 2, \ldots, m). \tag{19}$$

Hence, the stochastic programming problem (4) is reduced to single or multi-objective deterministic non-linear programming problems as

Min $\bar{y}$ (X) (20a)

Subject to the 'm' constraints given by equations in (19)
and $X \geq 0$.

Min σ_ψ (X) (20b)

Subject to the 'm' constraints given by equations in (19)
and $X \geq 0$.

$$\text{Min } \bar{y}, \qquad (20c)$$
$$\text{Min } \sigma_\psi$$

Subject to the 'm' constraints given by equations in (19)
and $X \geq 0$.

It is to be noted that (20a), (20b) and (20c) are referred to as E-model, V-model and combined E-V model respectively.

4. Fuzzy programming technique to solve the SNLP model (2)

To solve the above multi-objective programming problem (20), the first step is to assign, for each objective, two values U_k and L_k as upper and lower bounds for the k-th objective, where L_k = aspired level of achievement for the k-th objective, U_k = higher acceptable level of achievement for the k-th objective and $d_k = U_k - L_k$ = the degradation allowance for k-th objective (k = 1, 2, ……n.). Now, the stochastic programming problem (2) has completely defined in crisp environment. The steps of the fuzzy programming technique are as follows:

Step-1:
Solve the multi-objective programming problem as a single objective problem usin g, only one objective at a time and ignoring the other.

Step-2:
From the results of step-1, determine the corresponding values for every objective at each solution derived.

Step-3:
From step-2, we may find for each objective values L_k and U_k corresponding to the set of solutions.

For the multi-objective problem (20), a membership function $m_k(\bar{X})$, which may be linear or non-linear, corresponding to the k-th objective is defined as a linear membership function, for simplicity as,

$$\mu_k(\bar{X}) = \begin{cases} 1 & \text{if } Z_k < L_k \\ 1 - \dfrac{Z_k - L_k}{U_k - L_k} & \text{if } L_k < Z_k < U_k \\ 0 & \text{if } Z_k > U_k \end{cases} \qquad (21)$$

Here, $Z_k = \bar{y}(X)$ for k = 1,
$= \sigma_\psi(X)$ for k = 2.

According to Zimmerman (1978), the equivalent crisp non-linear programming problem for multi-objective programming problem (20c) as:

$$\text{Max } \alpha \tag{22}$$

Subject to

$$\frac{U_k - Z_k}{U_k - L_k} \ge \alpha, \qquad (for\ k = 1, 2),$$

$$\bar{g}_j - f_j(r_j)\left[\sum_{i=1}^{N}\left(\frac{\partial g_j}{\partial y_i}\bigg|_{X}\right)^2 \sigma^2_{y_i}\right]^{1/2} \ge 0, \qquad (j = 1, 2, \ldots, m),$$

and $X \ge 0$, $\alpha \in [0, 1]$.

5. Solution for proposed model

5. 1. Probabilistic model

Using the above algorithm the probabilistic model (2) is transformed to

$$\text{Max } \alpha \tag{23}$$

Subject to

$$\frac{U_1 - ETC(\bar{p}, Q)}{U_1 - L_1} \ge \alpha\ ,$$

$$\frac{U_2 - VTC(\bar{p}, Q)}{U_2 - L_2} \ge \alpha\ ,$$

$$\sum_{i=1}^{n} w_i Q_i \le W\ ,$$

$$\sum_{i=1}^{n} \bar{p}_i Q_i - \bar{B} - \phi_1(r_1)\left[\sum_{i=1}^{n} Q_i^2 \sigma_{p_i}^2 + \sigma_B^2\right]^{1/2} \le 0,$$

where $$ETC(\bar{p}, Q) = \sum_{i=1}^{n}\left[\frac{A_i}{\bar{p}_i^{\beta_i}}\left(\bar{p}_i + \frac{\bar{S}_i}{Q_i}\right) + \frac{\bar{H}_i Q_i}{2}\right]$$

and

$$VTC(\bar{p}, Q) = \left[\sum_{i=1}^{n}\left[\frac{A_i^2}{Q_i^2 \bar{p}_i^{2\beta_i}}\sigma_{S_i}^2 + \frac{Q_i^2}{4}\sigma_{H_i}^2 + \sigma_{p_i}^2\left\{\frac{A_i(1-\beta_i)}{\bar{p}_i^{\beta_i}} - \frac{\beta_i A_i \bar{S}_i}{Q_i \bar{p}_i^{\beta_i + 1}}\right\}^2\right]\right]^{1/2}$$

5. 2. Fuzzy-probabilistic model

In fuzzy set theory, the fuzzy objectives and fuzzy constraints are defined by their membership functions, which may linear and/or non-linear. Here, we assume $\mu_{ETC}(p, Q)$, $\mu_{VTC}(p, Q)$ and $\mu_W(p, Q)$ to be the linear membership functions for two objectives and one constraints respectively and these are

$$m_{ETC}(\bar{p},Q)=\begin{cases} 0 & for \quad ETC(\bar{p},Q) > C_0 + P_{ETC} \\ 1-\dfrac{ETC(\bar{p},Q)-C_0}{P_{ETC}} & for \quad C_0 \le ETC(\bar{p},Q) \le C_0 + P_{ETC} \\ 1 & for \quad ETC(\bar{p},Q) < C_0 \ , \end{cases} \tag{24}$$

$$m_{VTC}(\bar{p},Q)=\begin{cases} 0 & for \quad VTC(\bar{p},Q) > D_0 + P_{VTC} \\ 1-\dfrac{VTC(\bar{p},Q)-D_0}{P_{VTC}} & for \quad D_0 \le VTC(\bar{p},Q) \le D_0 + P_{VTC} \\ 1 & for \quad VTC(\bar{p},Q) < D_0 \ , \end{cases} \tag{25}$$

and

$$m_W(Q)=\begin{cases} 0 & for \quad \sum\limits_{i=1}^{n} w_i Q_i > W + P_W \\ 1-\dfrac{\sum\limits_{i=1}^{n} w_i Q_i - W}{P_W} & for \quad W \le \sum\limits_{i=1}^{n} w_i Q_i \le W + P_W \\ 1 & for \quad \sum\limits_{i=1}^{n} w_i Q_i < W \ , \end{cases} \tag{26}$$

Here, the expected goal for total average cost is C_o with tolerance P_{ETC}, the standard deviation goal for that is D_0 with tolerance P_{VTC} and space constraint goal is W with tolerance P_W.

Now, using fuzzy non-linear programming technique (Zimmermann (1976)), the solution of fuzzy-stochastic inventory model (3) is transformed to

Max α (27)

subject to

$$1- \frac{ETC(\bar{p},Q)-C_0}{P_{ETC}} \geq \alpha,$$

$$1- \frac{VTC(\bar{p},Q)-D_0}{P_{VTC}} \geq \alpha,$$

$$1- \frac{\sum_{i=1}^{n} w_i Q_i - W}{P_W} \geq \alpha,$$

$$\sum_{i=1}^{n} \bar{p}_i Q_i \leq B,$$

$\bar{p}_i$, $Q_i > 0$, (i = 1, 2, …….,n.)

and $\alpha \in [0, 1]$.

Here, ETC($\bar{p},Q$) and VTC($\bar{p},Q$) are known as in (23).

The above non-linear programming problems (23) and (27) are solved by a computer program based on reduced gradient method.

6. Numerical Example

To illustrate the model (2), we assume n = 2, A_1 = 100, A_2 = 120, $\hat{S}_1$ = (\$100, \$1), $\hat{S}_2$ = (\$120, \$1.2), $\hat{H}_1$ = (\$1, \$0.01), $\hat{H}_2$ = (\$1.5, \$0.015), β_1 = 0.85, β_2 = 0.8, w_1 = 2 sq. ft., w_2 = 3 sq. ft., W = 150 sq. ft., r_1 = 0.95, $\hat{B}$ = (\$1200, \$12) and $\hat{p}_i = (\bar{p}_i, .01\bar{p}_i)$ for i = 1, 2.

To illustrate the model (3) , we assume the input data of model (2), in addition to
P_W = 50, C_0 = \$475, P_{ETC} = \$75, D_0 = \$1.40, P_{VTC} = \$0.6.

Using this data, pay-off matrix for (2) is

$$\begin{array}{cc} & \begin{array}{cc} \text{ETC} & \text{VTC} \end{array} \\ \begin{array}{c} Q_I^1 \\ Q_I^2 \end{array} & \begin{pmatrix} \$481.03 & \$0.647 \\ \$491.34 & \$0.580 \end{pmatrix} \end{array}$$

The optimal results of stochastic and fuzzy-stochastic models are presented in table-1.

Table - 1.

Model	a	ETC	VTC	p_1	p_2	Q_1	Q_2
I	0.90	482.05	0.59	21.44	18.72	26.75	32.16
II	0.93	480.10	0.65	21.00	14.50	27.01	33.12

Here, optimal results of total average cost are less in the case of fuzzy stochastic model than that obtained for the stochastic model.

7. Conclusion

In this paper, we have formulated inventory problems in both random and random-fuzzy environments. Till now, stochastic inventory problems have been normally formulated using a special probability distribution for an inventory parameter and solved reducing them to equivalent crisp problems by integrating the distribution function. Here, we have formulated and solved the stochastic inventory problem completely in a different way. We have taken the unit cost, which is a decision parameter involved in objective and one constraint, inventory costs and limit imposed on investment constraint to be random. Assuming these to be normally distributed, the probabilistic inventory model has been reduced to equivalent crisp E- and V-models using chance constraint programming technique. Till now, none has formulated or attacked the inventory problems in the above manner.

Moreover, alongwith the above randomness, impreciseness has been introduced in the objective goal and constraint goal on storage area for an inventory model. Again, till now, very few inventory models have been formulated in such a mixed environment - fuzzy-random atmosphere.

Though simple EOQ models have been considered here, the technique illustrated in this paper can easily be applied to other inventory problems with deterioration, shortages, discount, fixed time horizon, etc. This technique is an appropriate tool to tackle the real-life inventory problems in realistic environments.

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Appendix

For single item

$$\text{Min TC(p,Q)} = \frac{A}{p\alpha}\left(p + \frac{S}{Q}\right) + \frac{HQ}{2}.$$

According to geometric programming method,

$$\text{Max d(w)} = \left(\frac{A}{w_{01}}\right)^{w_{01}} \left(\frac{AS}{w_{02}}\right)^{w_{02}} \left(\frac{H}{w_{03}}\right)^{w_{03}}$$

Subject to

$w_{01} + w_{02} + w_{03} = 1,$

$(1 - \alpha)w_{01} - \alpha w_{02} = 0,$

$- w_{02} + w_{03} = 0,$

and $0 \le w_{0i} \le 1$; for i = 1, 2, 3.

Therefor, we have $\dfrac{w_{01}}{\alpha} = \dfrac{w_{02}}{1-\alpha} = \dfrac{w_{03}}{1-\alpha} = \dfrac{1}{2-\alpha}$

Since $0 < w_{01} < 1$

i.e. $0 < \dfrac{\alpha}{2-\alpha} < 1$, which gives $0 < \alpha < 1$.